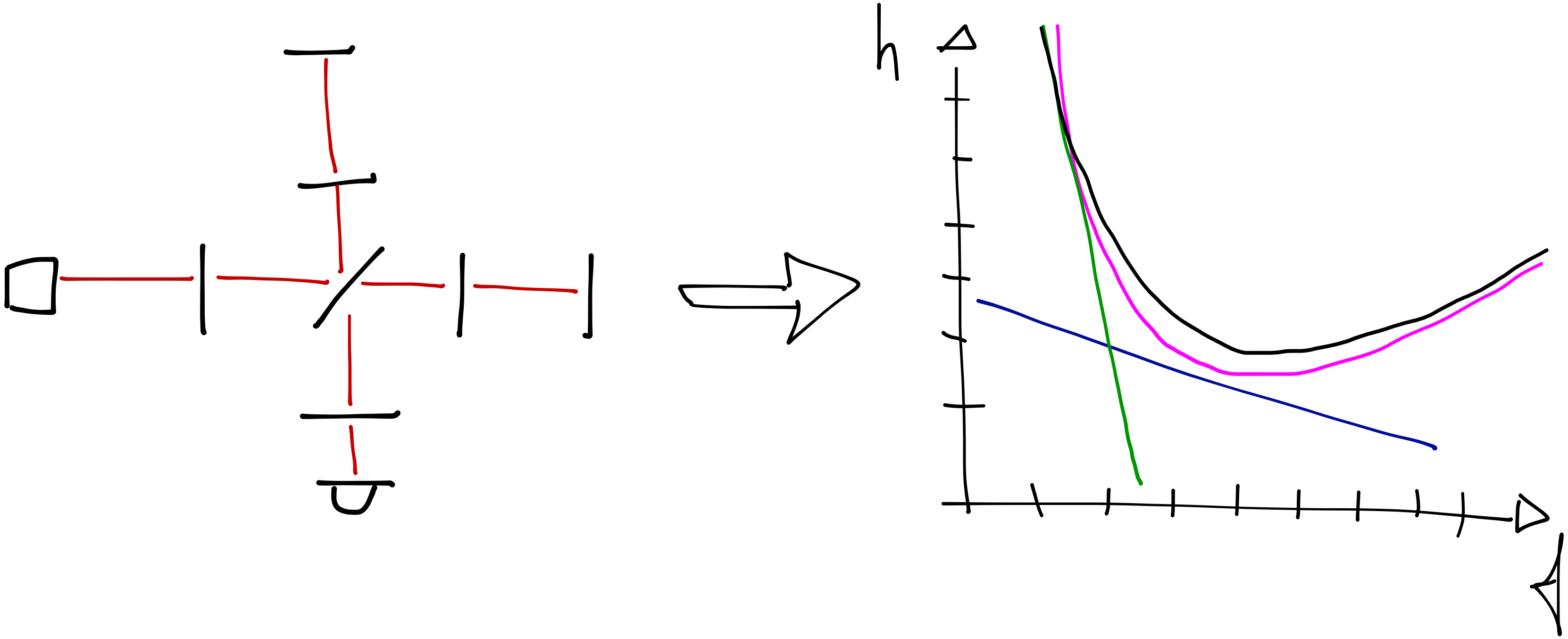
MICHELSON RESPONSE

07/2017 St. Andrews

MR 28/07/2017



This session: Michelson response + sensitivity

What is a signal?

What is noise?

How to compute a signal response?

How to use the frequency domain?

How to compute a sensitivity curve?

'Signal' and 'noise' are not well defined, there meaning changes with context.
One person's signal is another one's noise!

Possible mathematical definition:
Noise: output from a random (stochastic) process
Signal: output from a coherent process

Possible project definition:
Noise: unwanted output
Signal: wanted output

Can you think of examples where these definitions are compatible or not compatible?

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Signal and Noise in LIGO

In operation: output from any kind of gravitational wave is a signal. Everything else is noise. GWs can be coherent (cw, inspiral) or stochastic (burst, background)

During commissioning: we do not look for GWs. Instead we perform experiments to understand the behavior of the instrument. Then signals are typically created on purpose, such as coherent mirror motion, coherent modulation of the laser frequency, etc.

