

Gravitational Waveforms

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July 26, 2017

Gravitational-wave Detections

O1 (Sept 2015 - January 2016)

GW150914: Black-hole binary. (29, 36) solar masses

LVT151012: Black-hole binary. (13, 23) solar masses

GW151226: Black-hole binary. (7.5, 14) solar masses

O2 (December 2016 - August 2017)

GW170104: Black-hole binary. (19, 31) solar masses

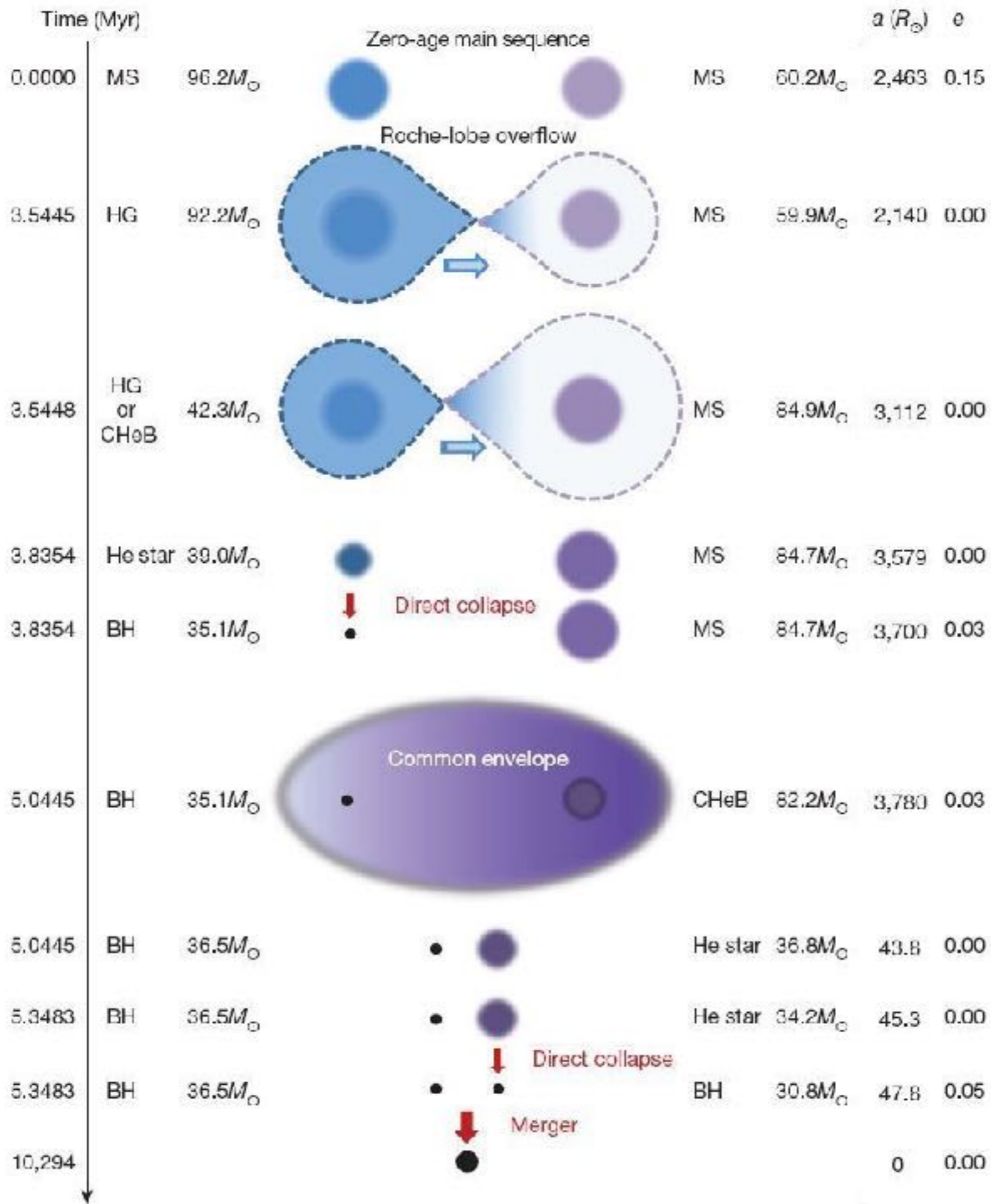
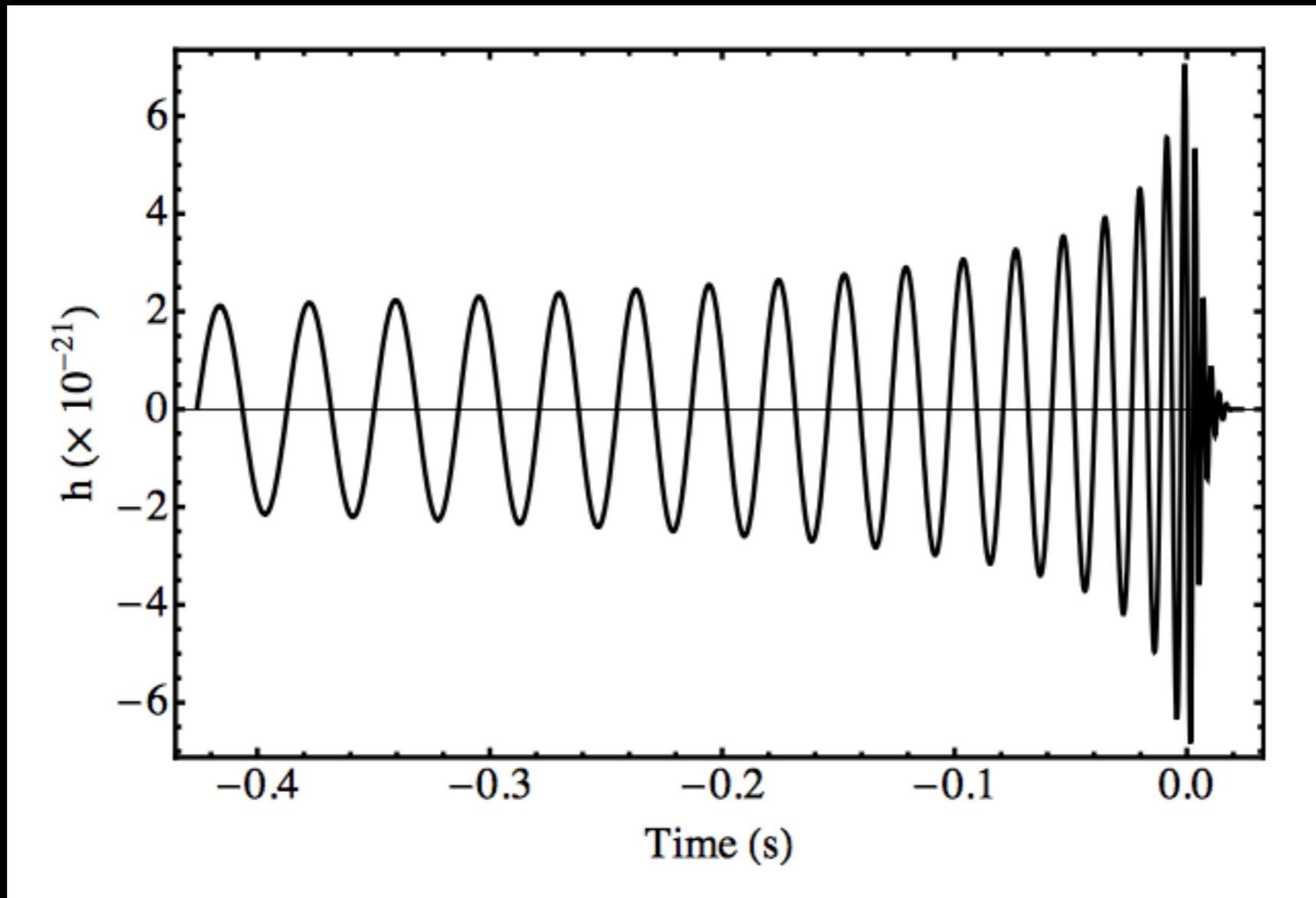


TABLE IV: Compact binary coalescence rates per Mpc^3 per Myr.^a

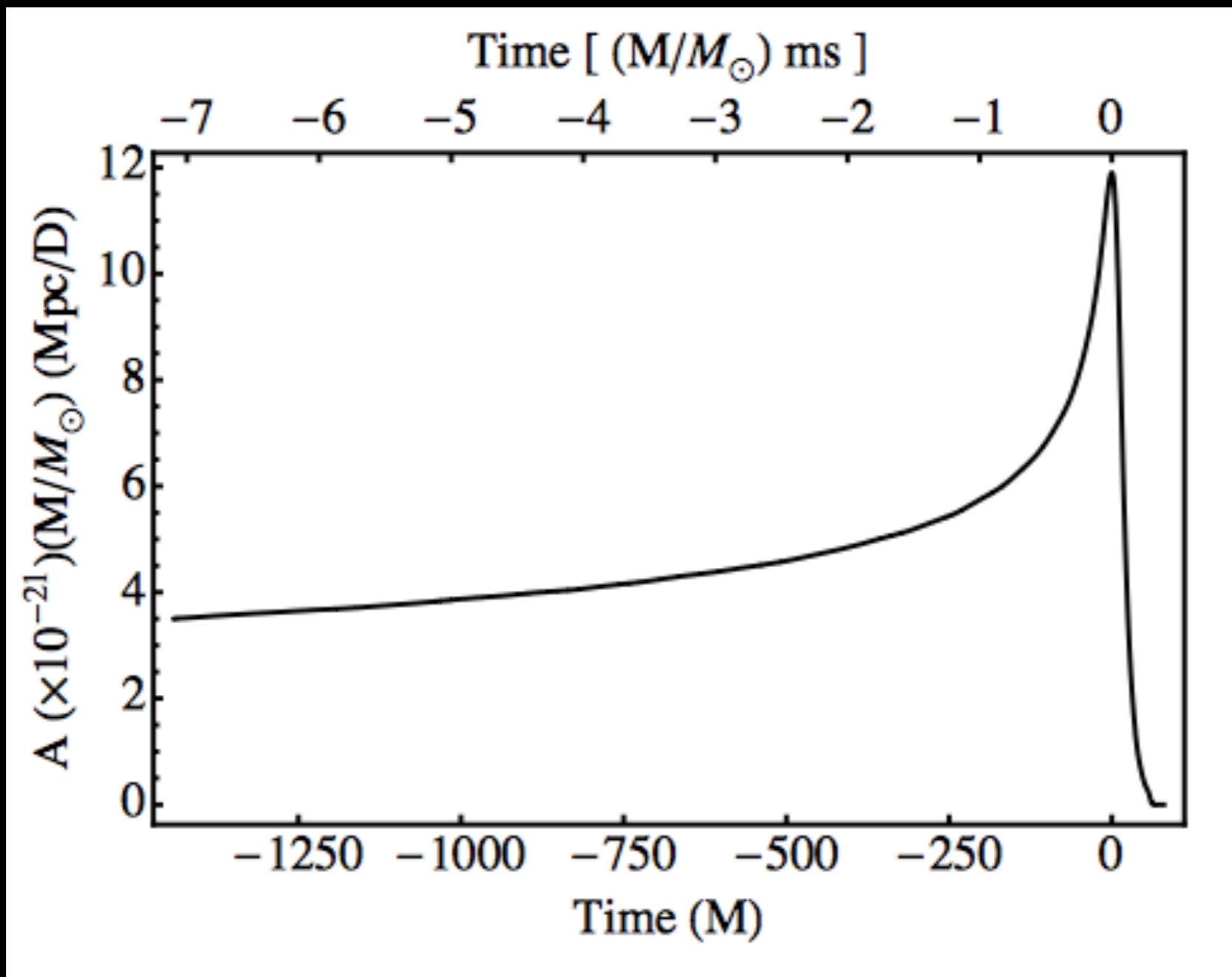
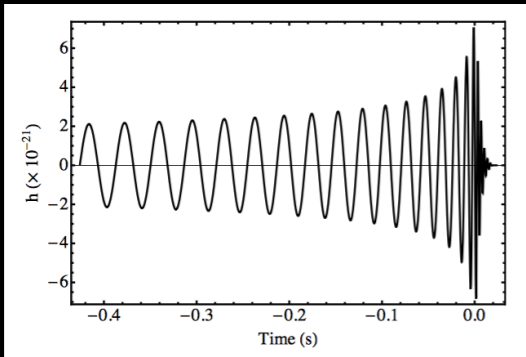
Source	R_{low}	R_{re}	R_{high}	R_{max}
NS-NS ($\text{Mpc}^{-3} \text{Myr}^{-1}$)	0.01 [1]	1 [1]	10 [1]	50 [16]
NS-BH ($\text{Mpc}^{-3} \text{Myr}^{-1}$)	6×10^{-4} [18]	0.03 [18]	1 [18]	
BH-BH ($\text{Mpc}^{-3} \text{Myr}^{-1}$)	1×10^{-4} [14]	0.005 [14]	0.3 [14]	

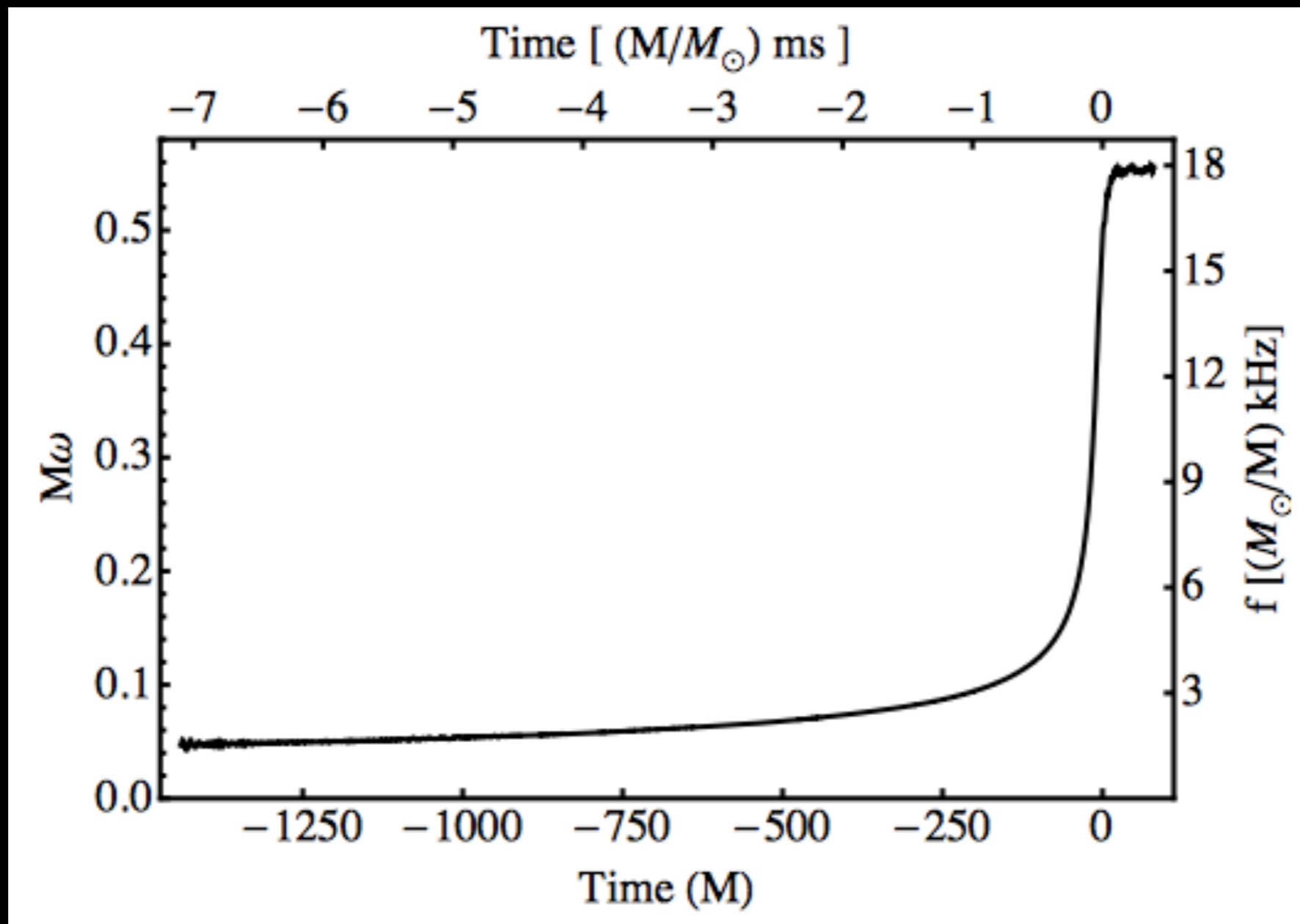
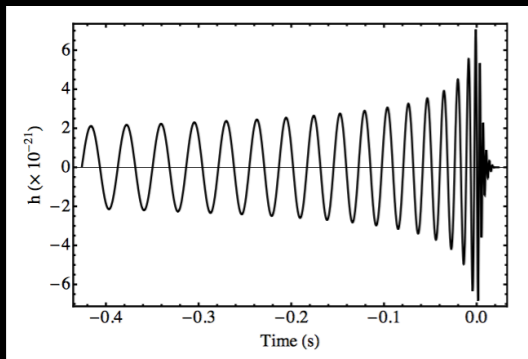
TABLE V: Detection rates for compact binary coalescence sources.

IFO	Source ^a	\dot{N}_{low} yr^{-1}	\dot{N}_{re} yr^{-1}	\dot{N}_{high} yr^{-1}	\dot{N}_{max} yr^{-1}
Initial	NS-NS	2×10^{-4}	0.02	0.2	0.6
	NS-BH	7×10^{-5}	0.004	0.1	
	BH-BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			$< 0.001^b$	0.01^c
	IMBH-IMBH			10^{-4d}	10^{-3e}
Advanced	NS-NS	0.4	40	400	1000
	NS-BH	0.2	10	300	
	BH-BH	0.4	20	1000	
	IMRI into IMBH			10^b	300^c
	IMBH-IMBH			0.1^d	1^e



(equal-mass, nonspinning, $50M_{\odot}$, face-on, 100Mpc)





$$\mathbf{a} = \mathbf{a}_N + \mathbf{a}_{PN} + \mathbf{a}_{SO} + \mathbf{a}_{2PN} + \mathbf{a}_{SS} + \mathbf{a}_{RR}, \quad (2.1)$$

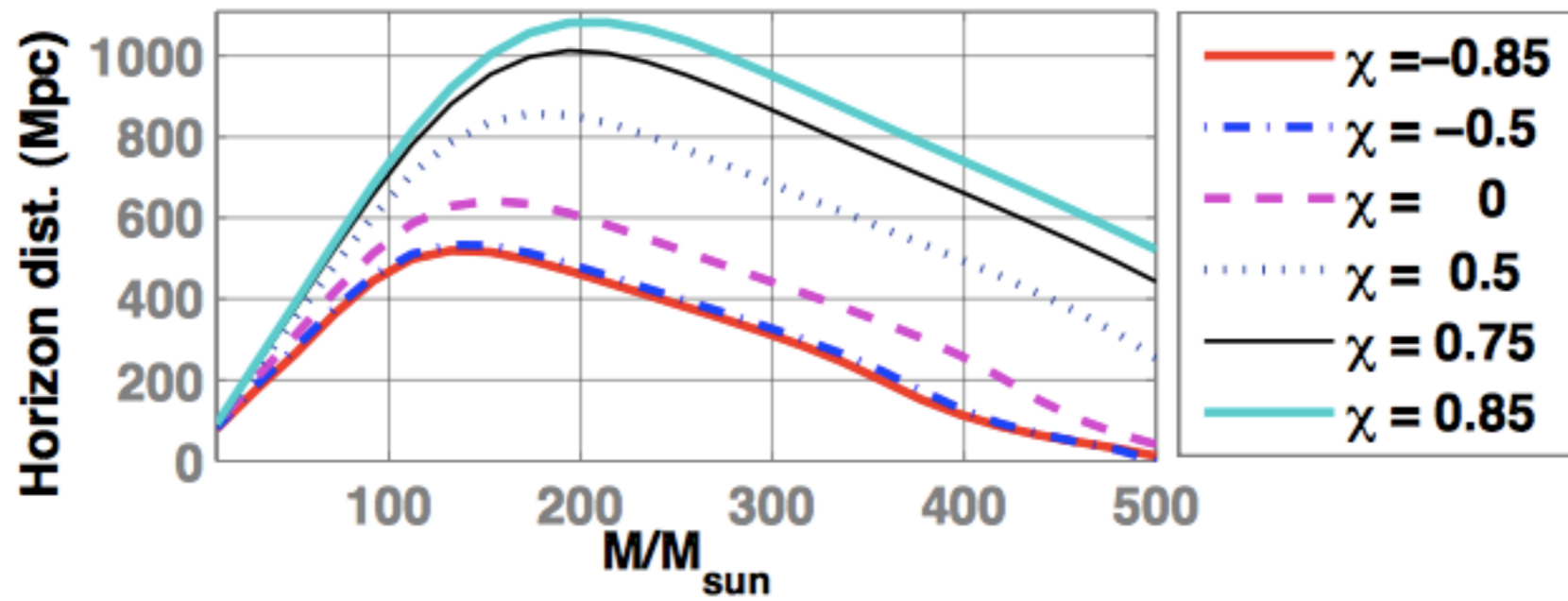
where

$$\mathbf{a}_N = -\frac{m}{r^2} \hat{\mathbf{n}}, \quad (2.2a)$$

$$\mathbf{a}_{PN} = -\frac{m}{r^2} \left\{ \hat{\mathbf{n}} \left[(1 + 3\eta)v^2 - 2(2 + \eta)\frac{m}{r} - \frac{3}{2}\eta\dot{r}^2 \right] - 2(2 - \eta)\dot{r}\mathbf{v} \right\}, \quad (2.2b)$$

