



Suspending the Test Masses of Gravitational Wave Detectors

Giles Hammond Institute for Gravitational Research University of Glasgow giles.hammond@glasgow.ac.uk



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- aLIGO suspensions and requirements
- Seismic noise & thermal noise
- Suspension construction
- Dissipation dilution
- Noise model of suspensions
- Warm and cold upgrades to aLIGO
- Summary





About Myself



• Suspension for Advanced gravitational wave detectors; silica, silicon, sapphire (now)



 aLIGO ISI (postdoc)



- Precision measurements of gravity; axion searches and Casimir force (PhD, postdoc)
 - MEMS gravimeters for gravity imaging (now)







aLIGO Layout and Core Optics





Suspensions in aLIGO





Suspension Inventory





Suspension System Functions

- Support the optics to minimise the effects of
 - thermal noise in the suspension
 - seismic noise acting at the support point
- Provide damping of low frequency suspension resonances (local control), and
- Provide means to maintain interferometer arm lengths (global control)
 - while not compromising low thermal noise of mirror
 - and not introducing noise through control loops
- Provide interface with seismic isolation system and core optics system
- Support optic so that it is constrained against damage from earthquakes
- Accommodate a thermal compensation scheme and other systems as required e.g. acoustic mode dampers, baffles, alignment fiducials, ancilliary tooling, vibration absorbers





Requirements:Test Masses

• Top-Level Requirements:

Requirement	Value
Suspension Thermal Noise	10 ⁻¹⁹ m/ $\sqrt{12}$ Hz at 10 Hz (longitudinal) 10 ⁻¹⁶ m/ $\sqrt{12}$ Hz at 10 Hz (vertical)*#
Residual Seismic Noise	10^{-19} m/ \sqrt{Hz} at 10 Hz (assumes seismic platform noise $2x10^{-13}$ m/ \sqrt{Hz})
Pitch and Yaw Noise	10 ⁻¹⁷ rad/√ Hz at 10 Hz (assumes beam centering to 1 mm)
Technical Noise Sources (e.g. electronic noise, thermal noise from bonds)	1/10 of longitudinal thermal noise for each source(since noise terms add in quadrature, each increases total by 0.5%)

*assumes 10⁻³ coupling vert. to long.

[#]except for highest bounce mode peak





Initial LIGO and aLIGO



single pendulum on steel wire

- hydraulic external pre-isolator (HEPI) (one stage of isolation)
- active isolation platform (2 stages of isolation)

 quadruple pendulum (four stages of isolation) with monolithic silica final stage



Initial LIGO and aLIGO

LIGO Advanced LIGO Wire clamps Steel 310 µm diameter steel suspension piano wire wires leading to upper metal suspension stages Ear Penultimate silica mass, 40 kg Silica fibres welded between the ears Silica test mass, 40 kg Silica test mass 10.7kg Ear Catcher and suspensions structure Catcher (some components structure omitted for clarity)



ASD and **RMS**





Seismic Noise in aLIGO

• There are two characteristic features that cause large motions over small frequency bands.

- Earth tides at $\approx 10^{-5}$ Hz (diurnal and semi-diurnal variation)
- Microseismic peak at \approx 0.16 Hz
- These are below the operating bandwidth for Advanced LIGO=> why the problem?
- Root Mean Square (RMS) motion of the test masses is important (m_{rms}) as well as the noise, or spectral density, at a particular frequency (m/ \sqrt{Hz})





Seismic Noise

- Seismic noise limits sensitivity at low frequencies - "seismic wall"
- Typical seismic noise at quiet site at 10 Hz is ~ few x 10^{-10} m/ \sqrt{Hz}
 - many orders of magnitude above target noise level
- Isolation required in vertical direction as well as horizontal due to cross-coupling
- Two-stage internal isolation platform has target noise level of $2 \times 10^{-13} \text{ m/}\sqrt{\text{Hz}}$ at 10 Hz.
- require 4 more stages, i.e. quadruple pendulum, to meet target of 10⁻¹⁹ m/ $\sqrt{\text{Hz}}$



Advantage of a double over single pendulum, same overall length



n

 Quad pendulum transfer function: predicted isolation ≈ 3 x 10⁻⁷ at 10 Hz



Active Seismic Isolation

- Sense motion with seismometers and actuate to drive their output to zero
- Advanced LIGO: Passive isolation above 10Hz, Active isolation from 100mHz-10Hz



BSC Seismic Isolation





Thermal Noise

Thermally excited vibrations of

- suspension pendulum modes
- suspension violin modes
- mirror substrates + coatings —

Use fluctuation-dissipation theorem to estimate magnitude of motion

To minimise:

- use low loss (high quality factor) materials for mirror and final stage of suspension (fused silica)
- use thin, long fibres to reduce effect of losses from bending
- use low loss bonding technique: hydroxide-catalysis bonding
- reduce thermal noise from upper stages by careful design of breakoff/attachment points







Origin of Thermal Noise

- Thermal noise is the statistical movement of particles driven by thermal energy
- The most common example is Brownian motion with $k_B T$ energy per degree of freedom distributed equally over the kinetic and potential energy
- A general theoretical description of fluctuations was developed by Callen, Welton & Greene (1950)
- A strong correlation between the loss in a system (dissipation) and its thermal noise was shown: Fluctuation-Dissipation theorem

• Both thermal energy and the dissipation in a system determine its thermal noise. $\langle V \rangle = 0$

 $S(\omega) = 4k_{B}T\Re[Z(\omega)]$

• example: Johnson Voltage noise

$$\langle V \rangle = 0$$
$$\langle V^2 \rangle = 4k_B TR\Delta f^{V=0}$$





The Fluctuation-Dissipation Theorem

 $k(1+i\phi)$

m

$F(\omega) = -k(1 + i\phi(\omega))x$

 If the resonant modes of the suspended mirror system in a gravitational wave detector are modelled as harmonic oscillators with internal damping, the equation of motion can be written as