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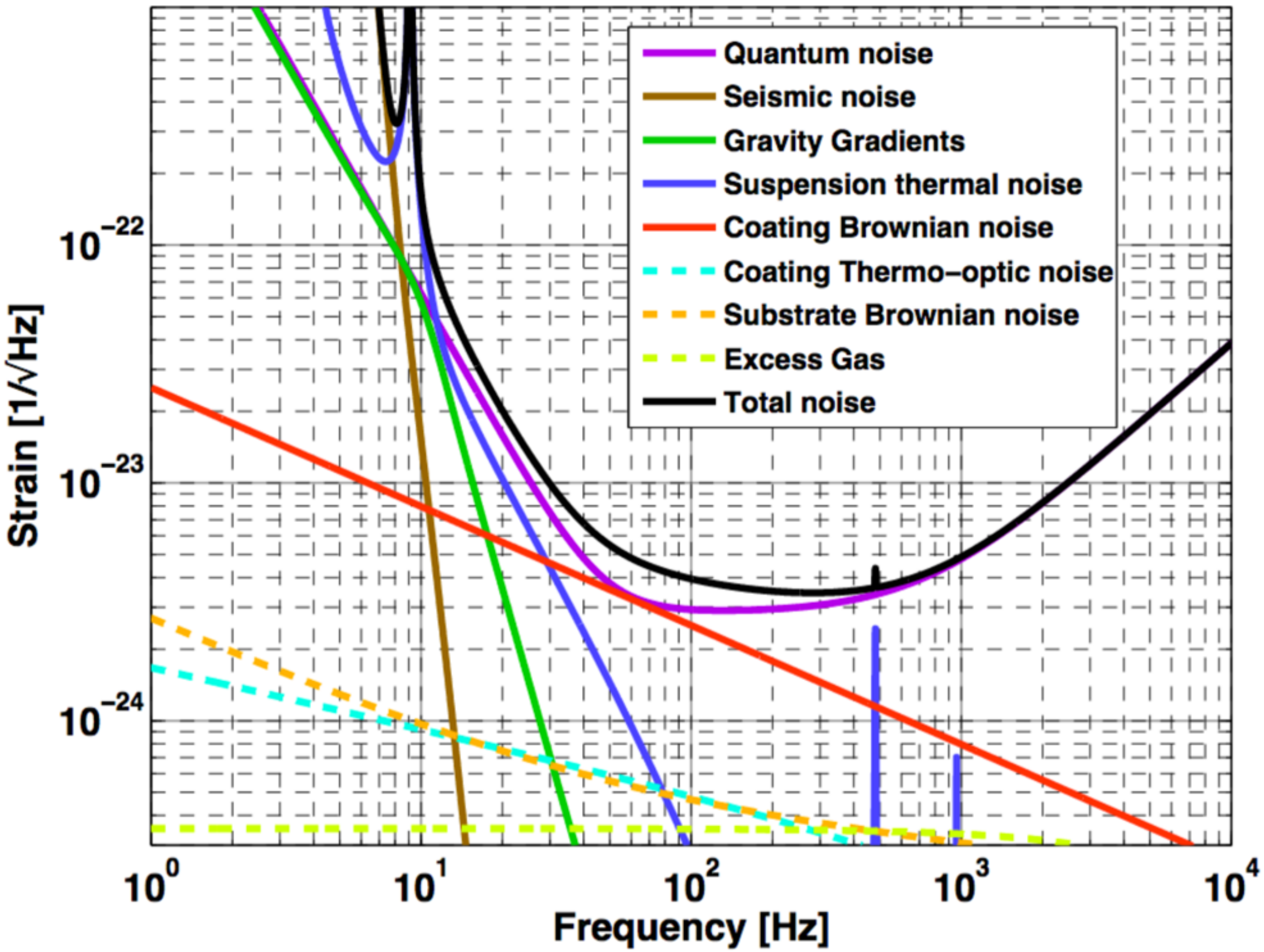
Projected (modelled) sensitivity to guide the instrument design

What are these?