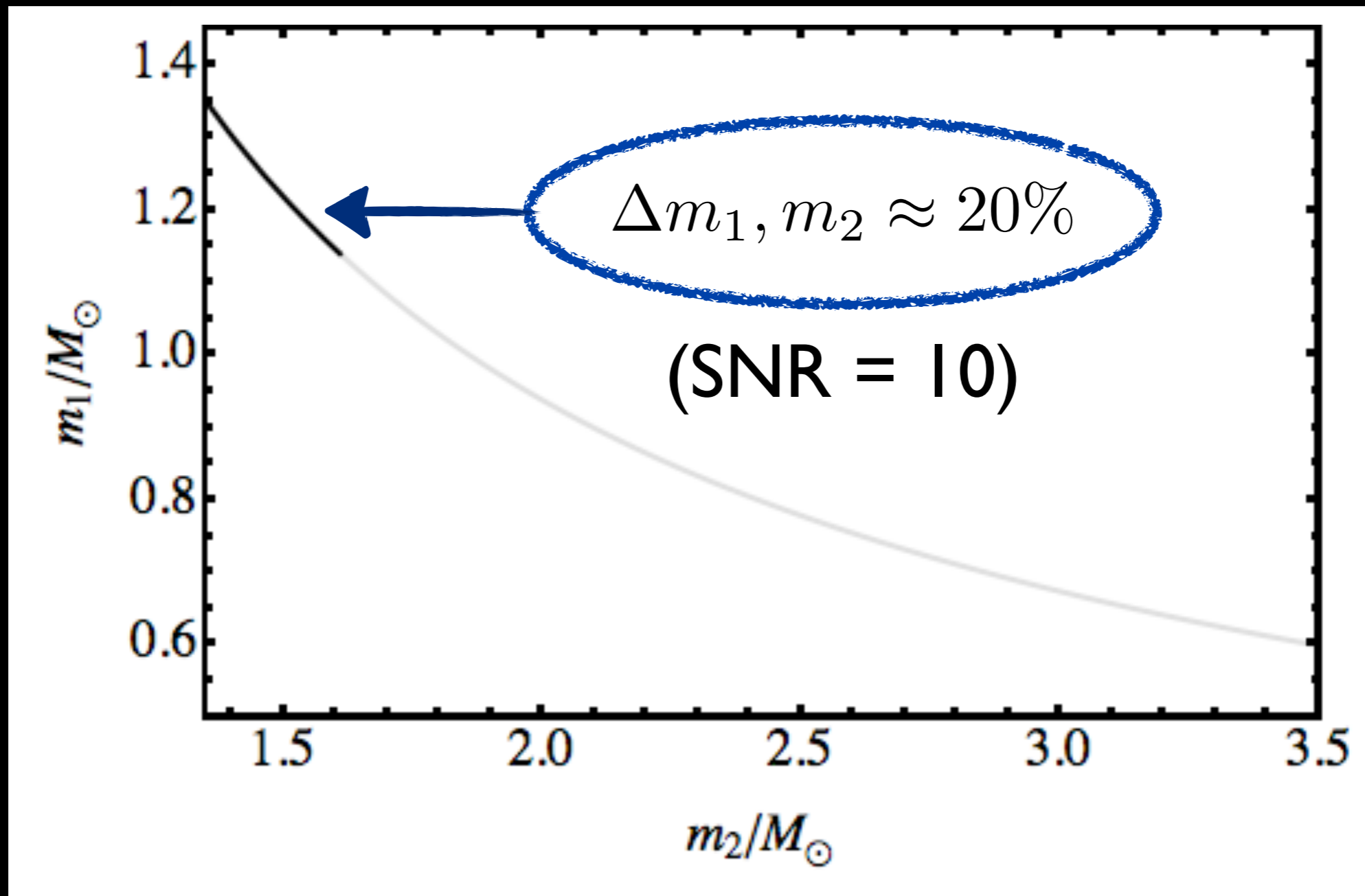
$$\mathbf{a}_{SO} = \frac{1}{r^3} \left\{ 6\hat{\mathbf{n}} \left[(\hat{\mathbf{n}} \times \mathbf{v}) \cdot \left(2\mathbf{S} + \frac{\delta m}{m} \mathbf{\Delta} \right) \right] - \left[\mathbf{v} \times \left(7\mathbf{S} + 3\frac{\delta m}{m} \mathbf{\Delta} \right) \right] + 3\dot{r} \left[\hat{\mathbf{n}} \times \left(3\mathbf{S} + \frac{\delta m}{m} \mathbf{\Delta} \right) \right] \right\}, \quad (2.2c)$$

[Kidder, PRD 52, p. 821 (1995)]



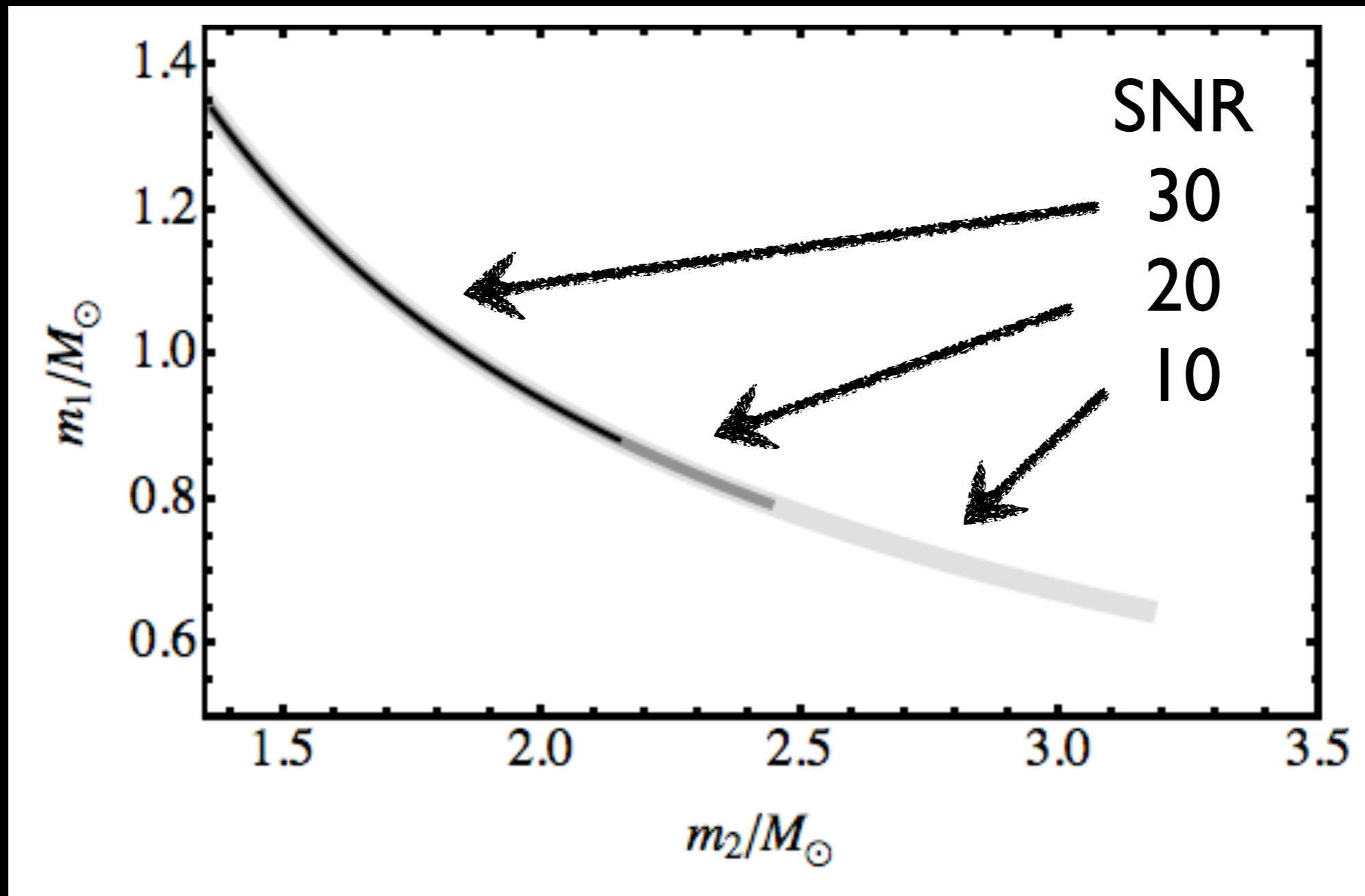
[Ajith, et. al., PRL 106, 241101 (2011)]

Mass measurements (non-spinning)

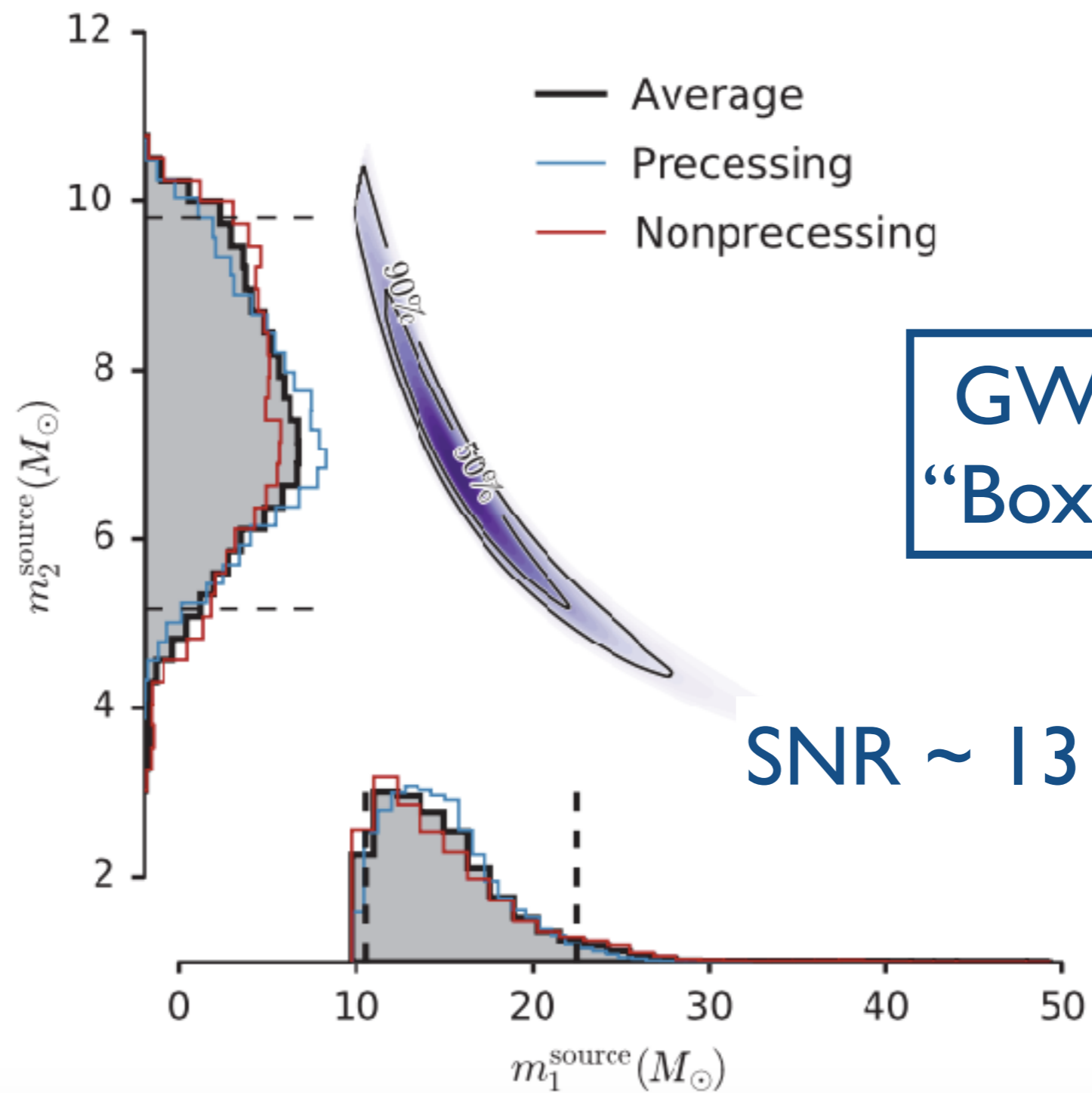


[Hannam, et. al., Ap.J.Lett, 766, 14 (2013)]

Aligned spins

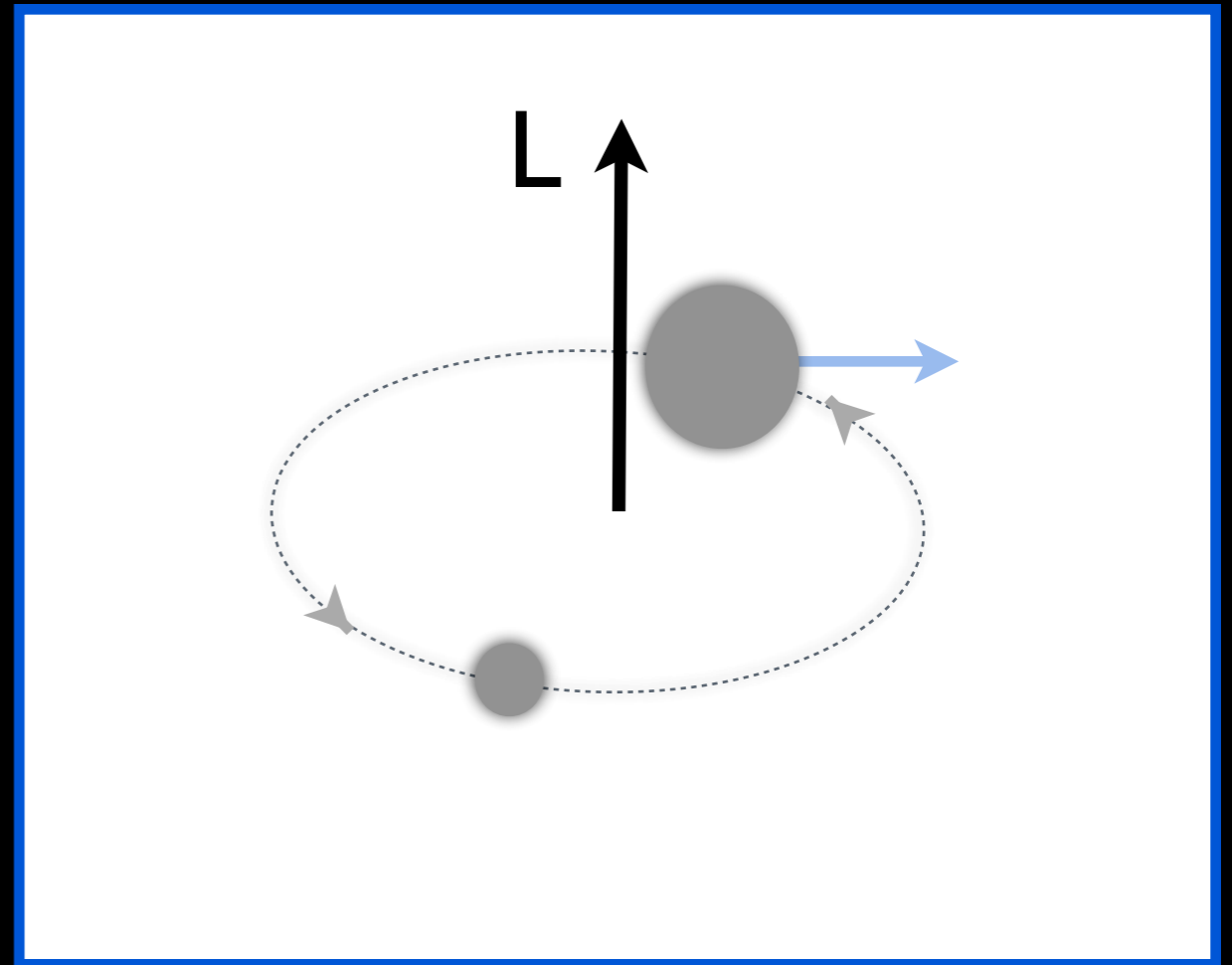
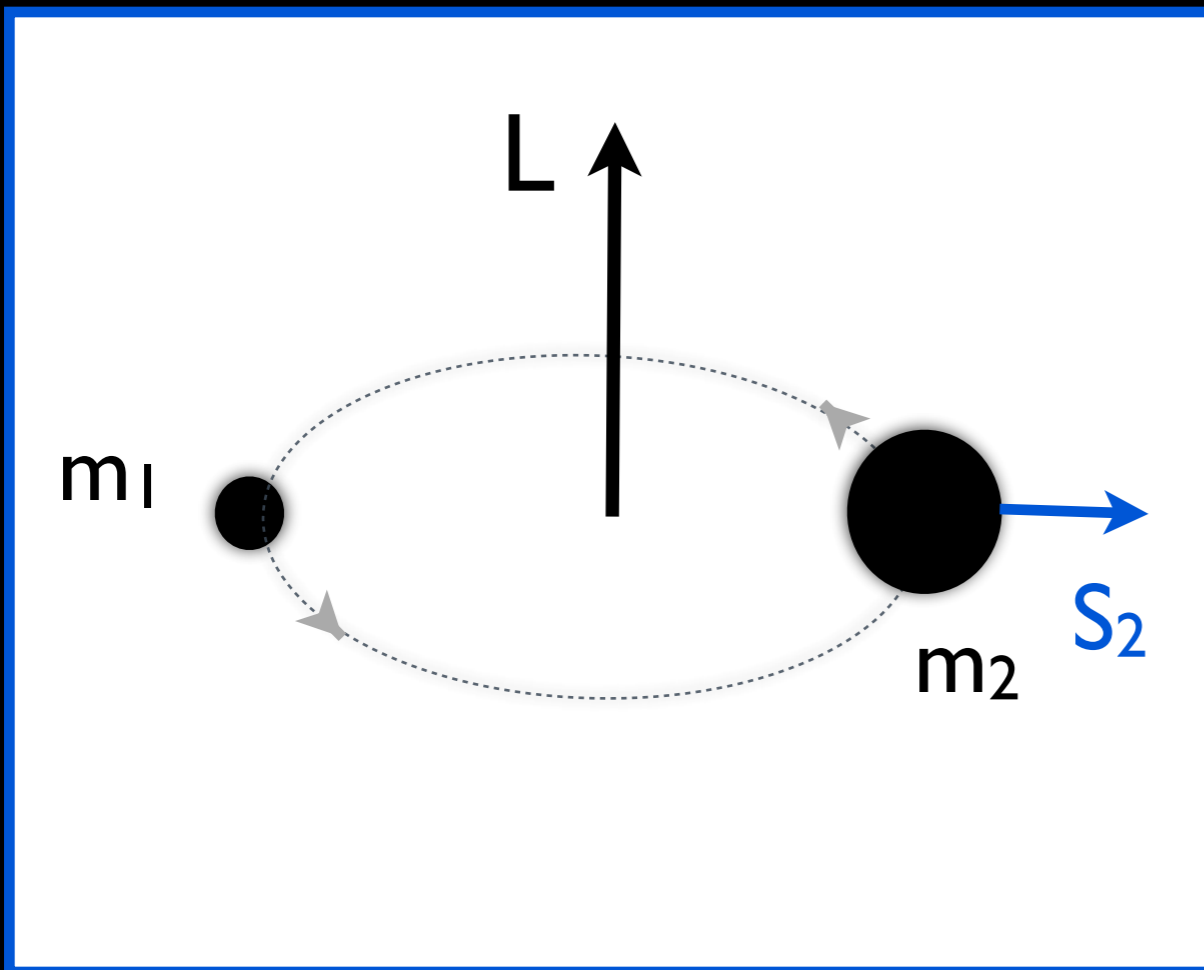


[Hannam, *et. al.*, *Ap.J.Lett*, 766, 14 (2013)]



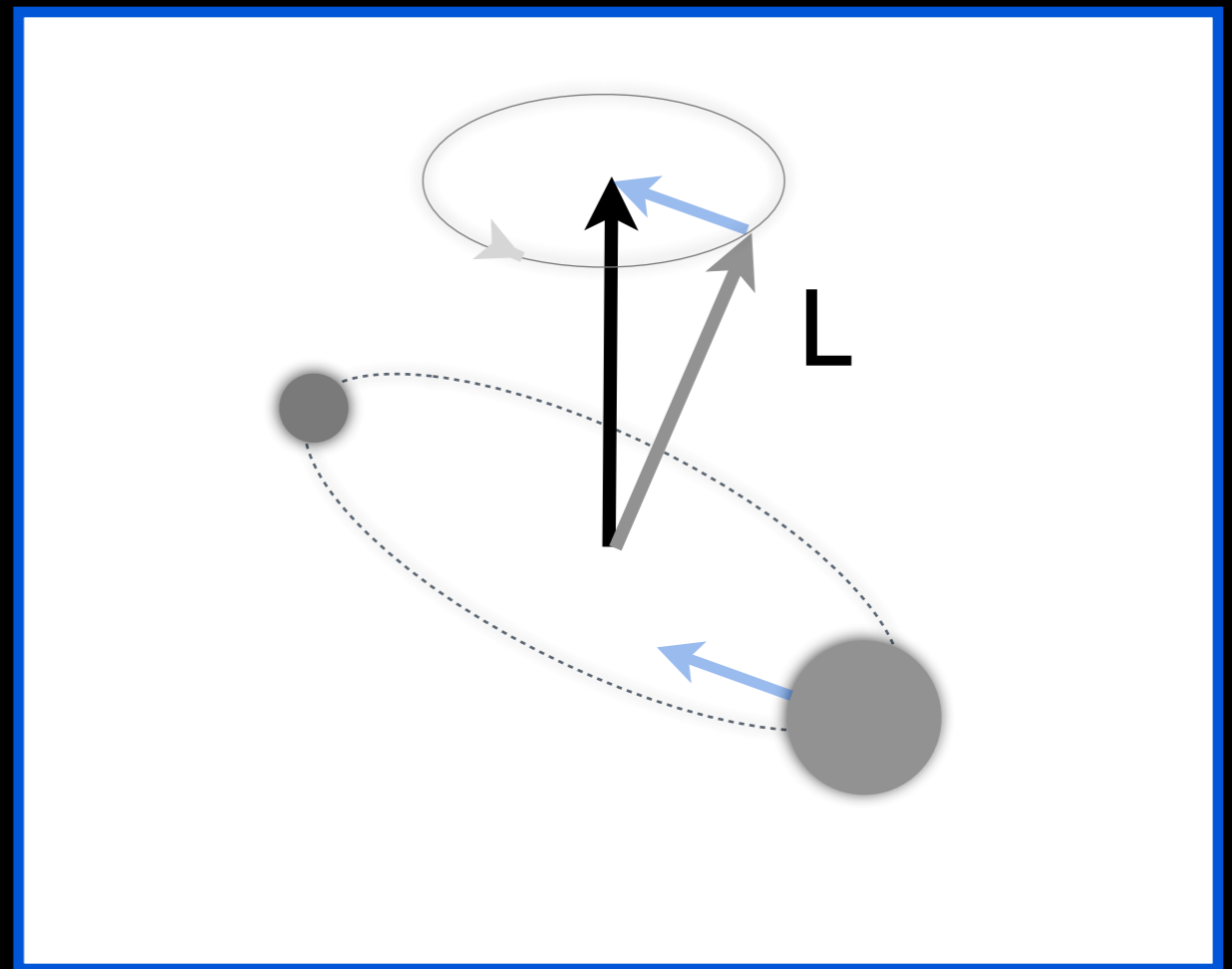
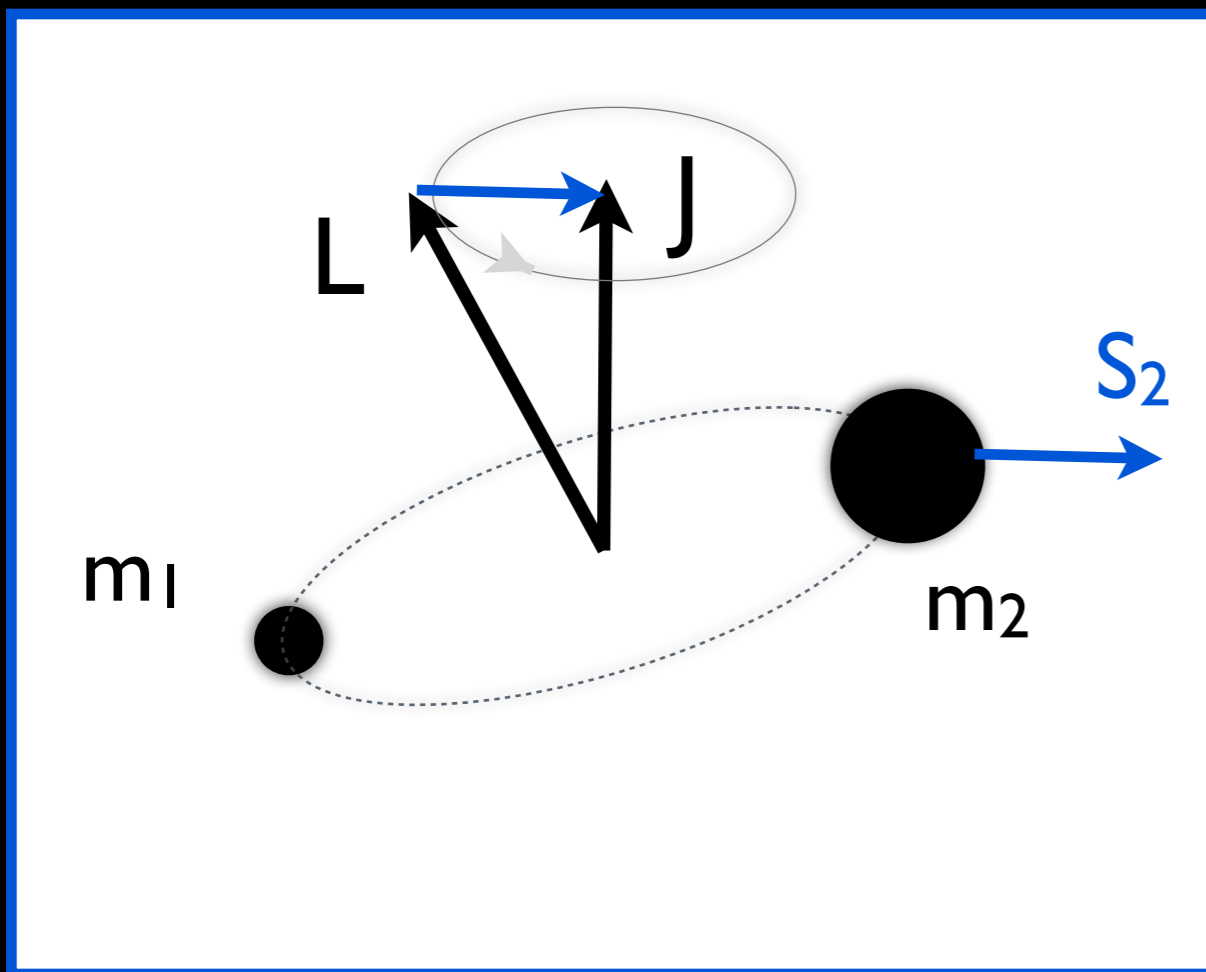
[LVC, PRL **116**, 24, 241104 (2016)]

Orbital precession



Newtonian gravity:
 L, S_1, S_2 remain fixed

Orbital precession



General relativity
(L, S_1, S_2) precess around J

Orientation dependence

$q=3$, $|\mathbf{S}_2| = 0.75$ (in plane)



Observer aligned
with J

Orientation dependence

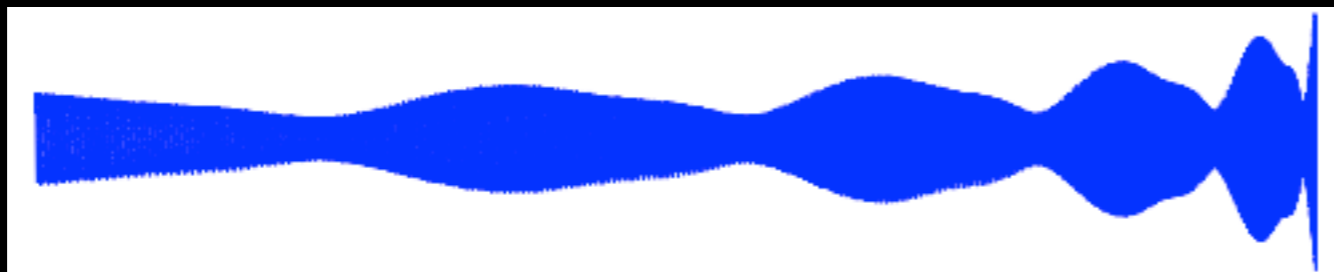
$q=3, |\mathbf{S}_2| = 0.75$ (in plane)