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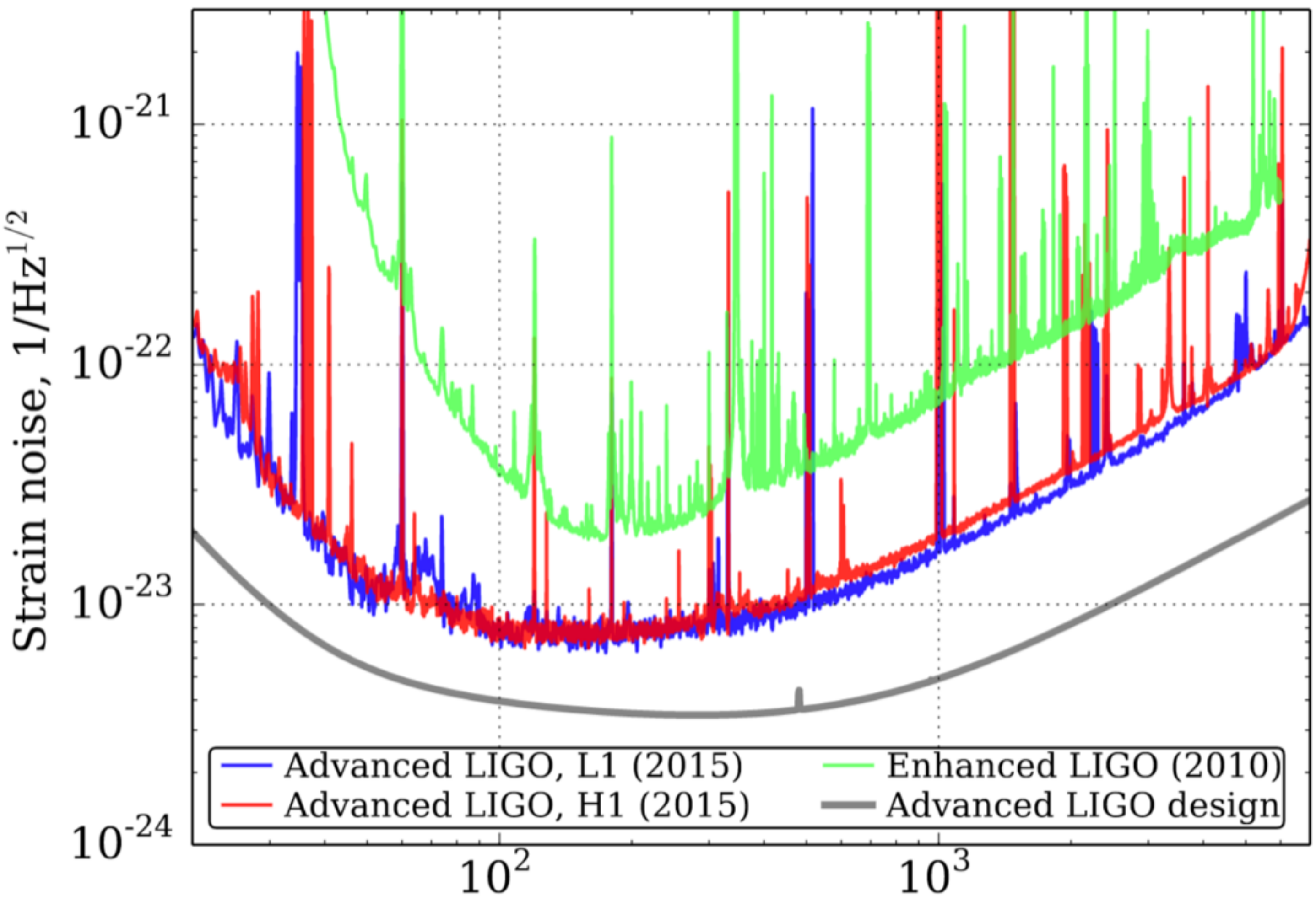
Measured sensitivity to compare performances



How to generate such plots?

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In certain conditions we can describe $n(t)$ by its frequency components, also called 'in the frequency domain'.

This is true for time invariant processes in linear systems

Simple and useful because:

- frequency components do not mix, can be computed separately
- components can be added (superposition principle)

LIGO is not always that simple, but approaches that state in 'science mode'.

Generally useful to understand the simpler frequency domain version of a system if possible

99% of all LIGO modelling in the frequency domain!

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Some basic features of the plot:

x-axis : f the Fourier frequency of a signal or noise component [Hz]