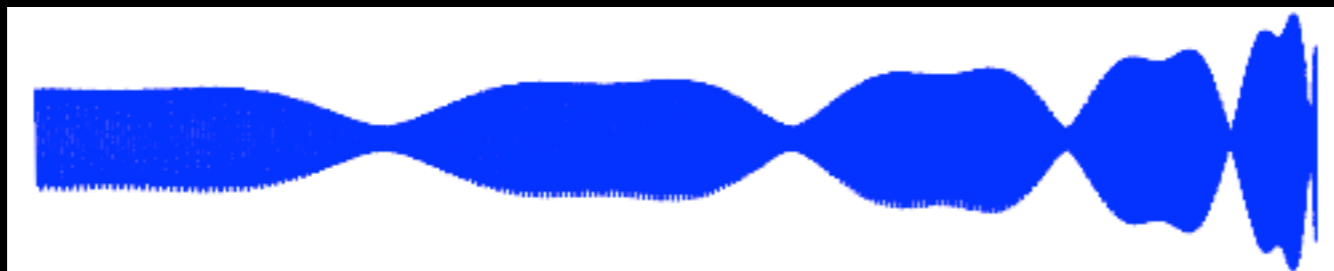
Observer aligned
with J



Observer inclined
 $\pi/6$ to J

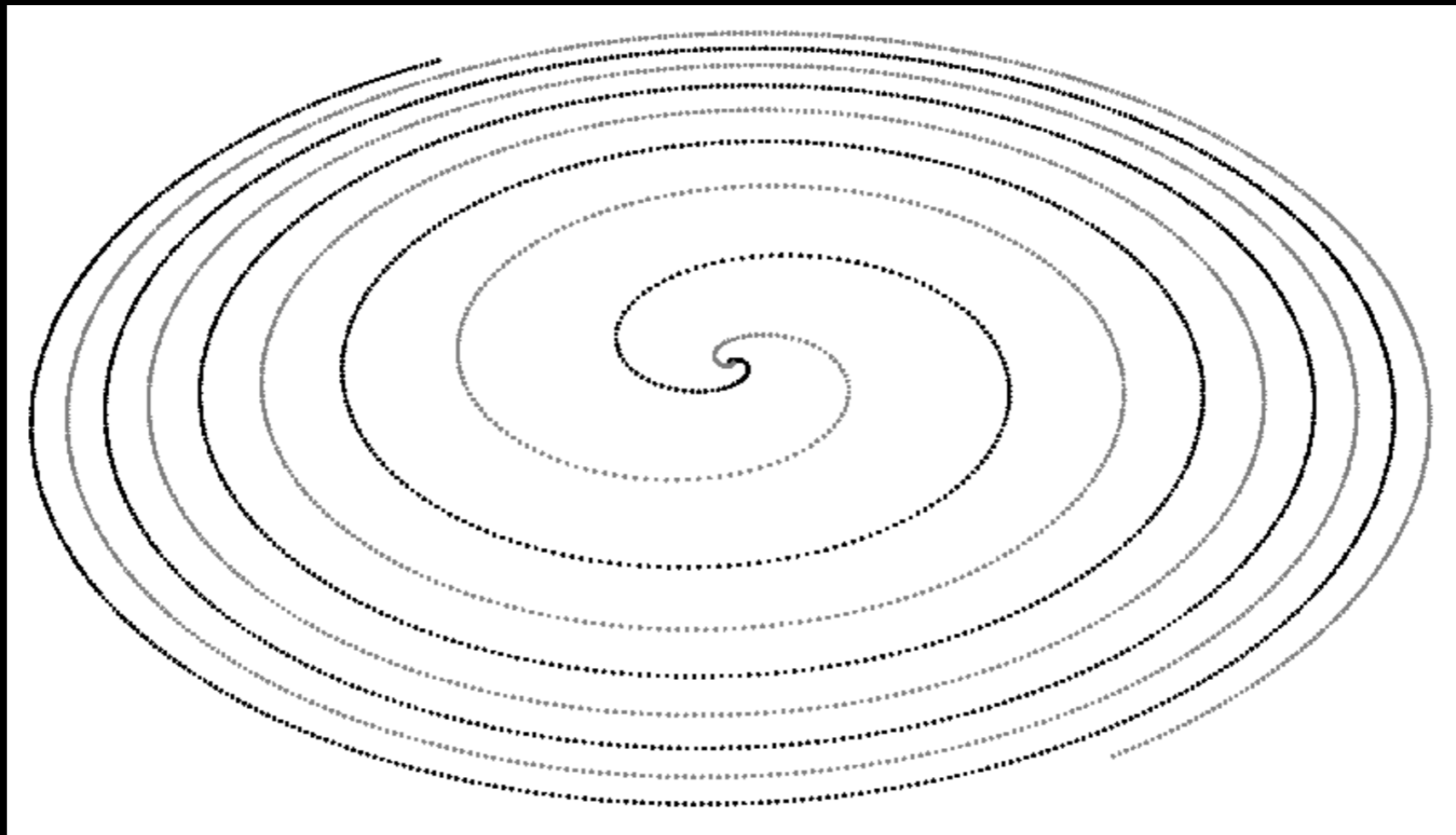


Observer inclined
 $\pi/3$ to J

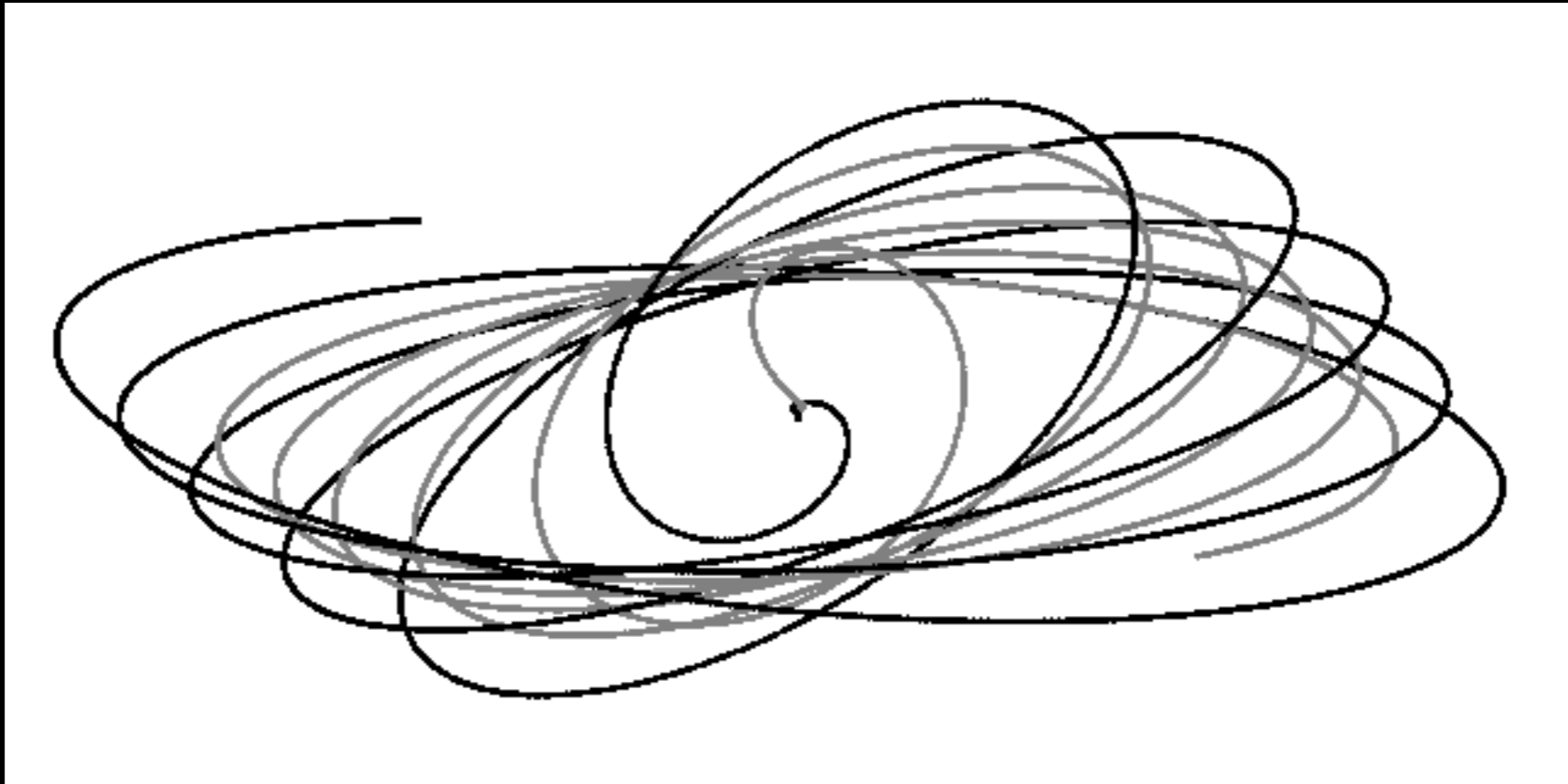


Observer inclined
 $\pi/2$ to J

Equal-mass nonspinning BBH consistent with GW150914

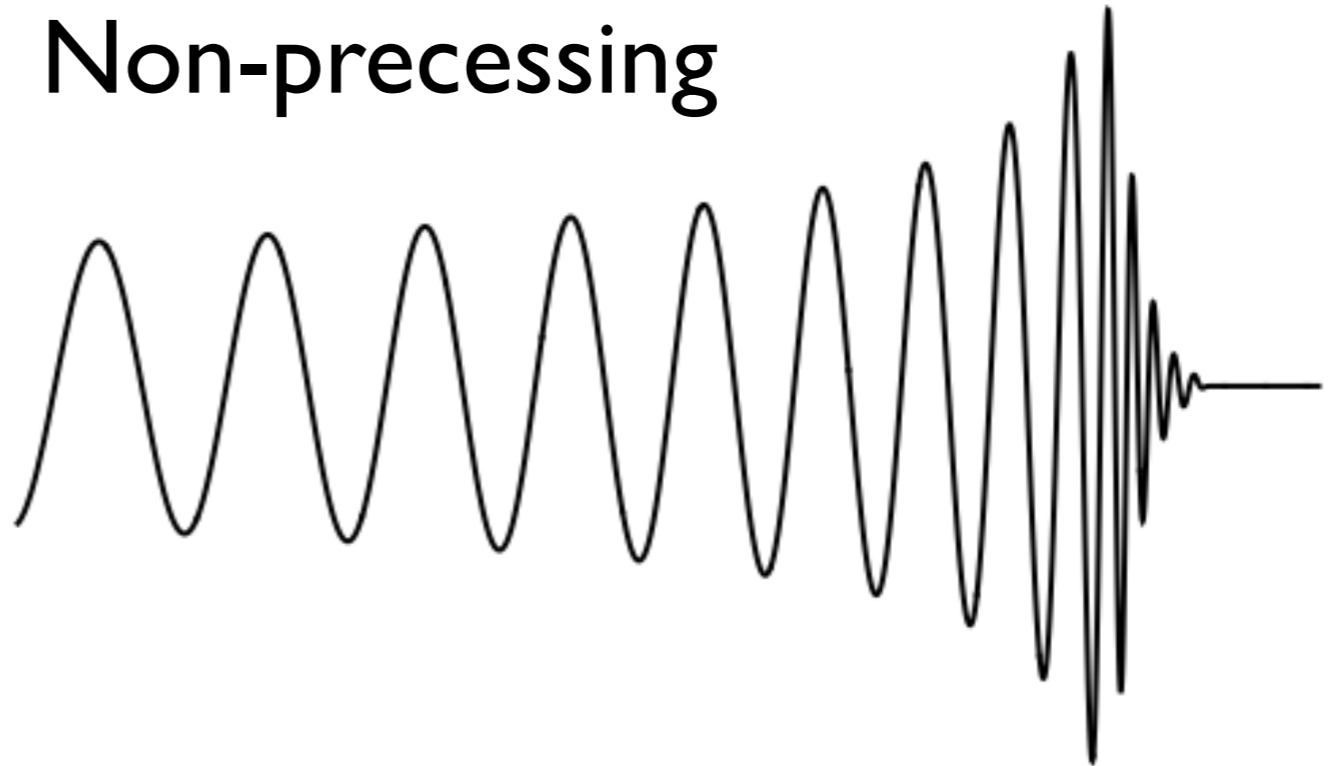


Unequal-mass precessing BBH consistent with GW150914

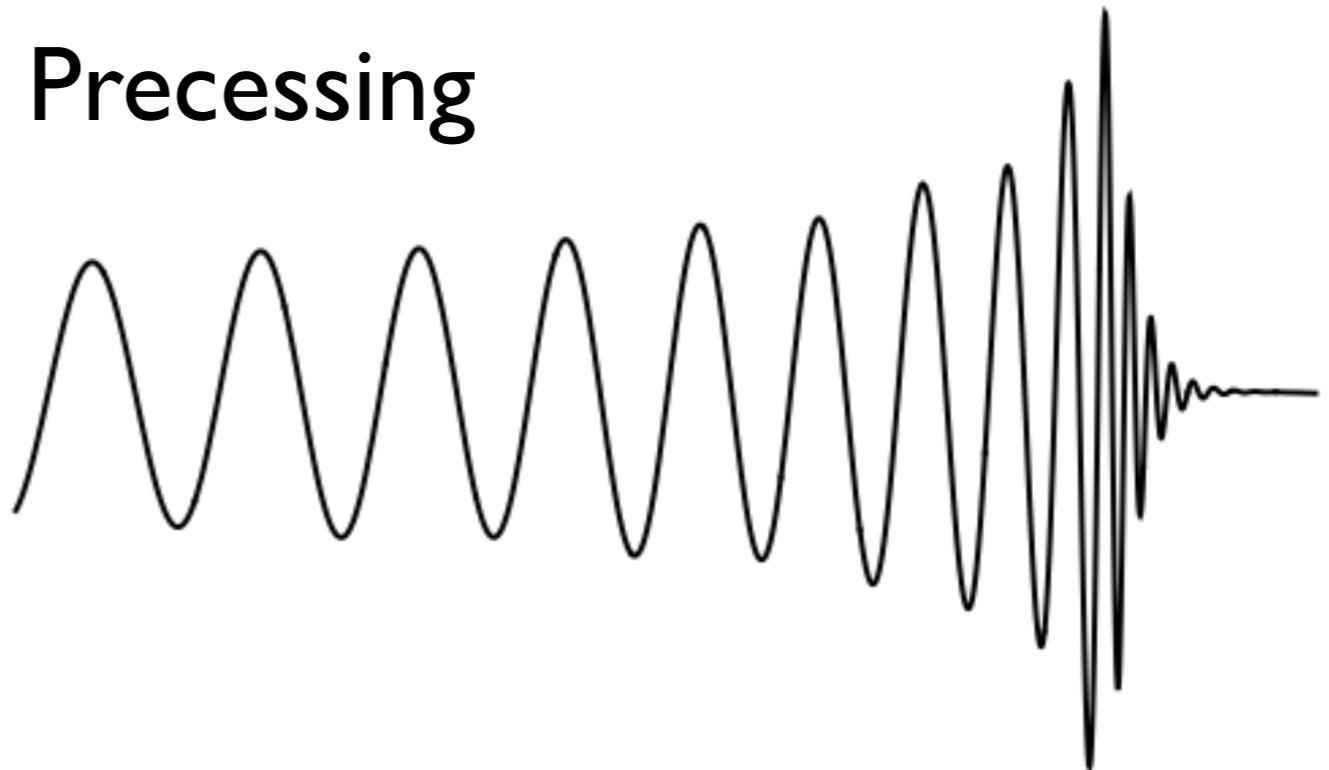


“Face-on”
to the
source

Non-precessing

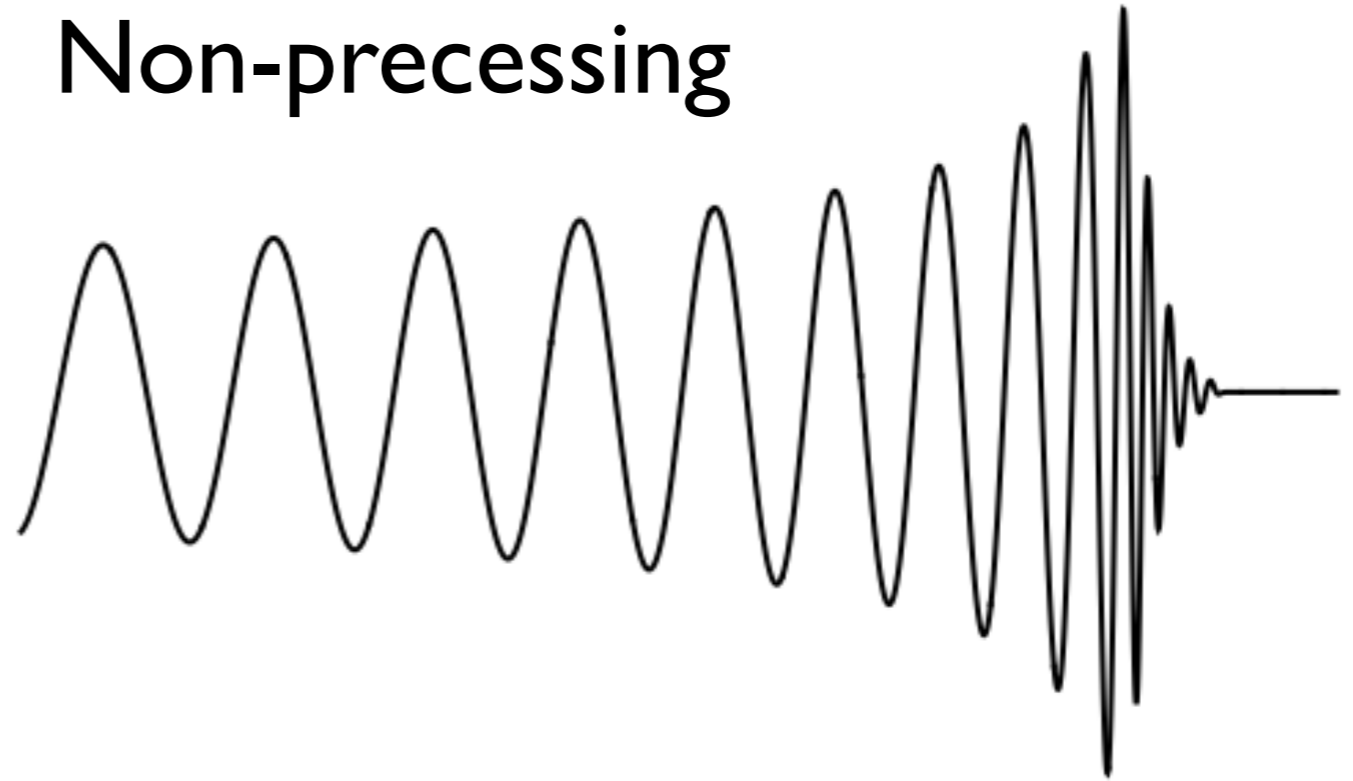


Precessing

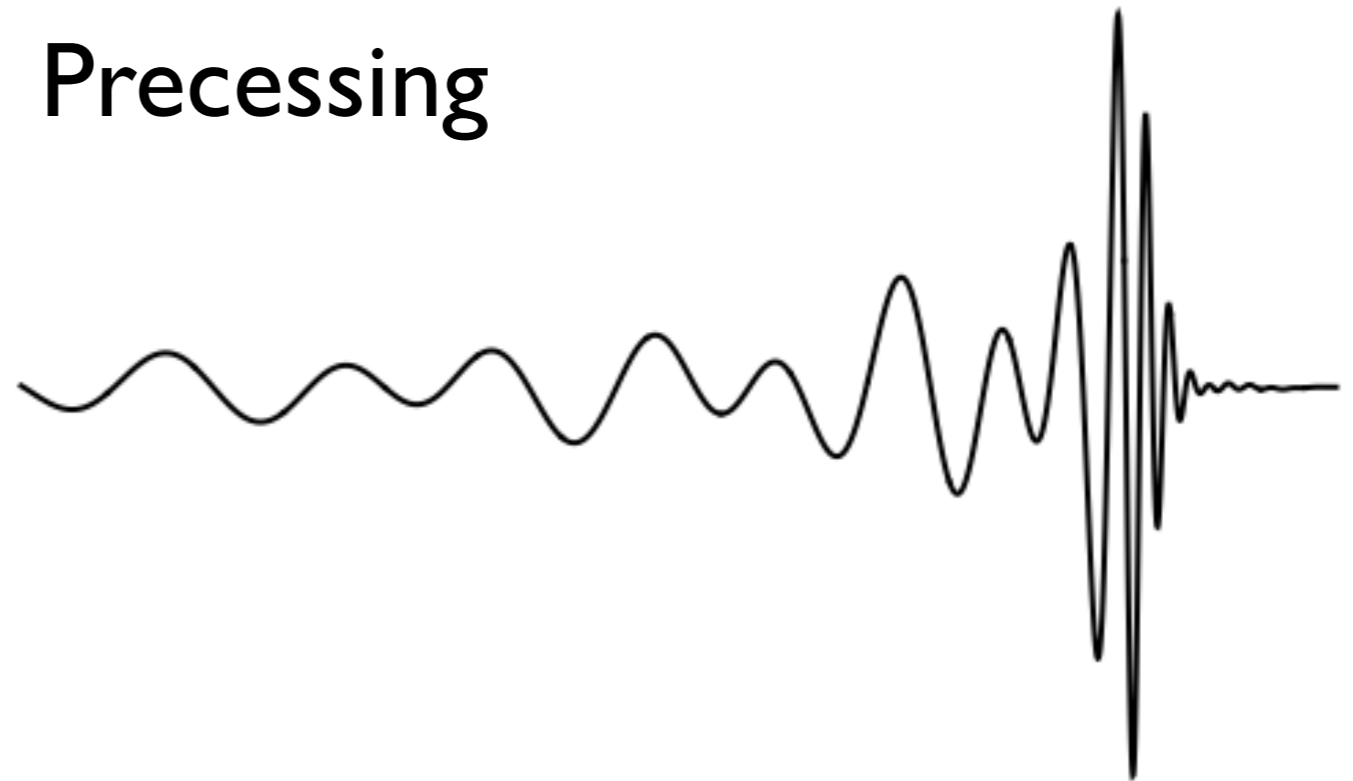


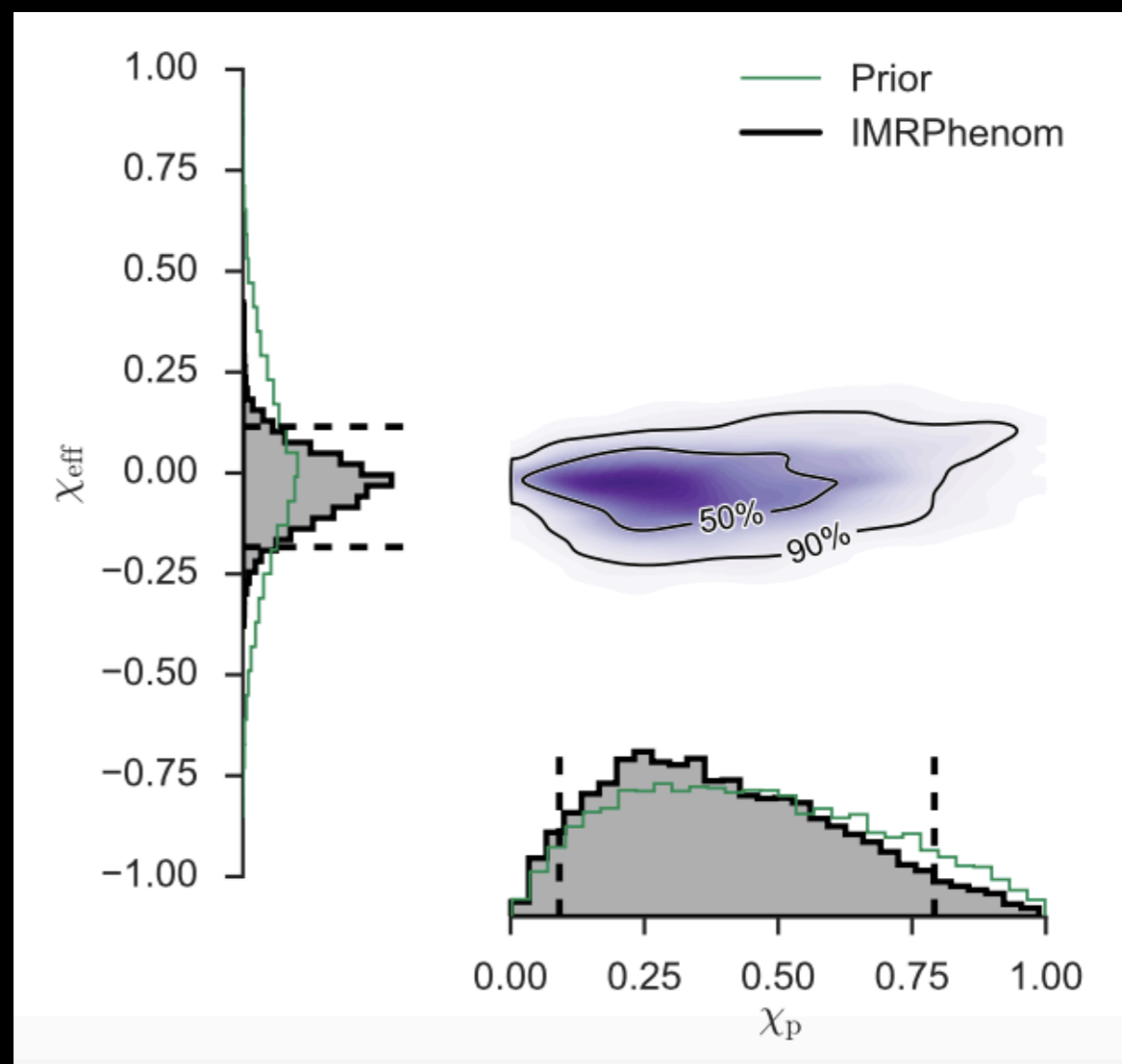
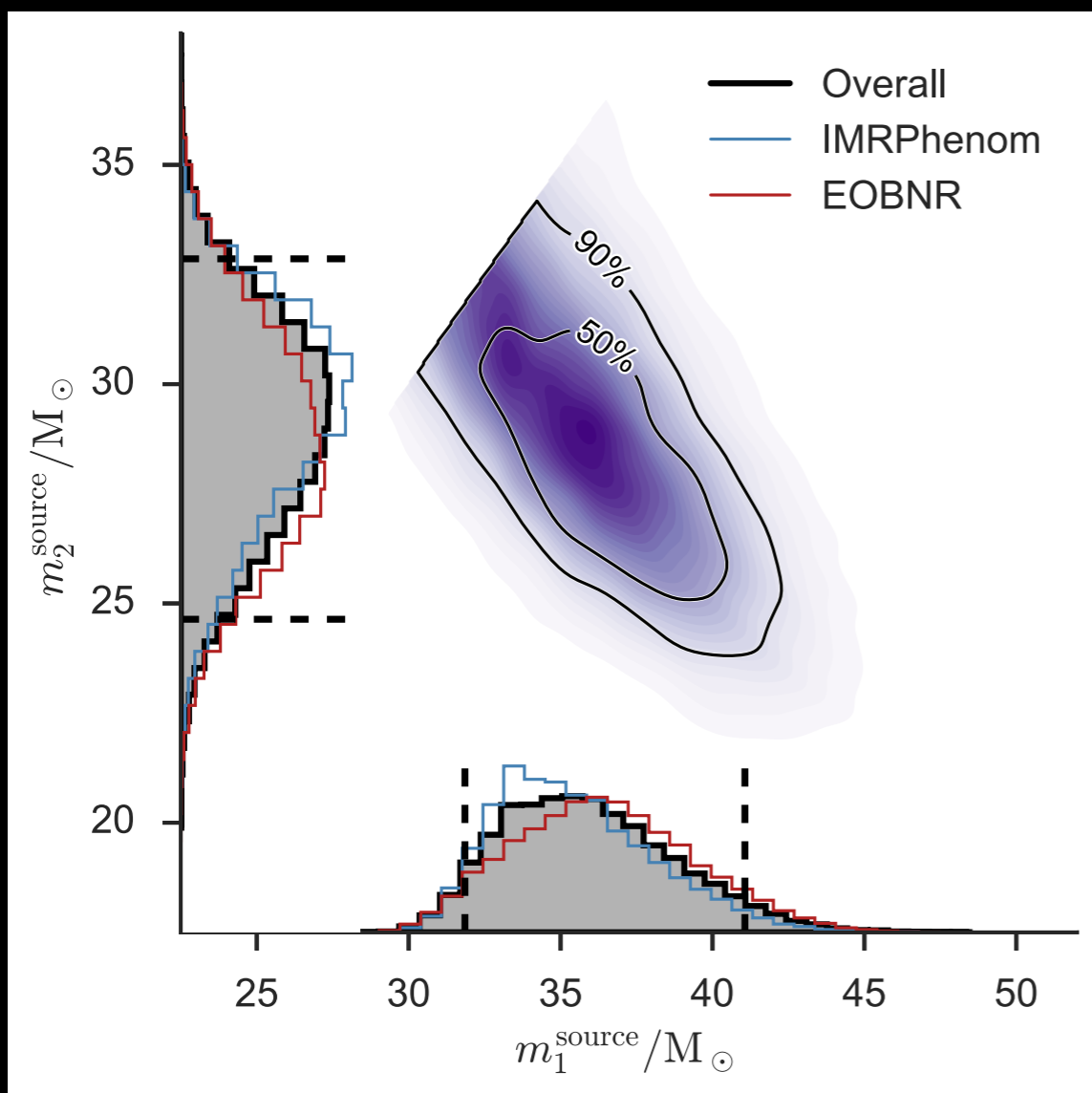
“Edge-on”
to the
source

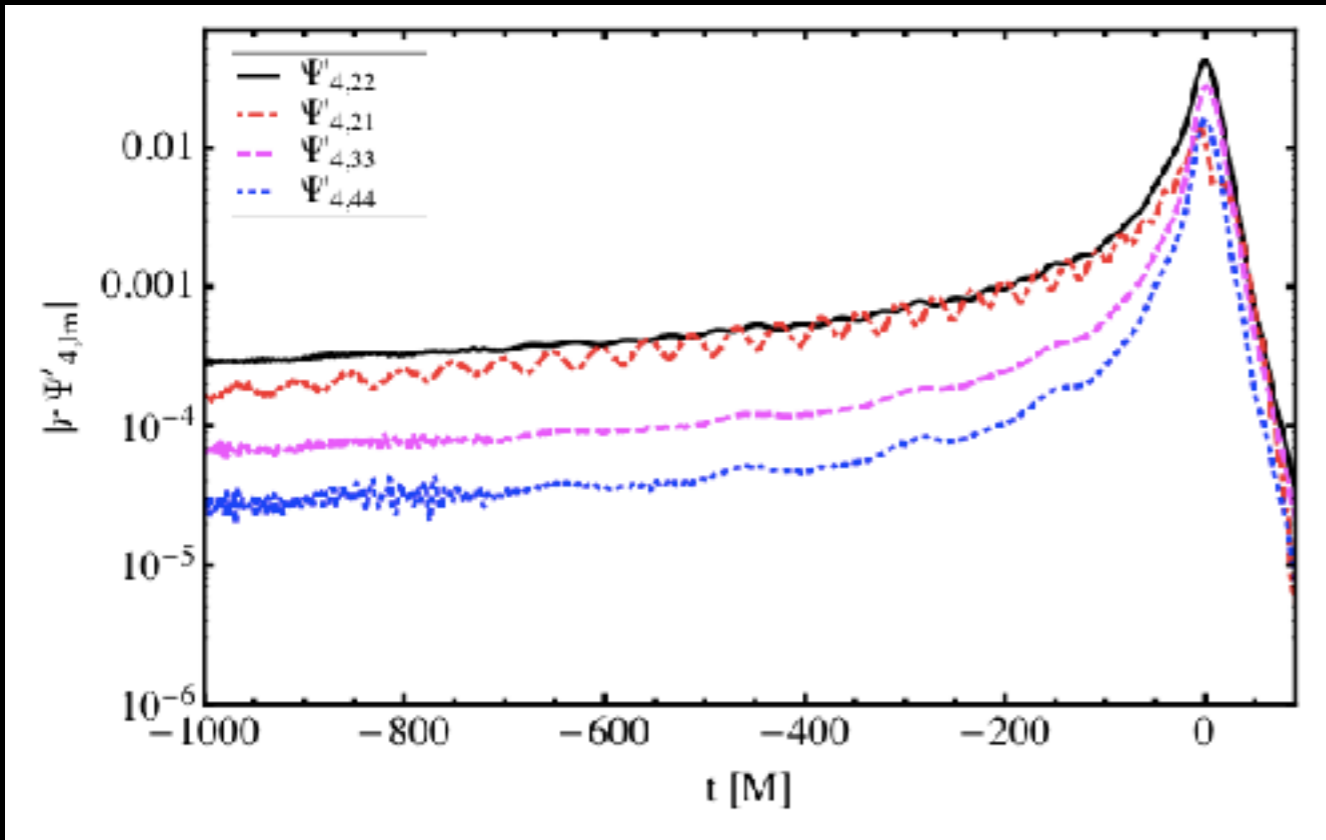
Non-precessing



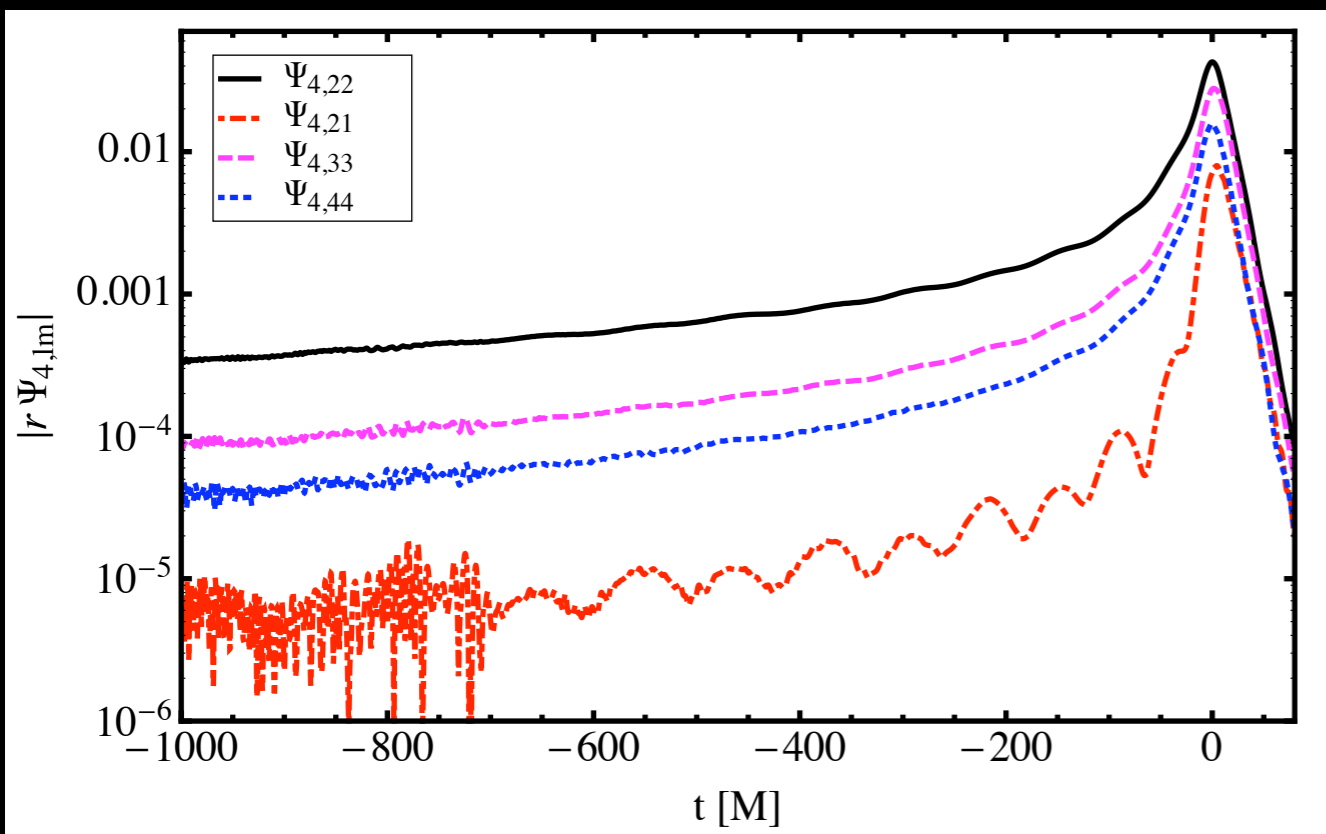
Precessing



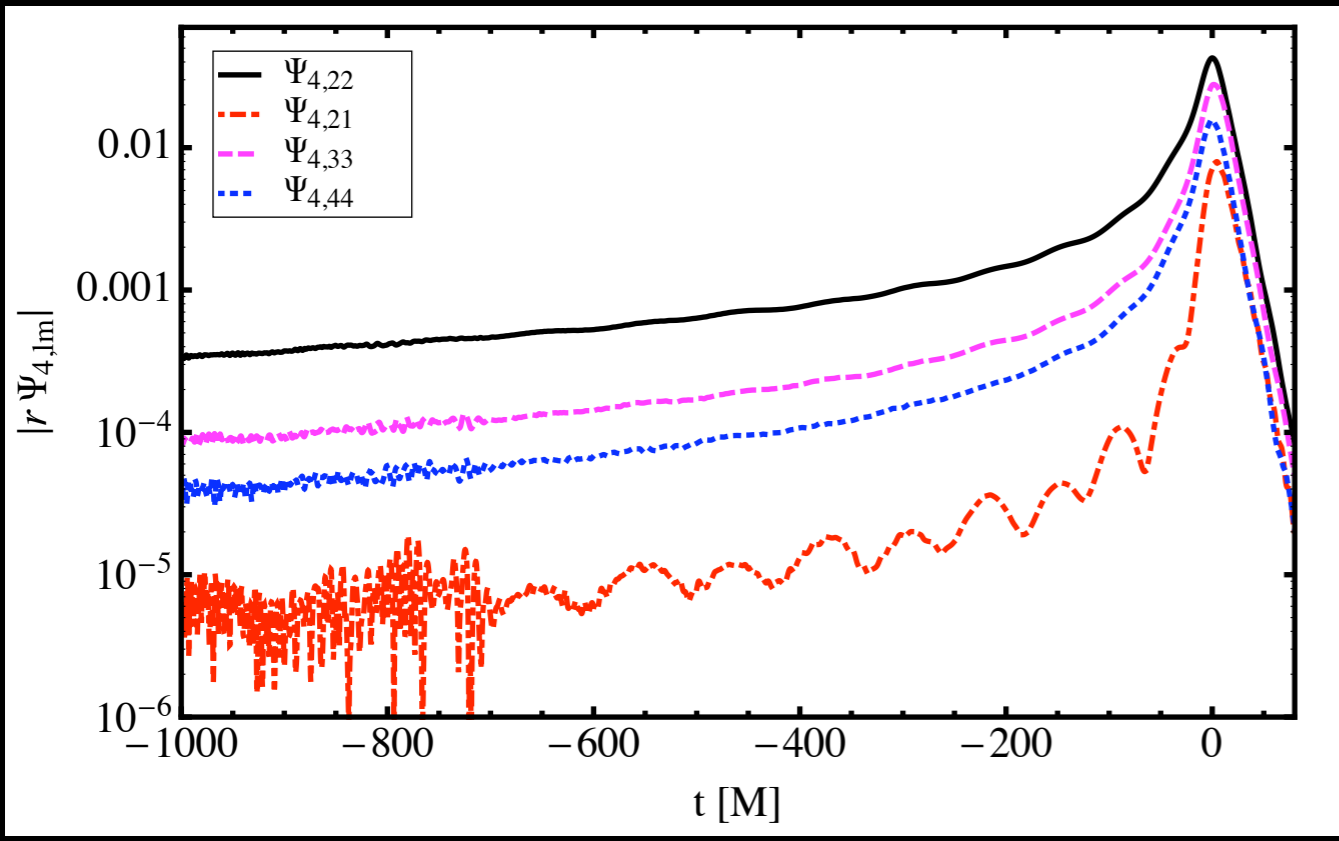
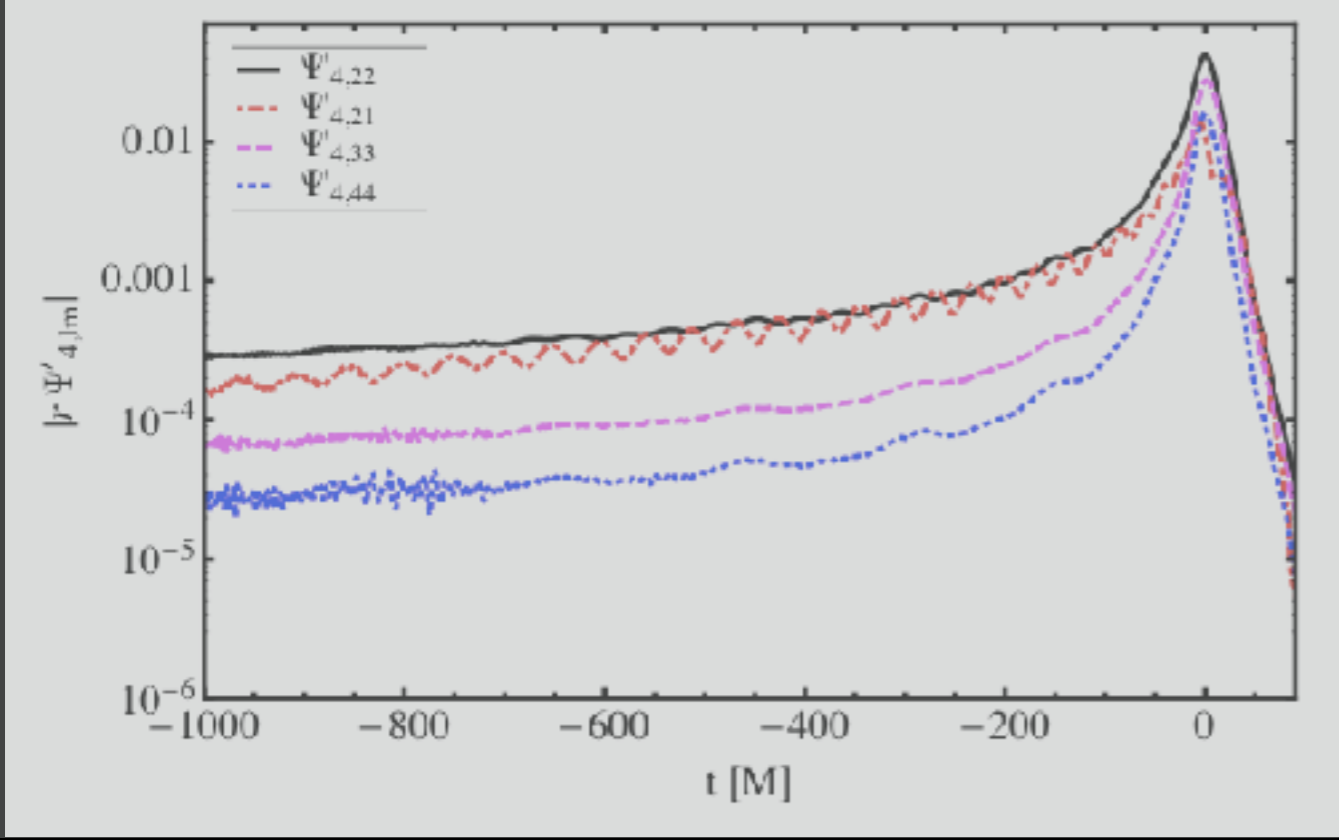




Frame aligned with
initial direction
of L

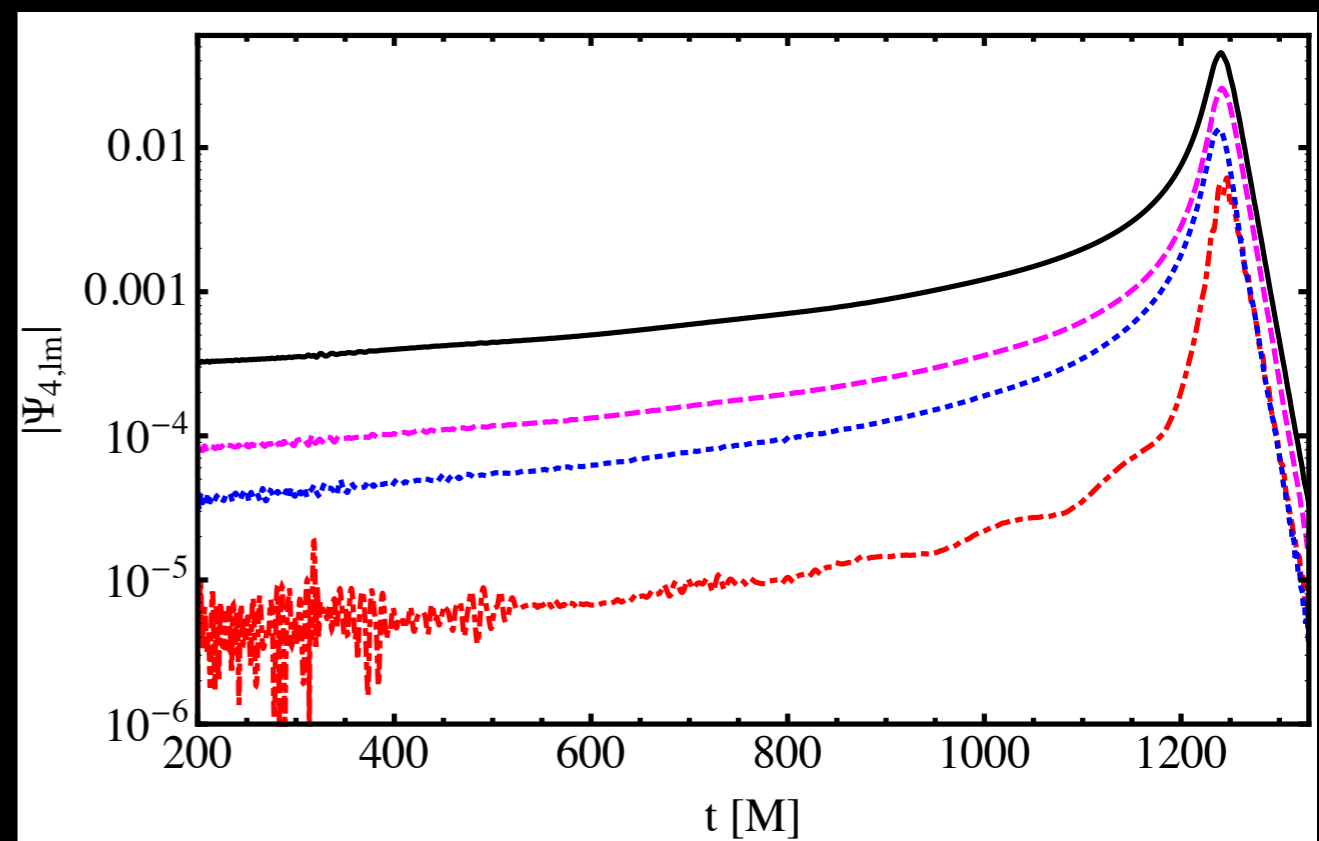
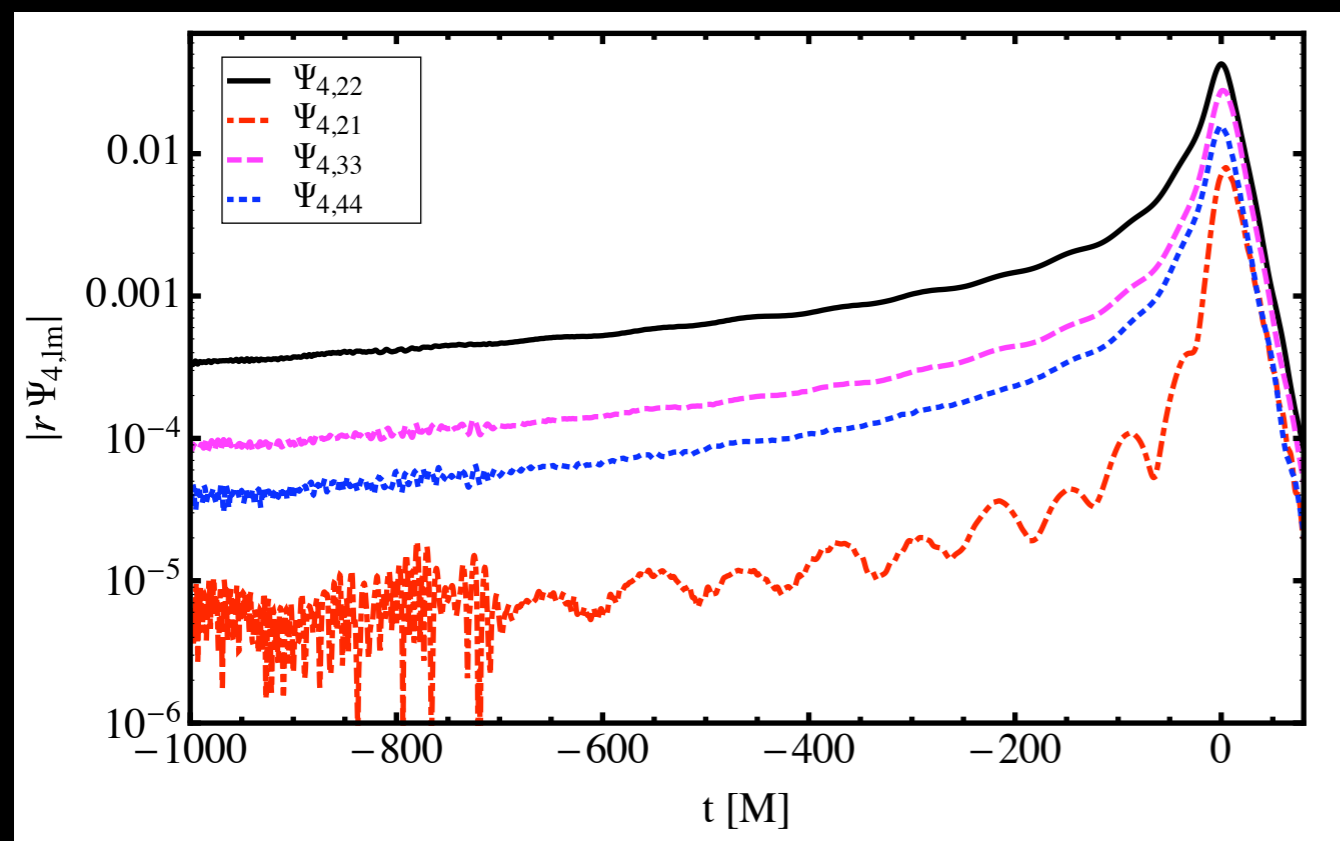
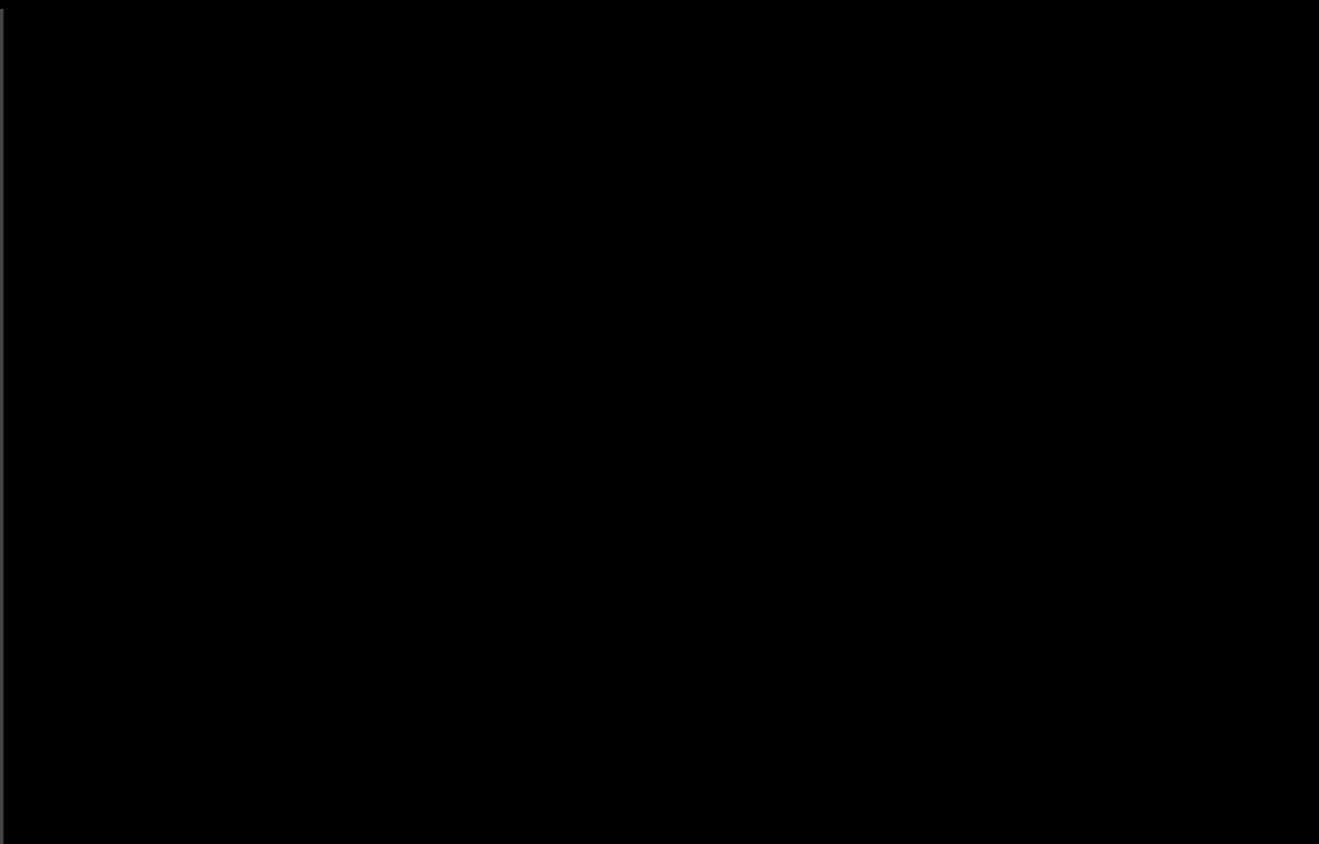
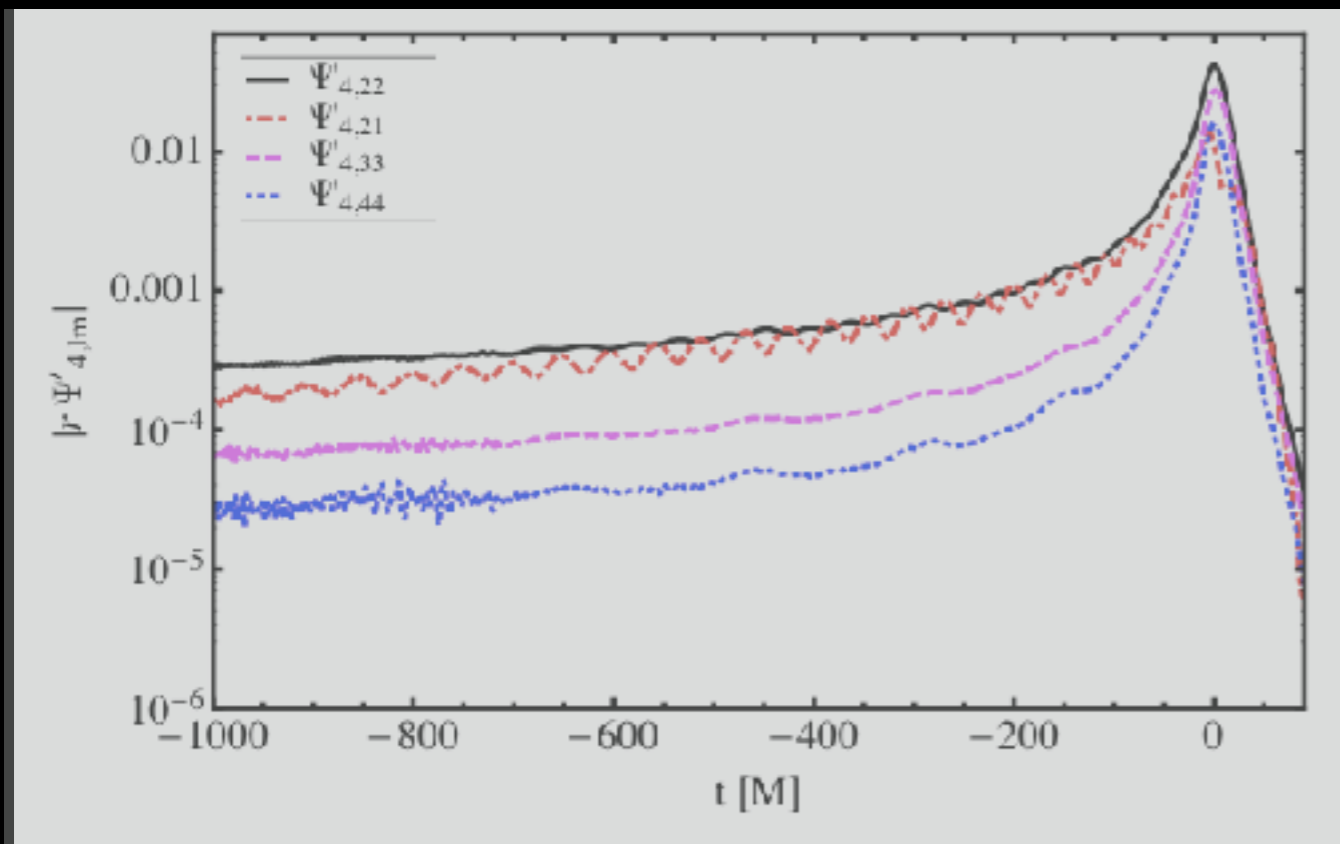