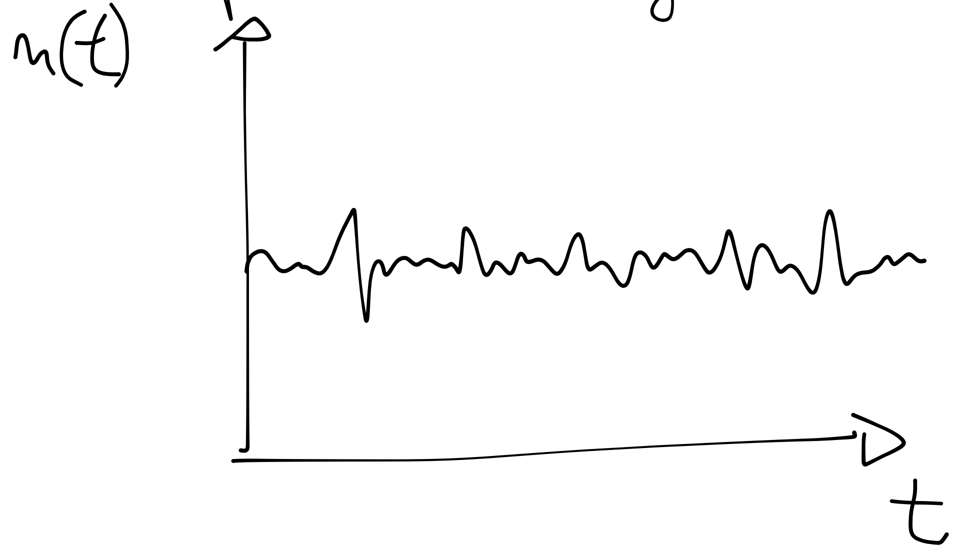
y-axis : h, gravitational wave strain, with $h = \Delta L / L$

with units $\left[1/\sqrt{\mathrm{Hz}}\right]$, it is an

Amplitude Spectral Density (ASD)

Very brief introduction into spectral densities

Lets start with a random process (stationary, i.e. time invariant)



How to get a description of 'n' in the frequency domain?
With the Fourier transform:

$$n(f) = FT(n(t)) = \int_{-\infty}^{\infty} n(t)\, e^{-i2\pi f t}\, dt$$

We measure for a limited time T and only get:

$$n_T(f) = \int_{-T/2}^{T/2} n(t)\, e^{-i2\pi f t}\, dt$$

Most random processes have average constant power so that
$n_T(f) \sim T$, ie different results for different measurement times
Instead a more useful spetral representation is the
power spectral density:

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Power Spectral Density (PSD)

$$S_n(f) = \lim_{T\to\infty} \frac{2}{T} \left| \int_{-T/2}^{T/2} \left(n(t) - \bar{n}\right) e^{-i2\pi f t}\, dt \right|^2$$

mean of $n(t)$

Amplitude Spectral Density (ASD):

$\sqrt{S_n(f)}$, used because has often intuitive units

In practise:

- measure $n(t)$ over T
- remove mean $\bar{n}(t)$
- compute FFT
- square, multiply by $\frac{2}{T}$, square root

Depends on FFT function!

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Units of noise curves

Example $\quad n(t) \quad [m]$

$\tilde{n}(f) \quad [m \cdot s] = \left[\frac{m}{Hz}\right]$ Fourier transform

$S_n(f) \quad \left[\frac{m^2}{Hz^2} \cdot \frac{1}{s}\right] = \left[\frac{m^2}{Hz}\right]$ power spetral density

$\sqrt{S_n(f)} \quad \left[\frac{m}{\sqrt{Hz}}\right]$ amplitude spetral density

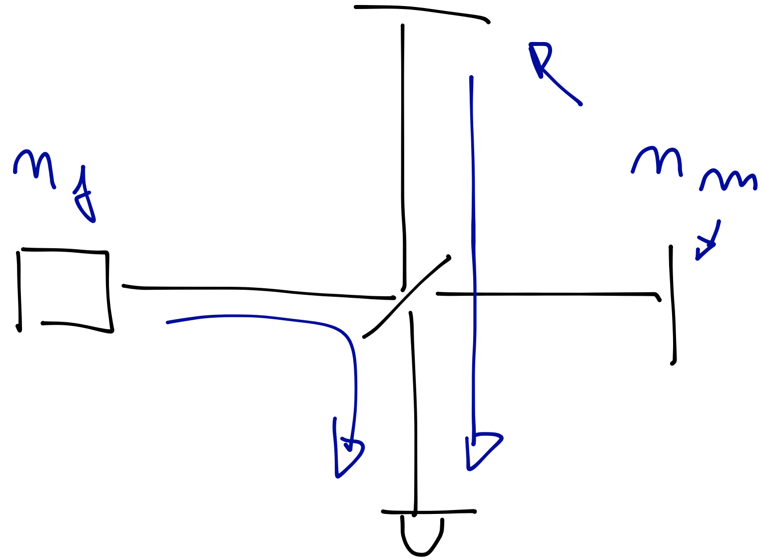
JR

LIGO Design example: want to compare laser frequency noise to a mirror motion noise.

$ASD_f \left[\frac{Hz}{\sqrt{Hz}}\right]$

$ASD_m \left[\frac{m}{\sqrt{Hz}}\right]$

How to compare?