Co-precessing
frame



Just a second...

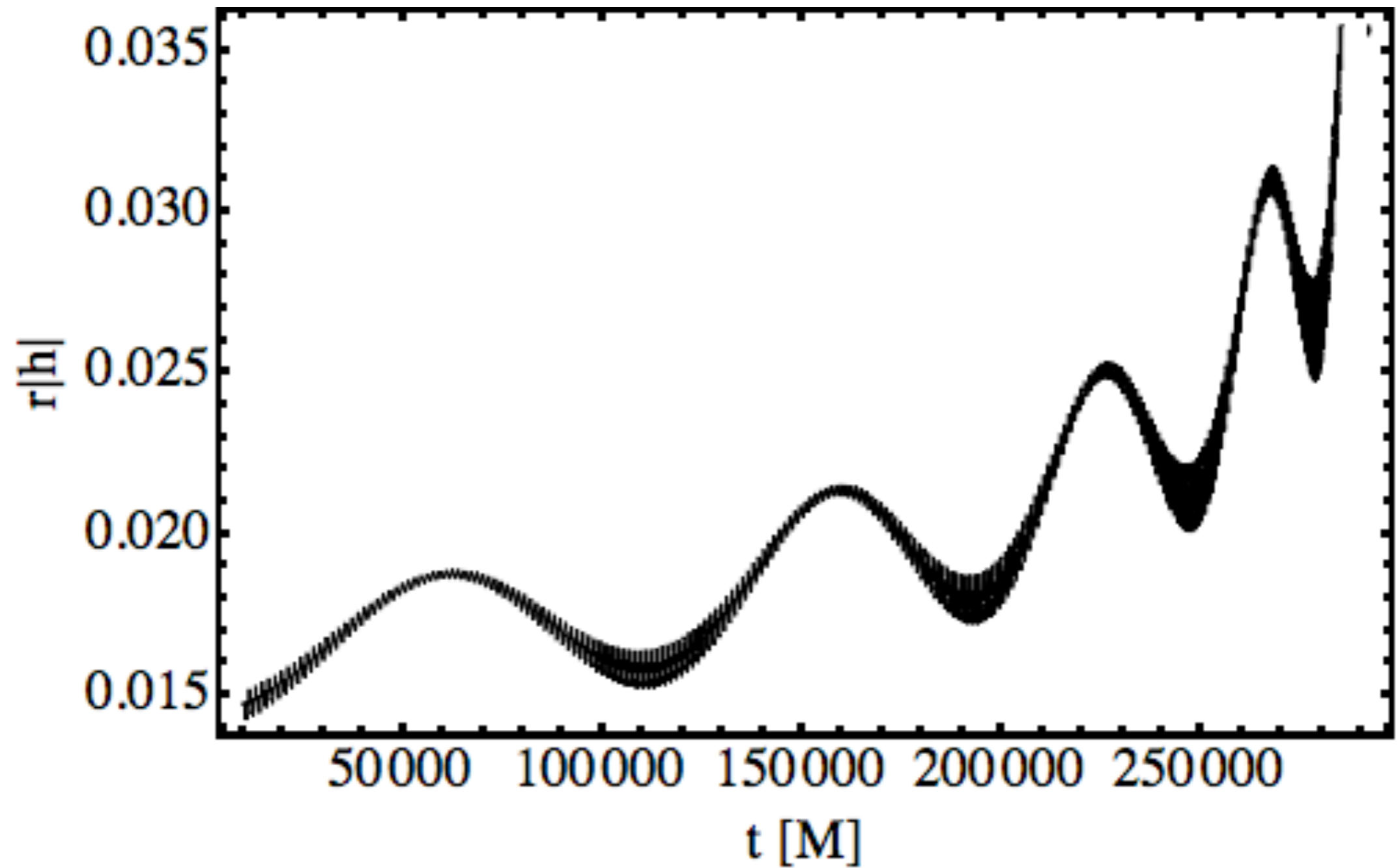
$q=3$ precessing-binary
in co-processing frame



$q=3$ precessing-binary
in co-precessing frame

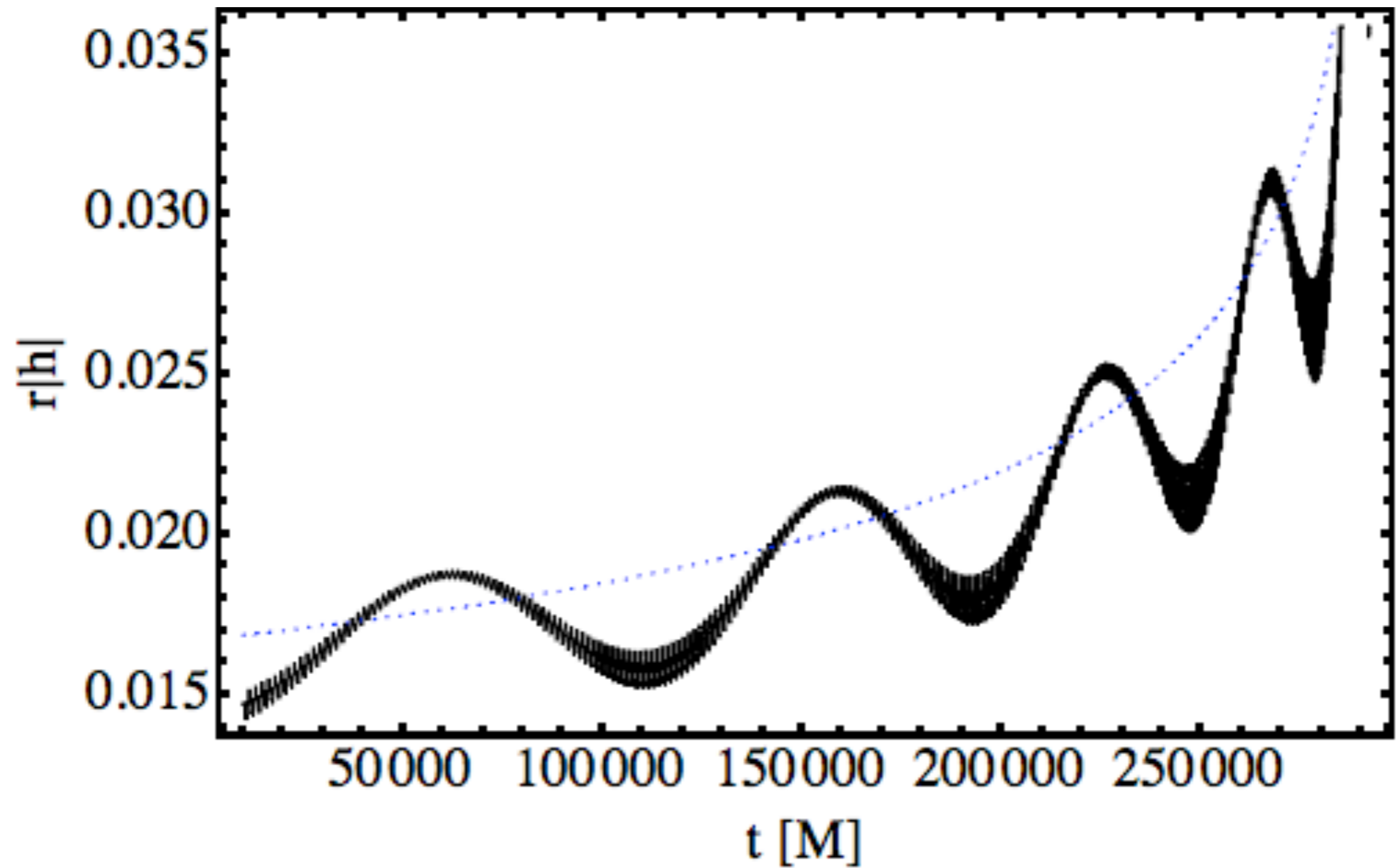
$q=3$ non-precessing
binary

(Inclination 2.8 rad)



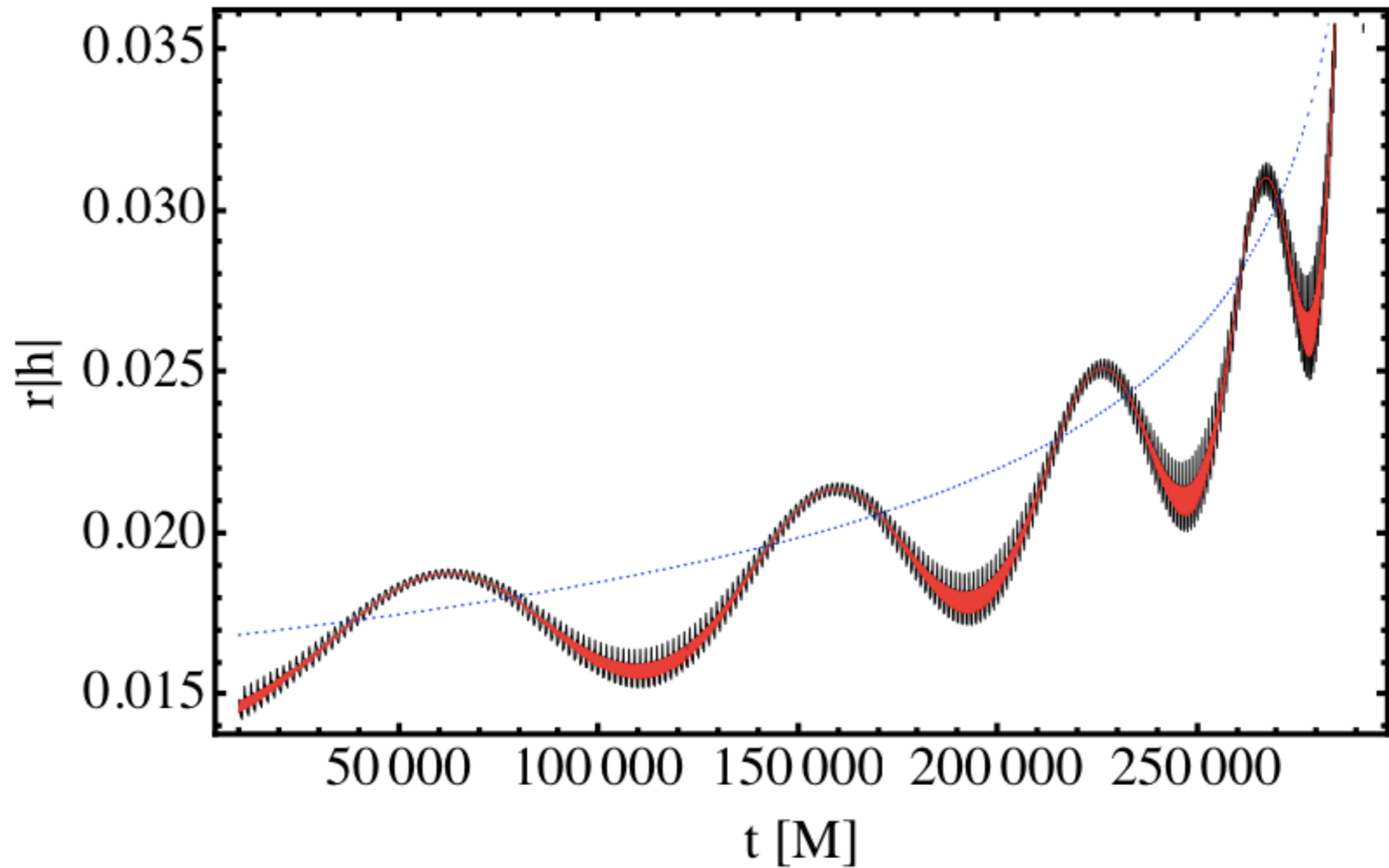
[Schmidt, Hannam, Husa (2012)]

(Inclination 2.8 rad)



[Schmidt, Hannam, Husa (2012)]

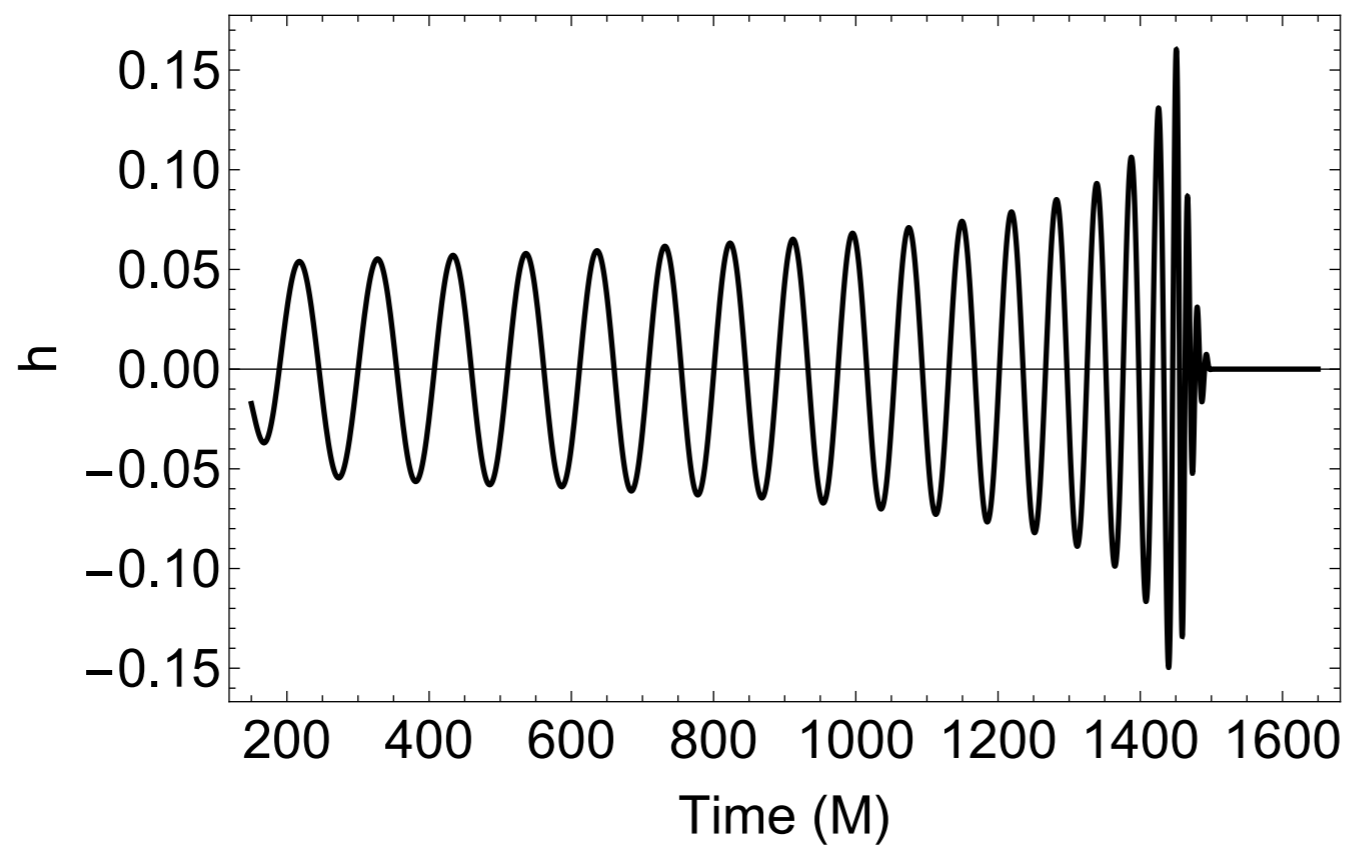
(Inclination 2.8 rad)



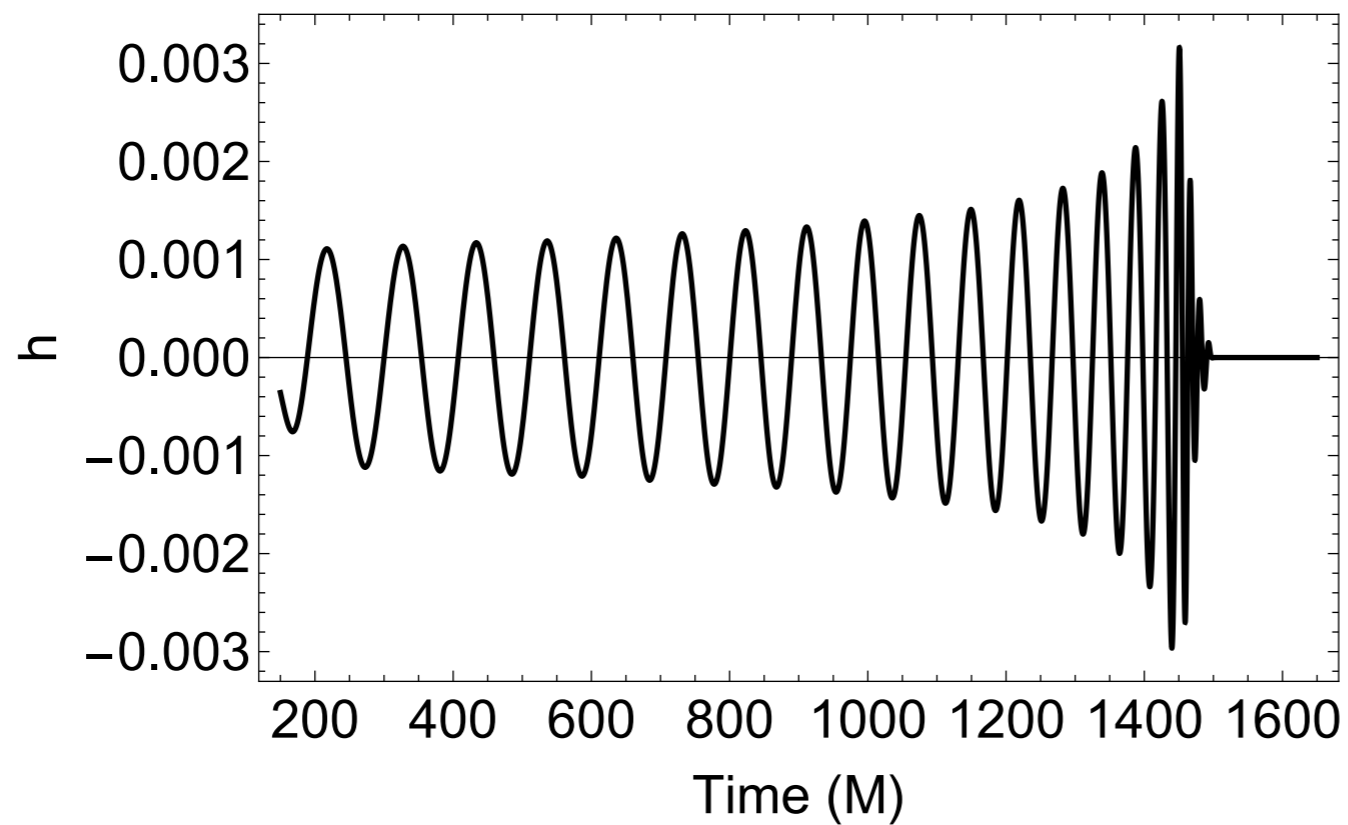
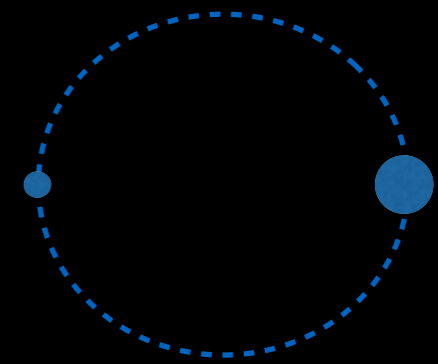
[Schmidt, Hannam, Husa (2012)]

(2,2) and (2,-2) only

(q=4)



(face-on)

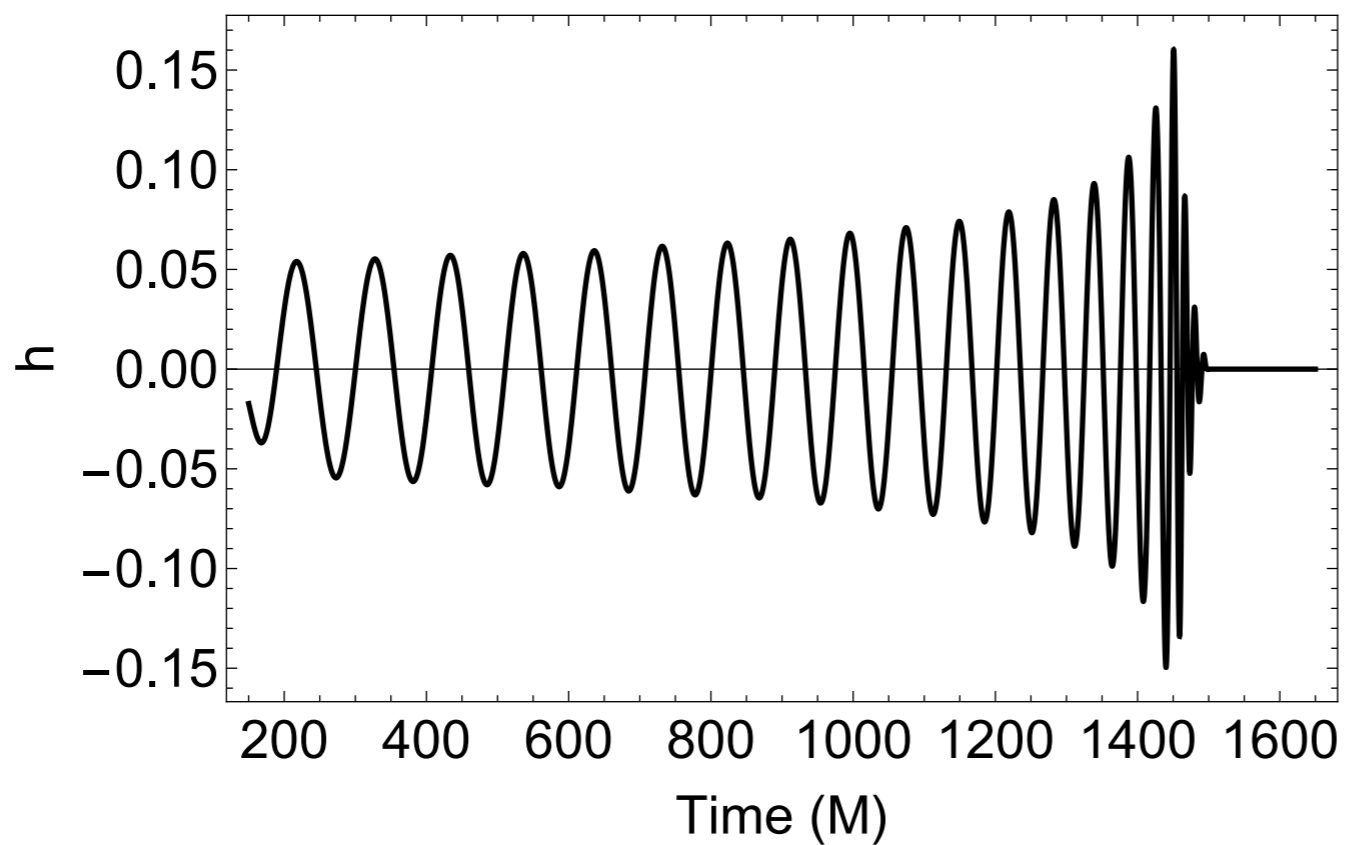


(edge-on)

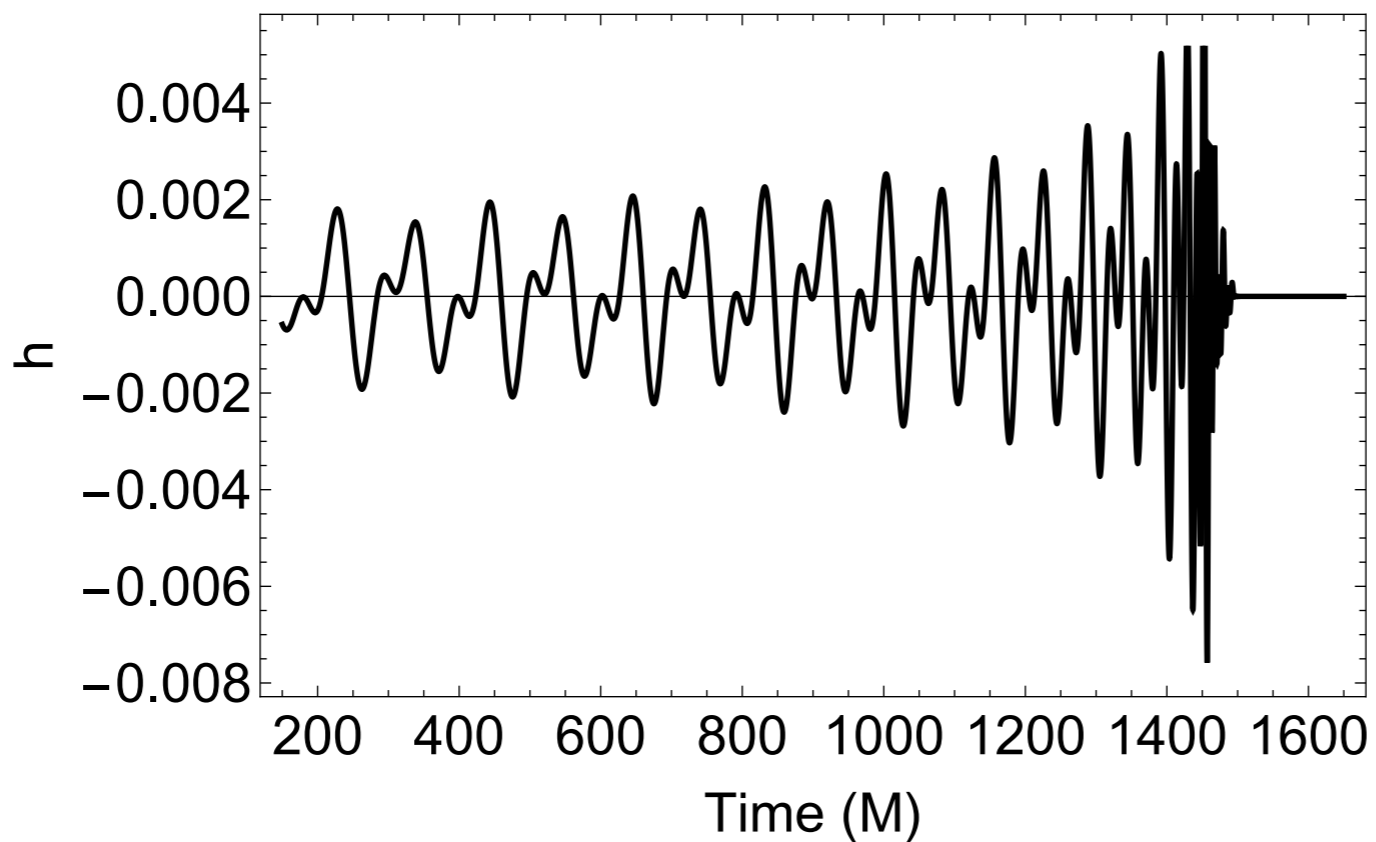
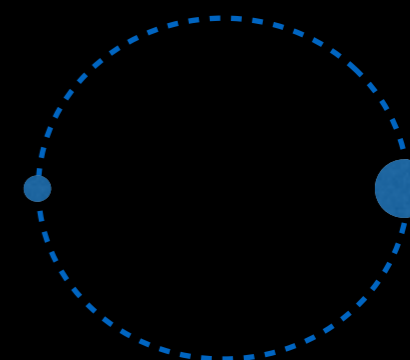


all modes

($q=4$)

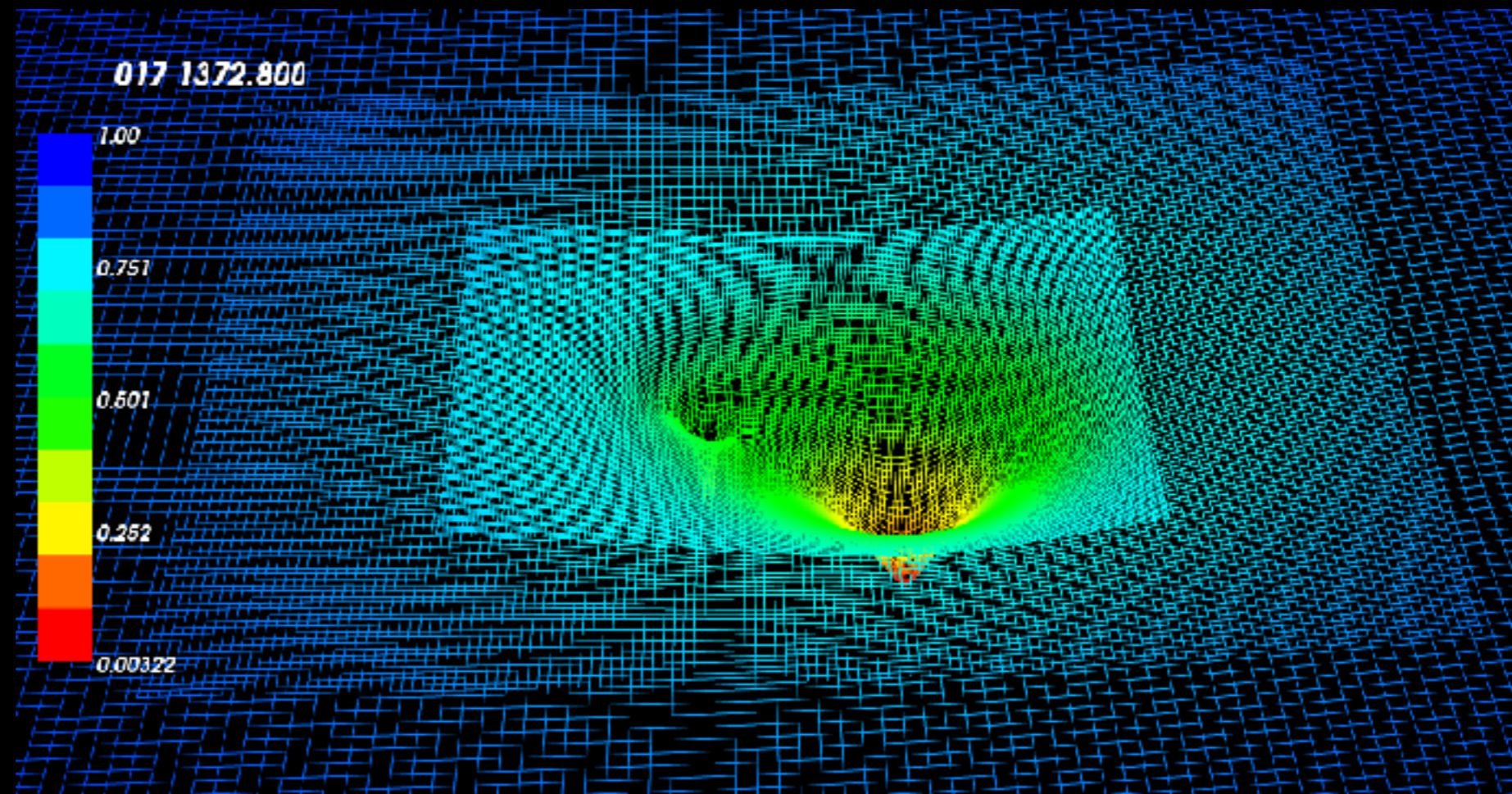
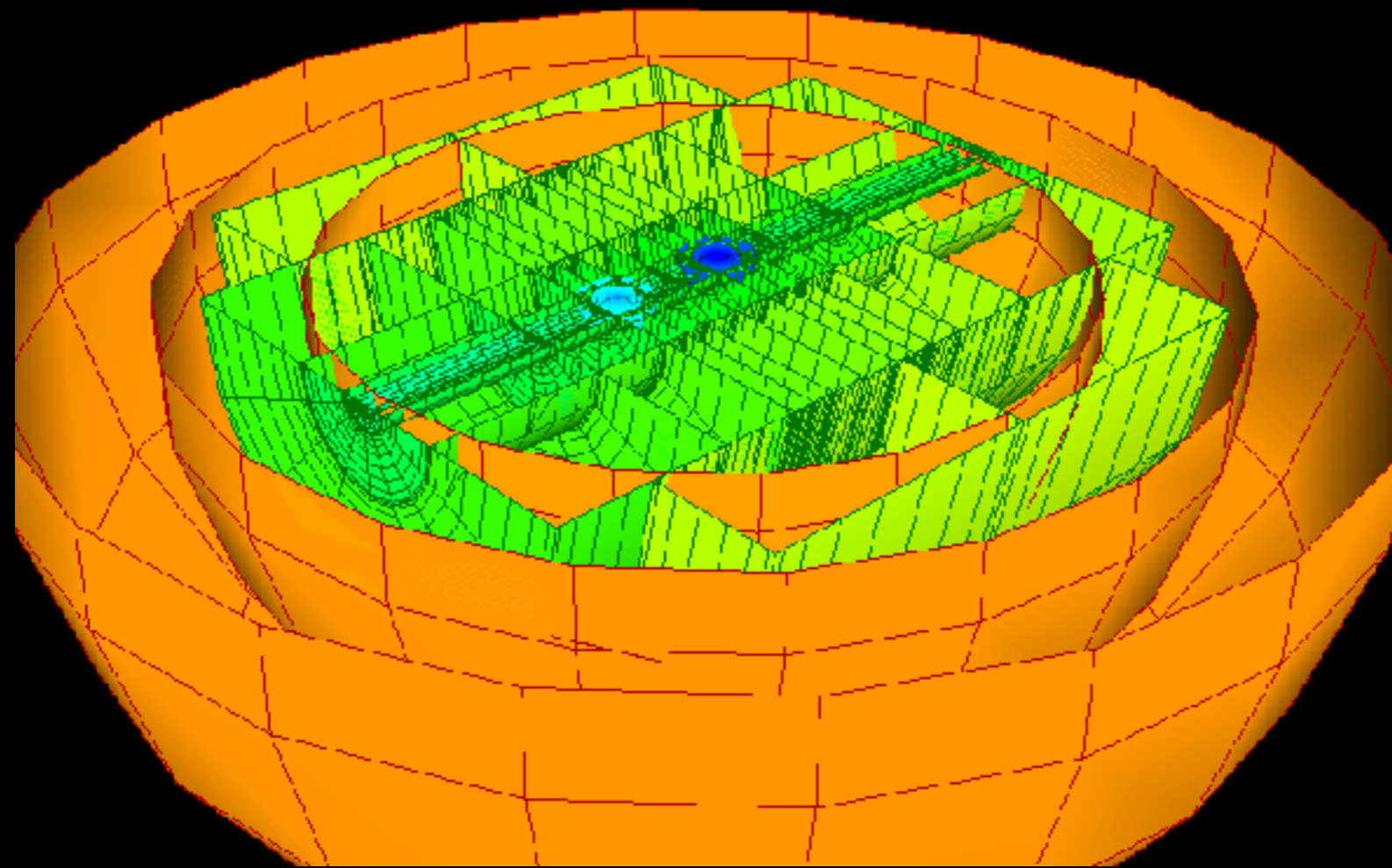


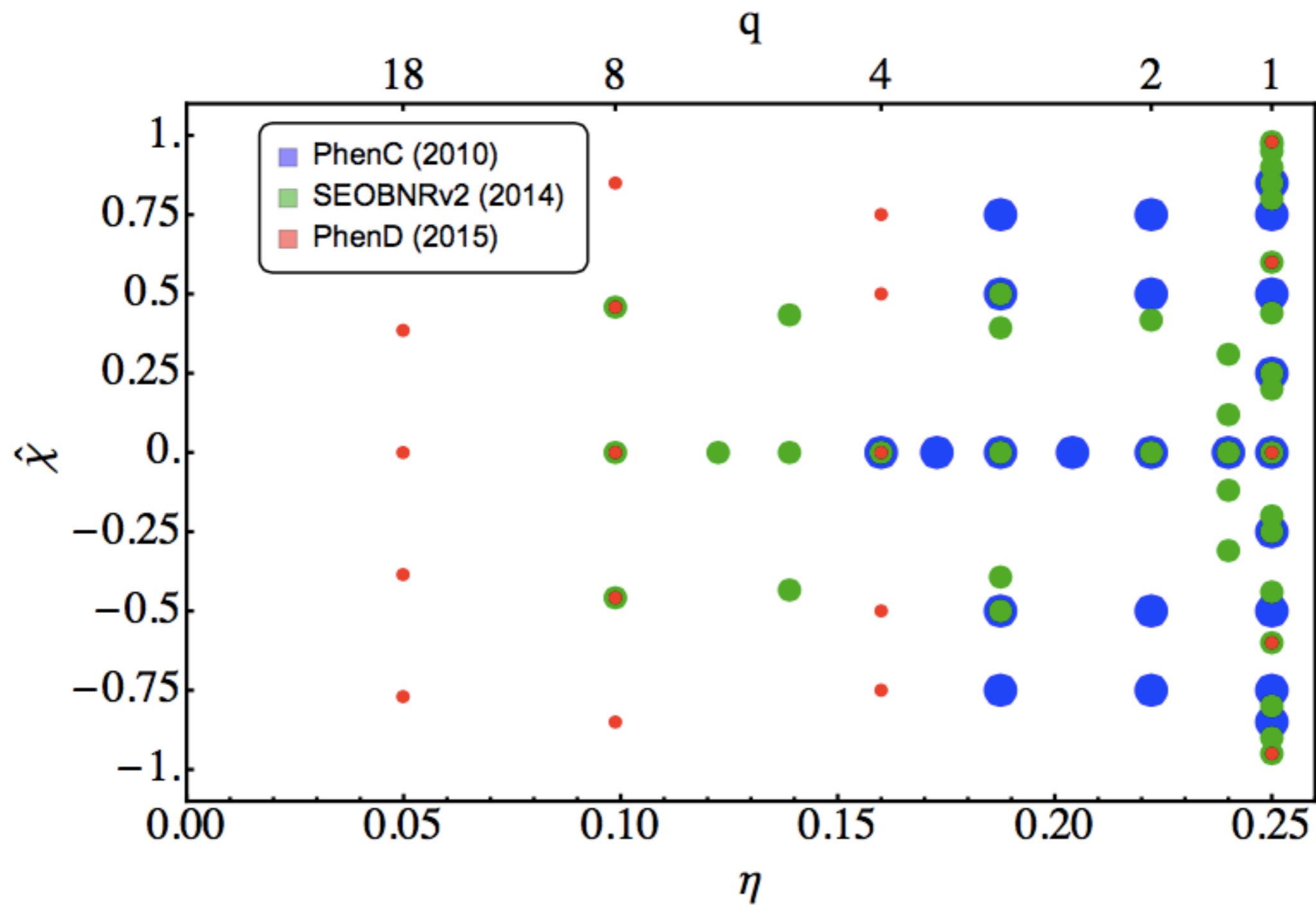
(face-on)