Idea 1: Can compare the noise each would generate in the output.

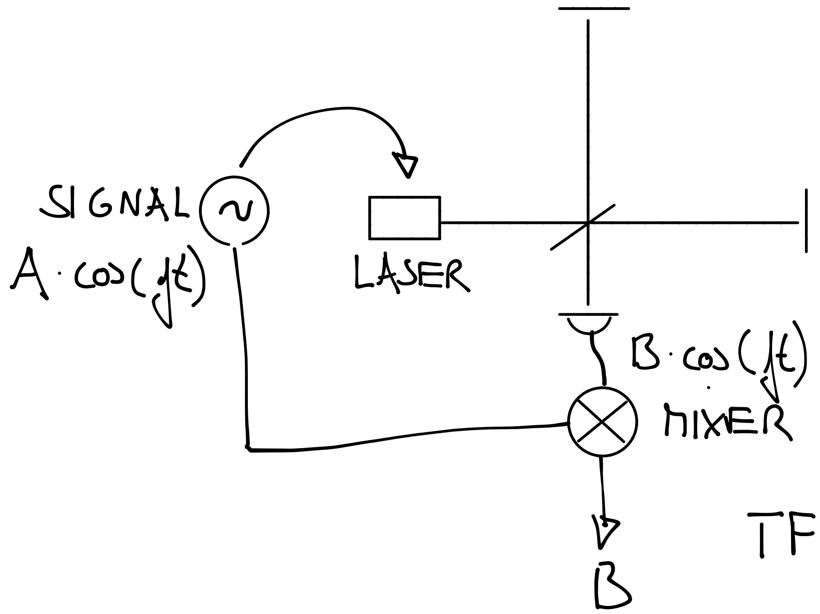
$ASD_{out} \left[\frac{V}{\sqrt{Hz}}\right]$ or in FINESSE $\left[\frac{W}{\sqrt{Hz}}\right]$

Need to find a way to compute output spectrum from input spectrum → need Transfer Function

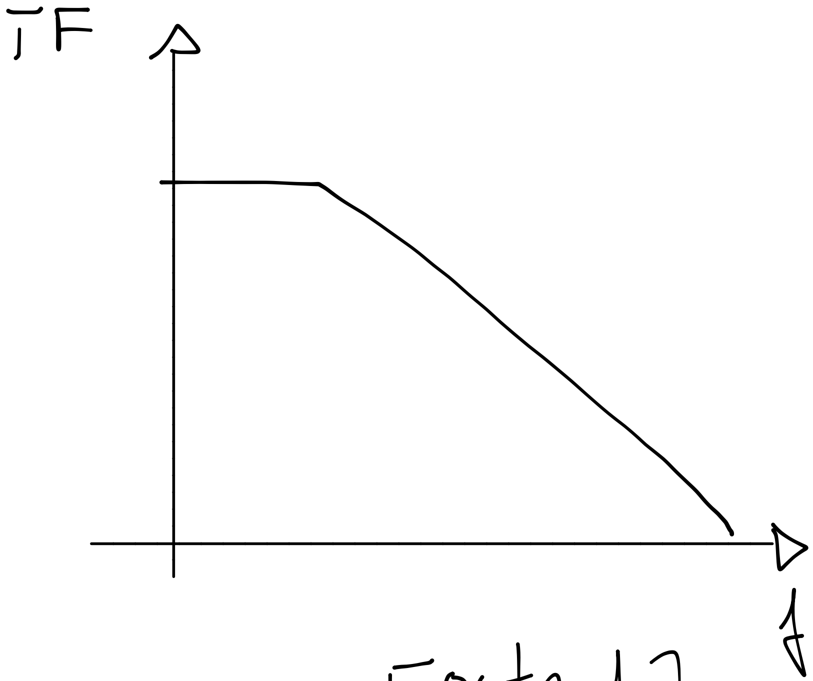
$$ASD_{out}(f) = ASD_m(f) \cdot TF(f)$$

function of the interferometer

Measure a transfer function



$$TF(f) = \frac{B}{A}$$



Units : $\left[\frac{\text{output}}{\text{input}}\right]$, in this case TF $\left[\frac{W}{Hz}\right]$

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Recipe for sensitivity plot:

For each noise:

- obtain input spetrum (measure, or theoretical prediction)
- compute transfer function to output TF_N $\left[\frac{W}{Noise}\right]$
- compute transfer function for GW signal to output TF_{GW} $\left[\frac{W}{h}\right]$
- compute noise in units of h as $n(f) \cdot \frac{TF_N}{TF_{GW}}$
- add curve to plot