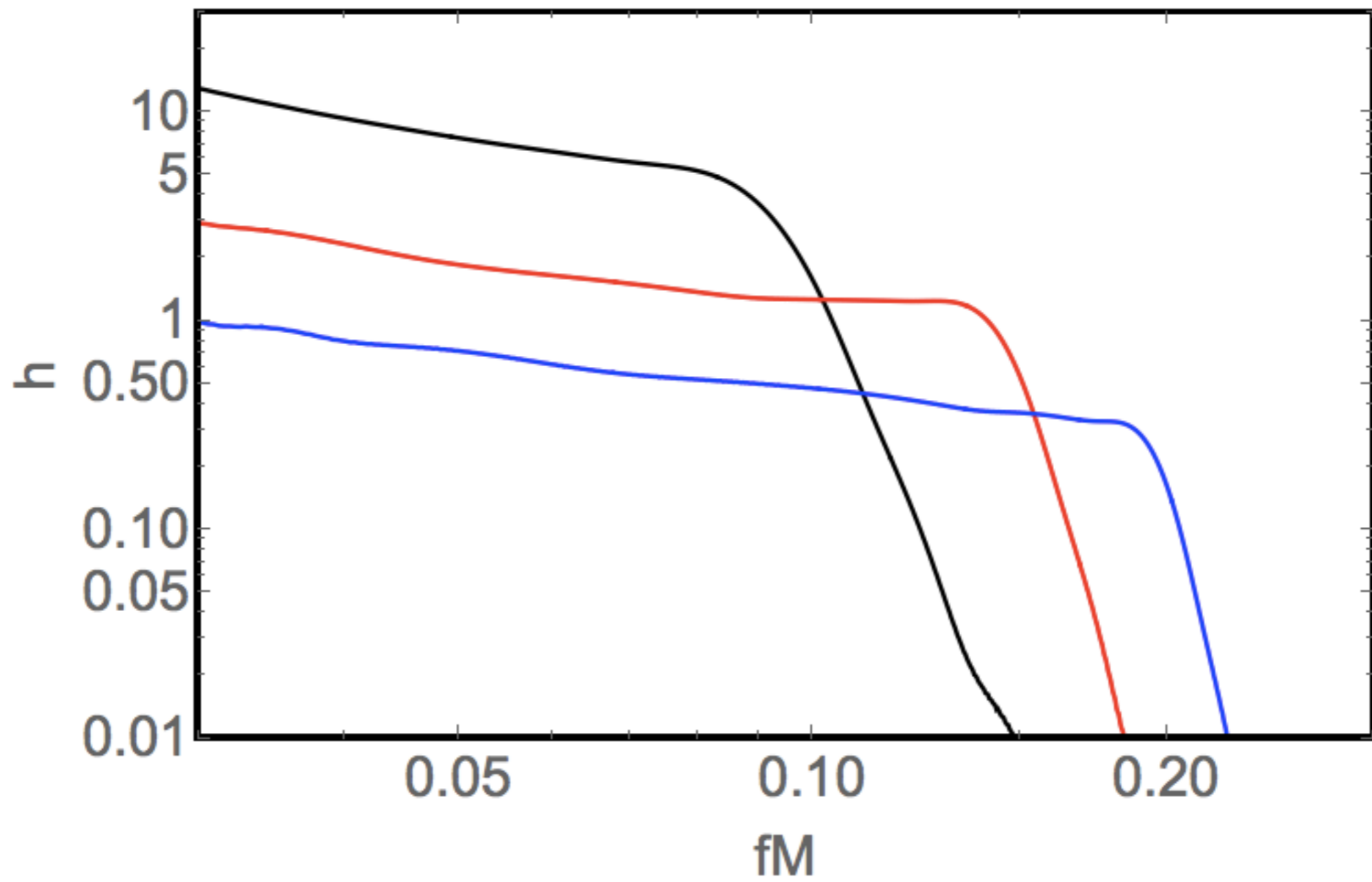
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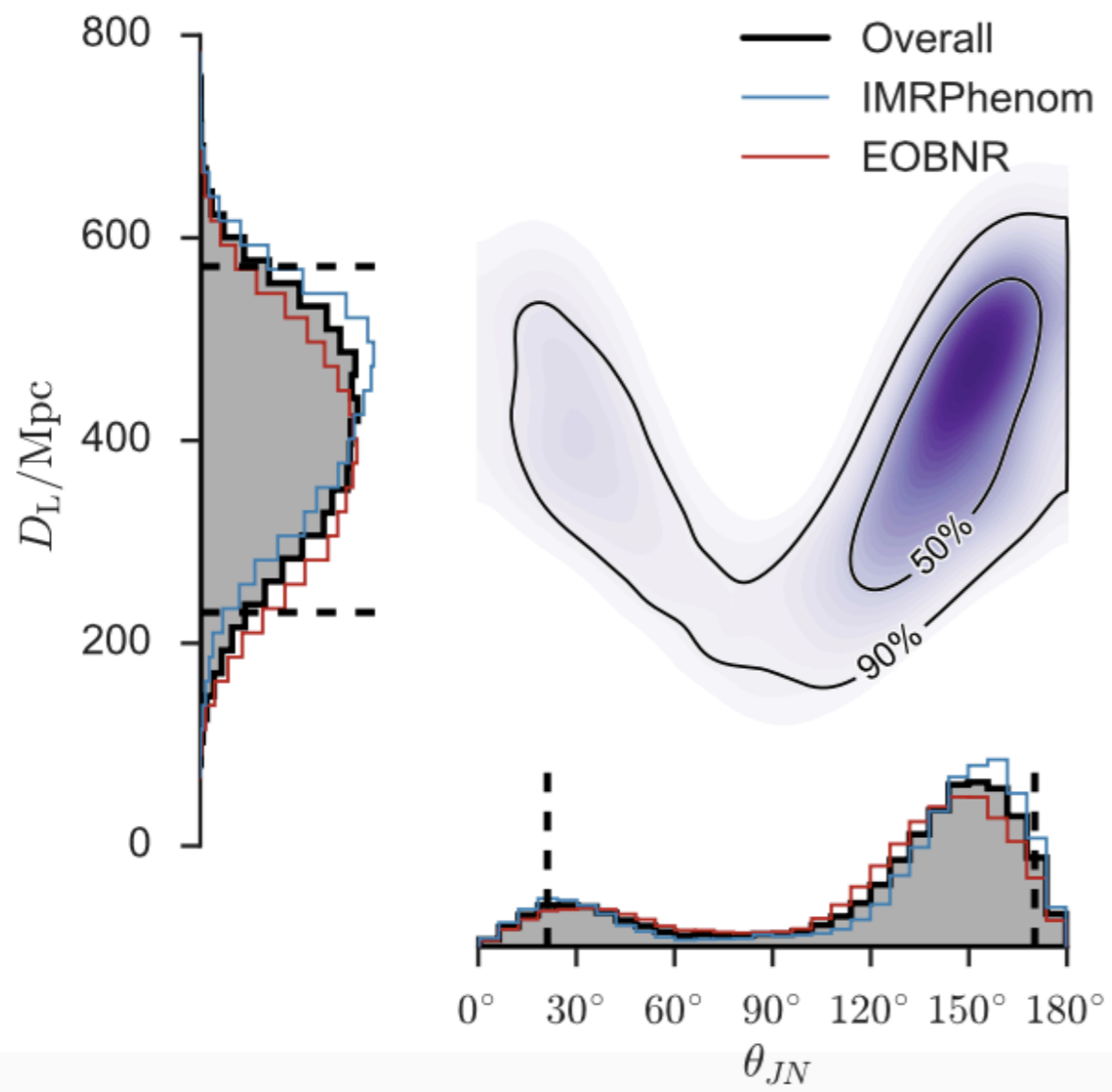


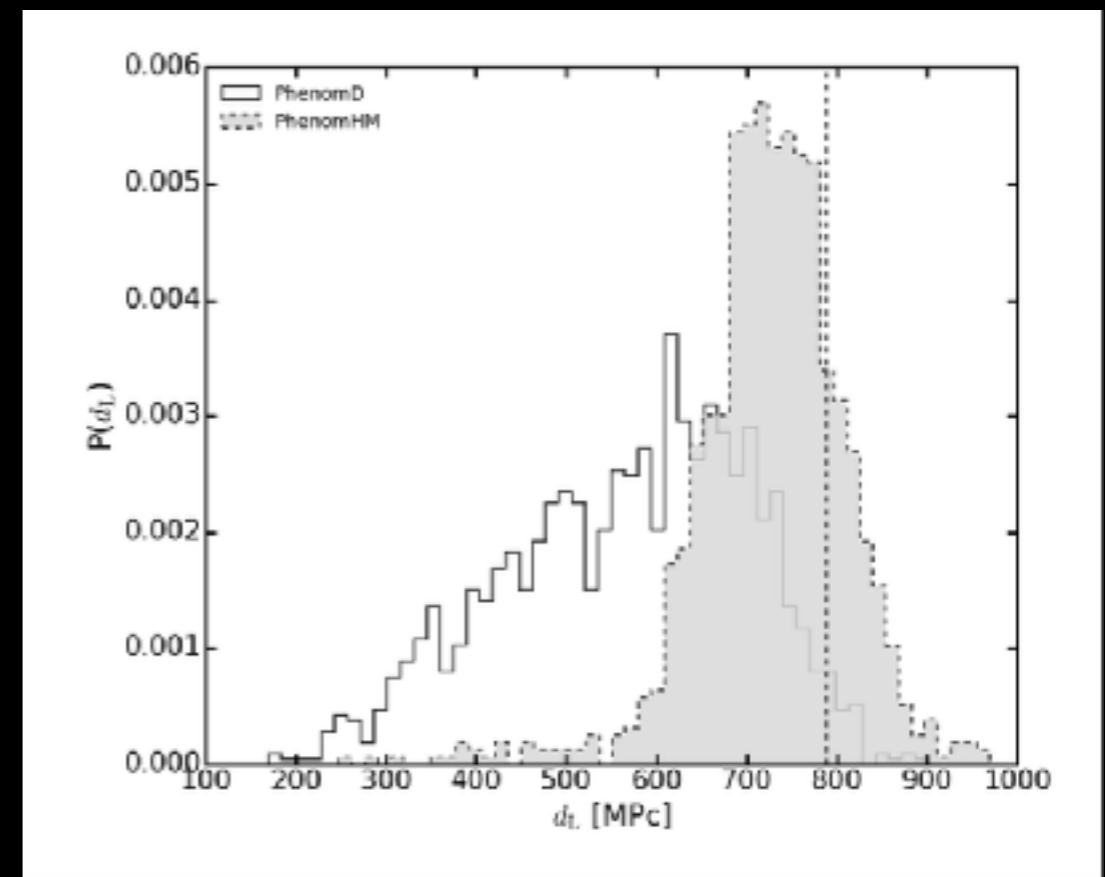
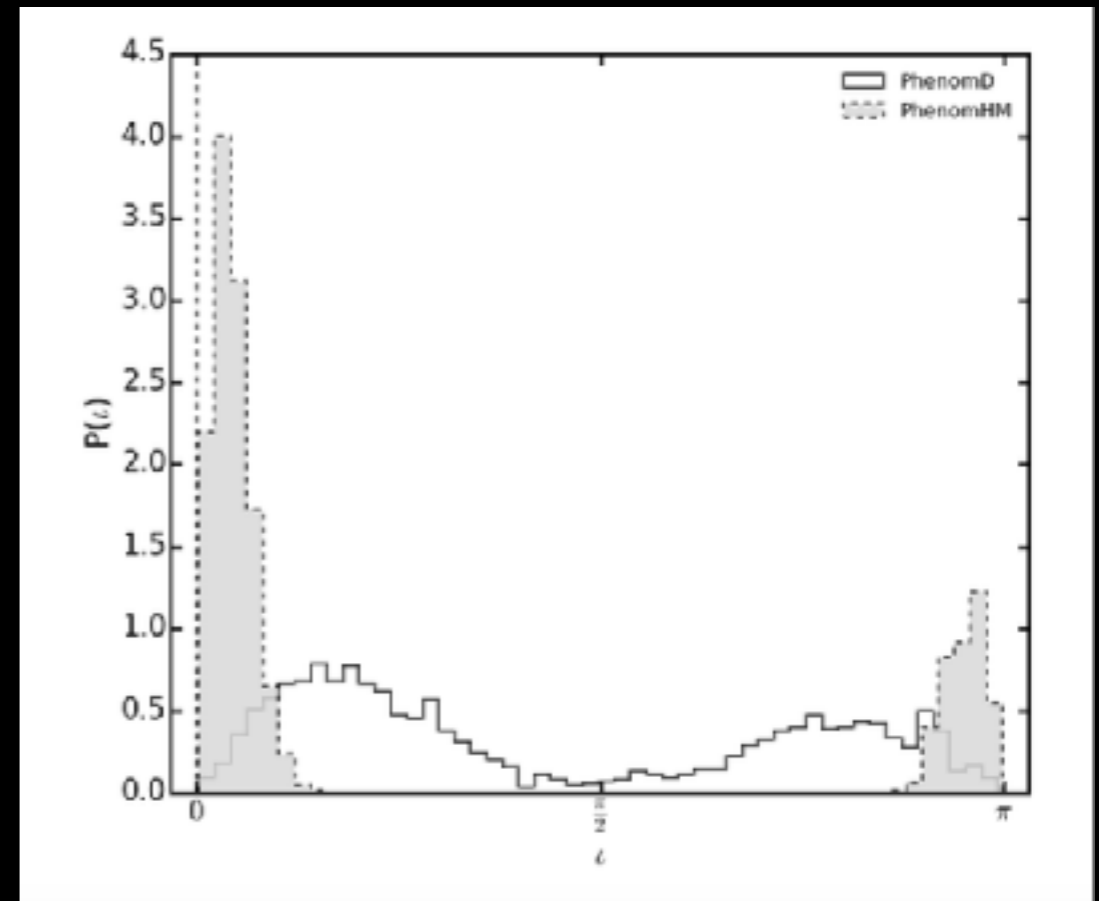
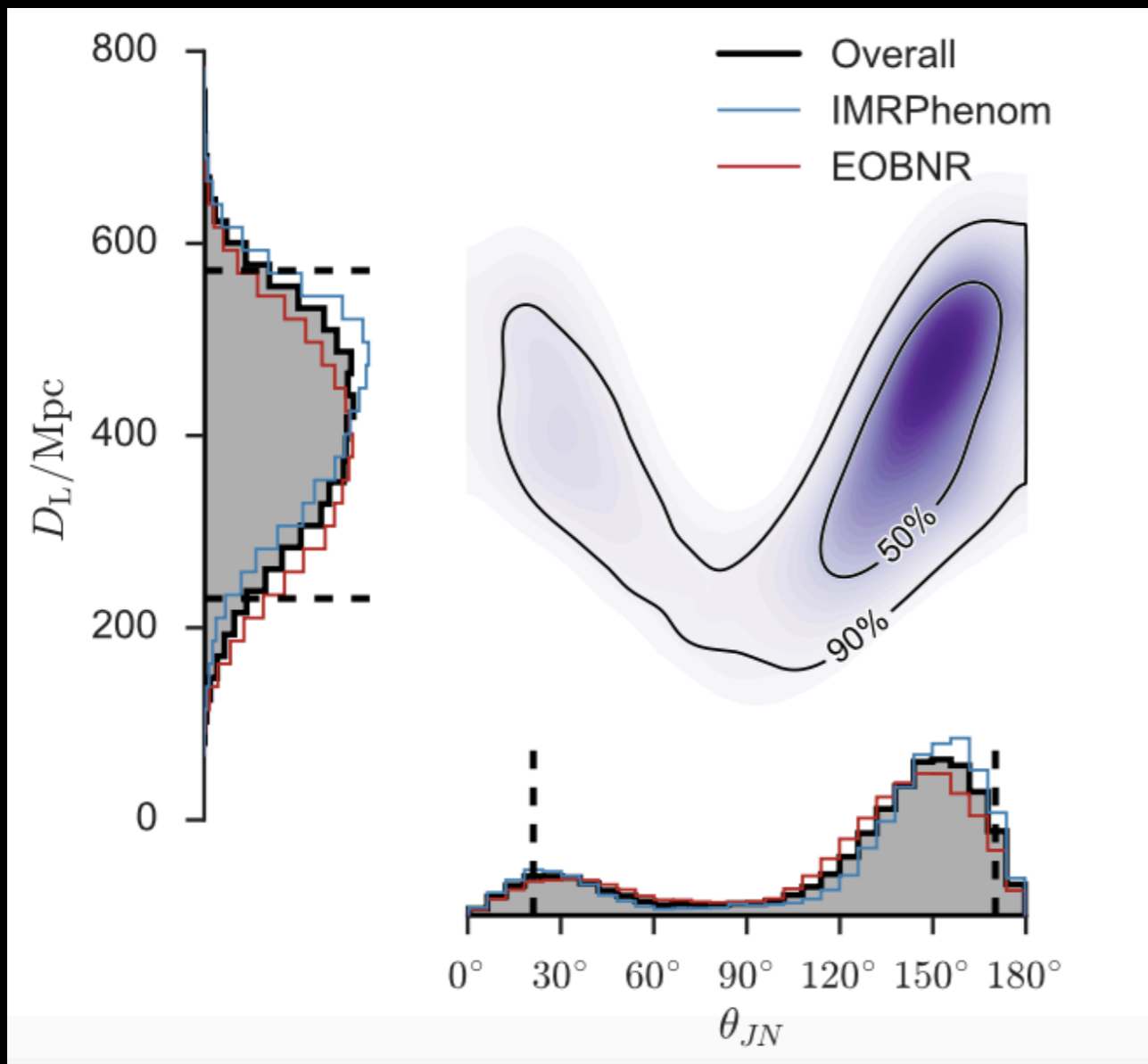


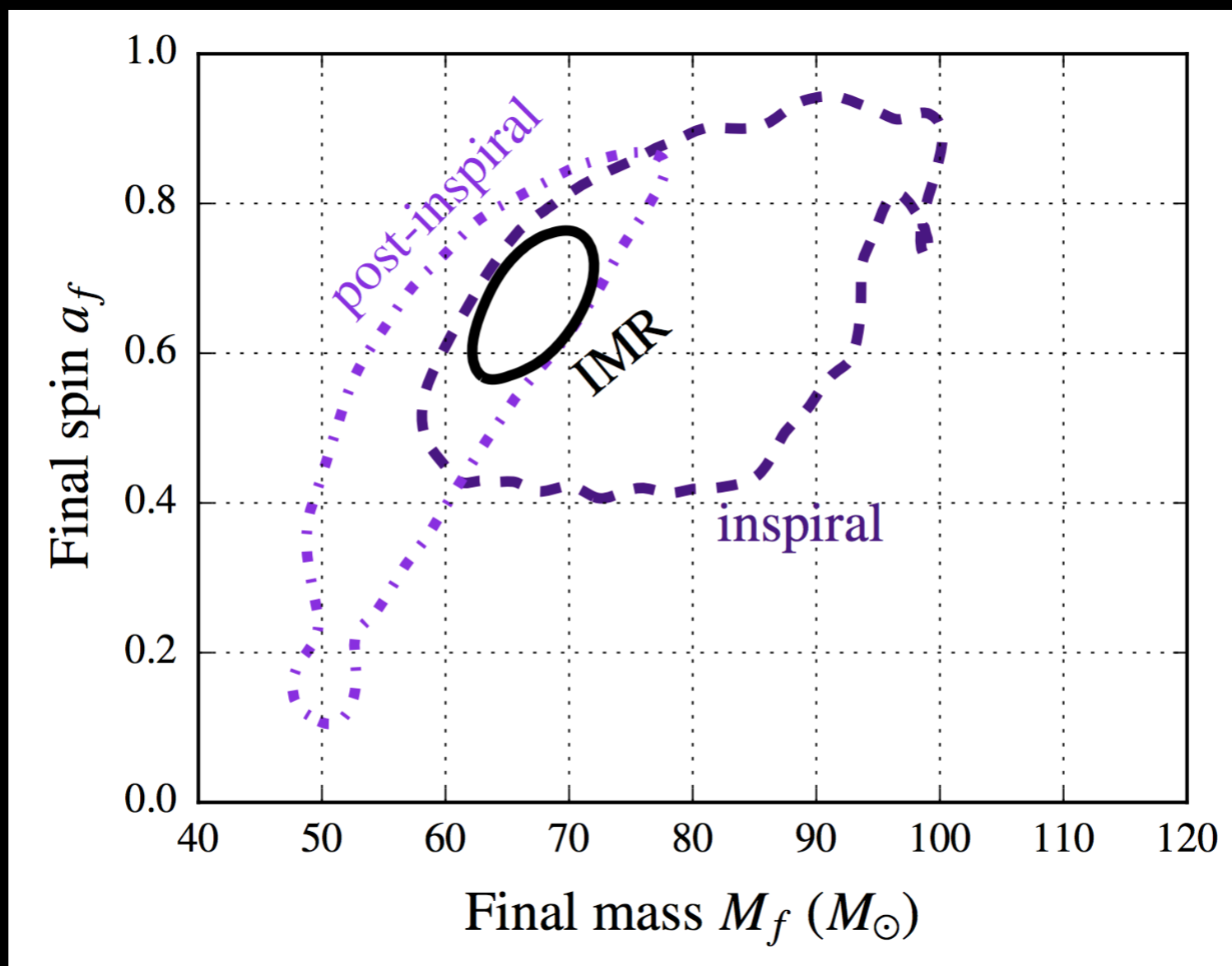


[Khan, *et. al* (2016)]









[LVC, PRL **116**, 22, 221101 (2016)]