Sum (squared) all curves for 'total noise'

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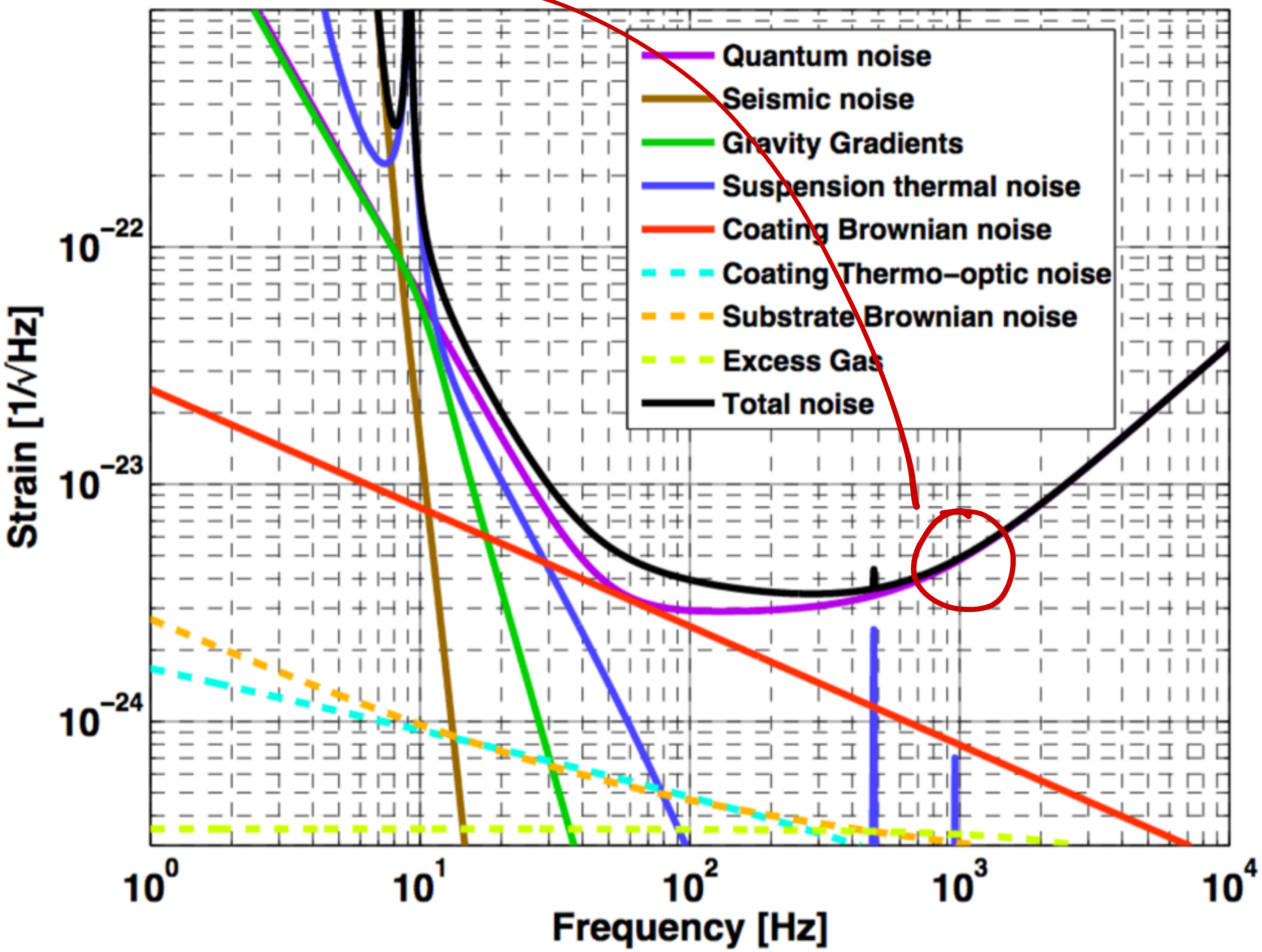
At 1000 Hz, the noise (quantum) has a magnitude comparable to a GW signal of $5 \cdot 10^{-24} \frac{1}{\sqrt{Hz}}$. Note: easy to compare noise to noise.



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Summary

- for noise we use ASD to quantify + compare
- we use signal TFs to project noise from one point in the system to other
- sensitivity plot are noise curves projected to GW signal