$F(\omega) = m\ddot{x} + k(1 + i\phi(\omega))x$

- Using the sHomework: go through displacement to the velocity of a harmonic oscillator $x \propto e^{i} \mathcal{G}, v = i\omega x, \dot{x} = i\omega \mathcal{G}$ this derivation $F(\omega) = i\omega mv - i - (1 + i\phi(\omega))v$
- The improvence Z can then be calculated





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• Using the standard expressions relating the acceleration and displacement to the velocity of a harmonic oscillator $x \propto e^{i\omega t}$, $v = i\omega x$, $\ddot{x} = i\omega v$

$$F(\omega) = i\omega mv - i\frac{k}{\omega}(1 + i\phi(\omega))v$$

• The impedance Z can then be calculated

$$Z(\omega) = \frac{F(\omega)}{v}$$

 $=i\omega m - i\frac{k}{\omega}(1+i\phi(\omega))$





The Fluctuation-Dissipation Theorem

- The impedance Z can be simplified to $Z(\omega) = i \left(\omega m \frac{k}{\omega} \right) + \phi(\omega) \frac{k}{\omega}$
- This expression can be inverted to give the admittance

$$Y(\omega) = \frac{\omega}{i(\omega^2 m - k) + \phi(\omega)k}$$

• Then rationalise the denominator by multiplying by numerator and denominator by the complex conjugate $k\phi(\omega) - i(\omega^2 m - k)$ gives

$$Y(\omega) = \frac{\phi(\omega)k\omega - i\omega(\omega^2 m - k)}{(\omega^2 m - k)^2 + (\phi(\omega)k)^2}$$



 Can now calculate the power spectral density of displacement thermal noise of an oscillator with mass *m* at temperature *T* associated with a resonant mode of frequency w₀

$$S_{x}(\omega) = \frac{4kT}{\omega^{2}} \Re \left[Z^{-1}(\omega) \right] = \frac{4kT}{\omega^{2}} \Re \left[Y(\omega) \right]$$

- where $\omega_0^2 = {k / m}$ and $\phi(\omega)$ is the mechanical dissipation of the oscillator

$$S_{x}(\omega) = \frac{4kT}{\omega} \left[\frac{\phi(\omega)k}{(\omega^{2}m - k)^{2} + (\phi(\omega)k)^{2}} \right]$$

$$S_{x}(\omega) = \frac{4kT}{m\omega} \left[\frac{\phi(\omega)\omega_{0}^{2}}{\left(\omega^{2} - \omega_{0}^{2}\right)^{2} + \phi^{2}(\omega)\omega_{0}^{4}} \right]$$



- Thermal noise of the optical materials and suspension elements manifests as statistical fluctuations of the front surface which is sensed by the laser beam:
 - coatings and suspensions are important areas of R&D.
- The fluctuation of the surface can be produced by two possible mechanisms:
 - Brownian motion of the surface Brownian thermal noise
 - statistical temperature fluctuations within the test mass cause local changes of the surface position (thermal expansion) – Thermoelastic noise
- The origin is the thermal energy that is stored in the atoms of the system so improvements require:
 - ultra pure materials with low mechanical loss
 - lower temperature





Mechanical Loss

- Thermal energy ($k_{\rm B}T$) drives resonant modes
- Width of resonance related to mechanical loss of material, *φ*:
- Mechanical loss is energy dissipation by internal friction in material

$$\frac{\Delta\omega}{\omega_0} = \phi = \frac{1}{Q}$$

- Lower loss material \rightarrow lower off resonance thermal noise
- Use of fused silica (ϕ <10⁻⁷) mirror substrates and suspension fibres in room temperature GWDs







aLIGO Quadruple Suspension

• The input test masses (ITM) and end test masses (ETM) of Advanced LIGO will be suspended via a quadruple pendulum system

• Seismic isolation: use quadruple pendulum with 3 stages of maraging steel blades for horizontal/vertical isolation

• Thermal noise reduction: monolithic fused silica suspension as final stage

• **Control noise minimisation:** use quiet reaction pendulum for global control of test mass position

• Actuation: Coil/magnet actuation at top 3 stages, electrostatic drive at test mass







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aLIGO Monolithic Stage





Local and Global Control

- Local control: damping at top mass, separately for each chain, using BOSEMs (next slide) with 10mm x 10mm NdFe magnets
- Global control (length and angular) at all four levels
 - BOSEMs with 10mm x 10mm NdFe magnets at top mass
 - BOSEMs with 10mm x 10mm SmCo magnets acting between chains at upper intermediate mass
 - AOSEMs** with 2mm x 6mm SmCo magnets acting between chains at penultimate mass
 - Electrostatic drive (ESD) using gold pattern on reaction mass/compensator plate at test mass level

•*BOSEM = Birmingham Optical Sensor/Electromagnetic Actuator •*AOSEM = ALIGO OSEM = modified initial LIGO OSEM





Monolithic Suspension Construction

Hydroxide catalysis bonding of the ears



- - Fibre pulling with CO₂ laser









Laser Pulling





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400µm diameter (800MPa for 10kg mass per fibre) for the remainder of the fibre to lower bounce mode (<10Hz) and increase violin modes (>500Hz)



Monolithic Suspensions

- Monolithic suspensions & signal recycling pioneered in GEO-600 \rightarrow upscaled to aLIGO







aLIGO Suspensions













Suspension Thermal Noise (Dilution)



Profile fused silica fibre



• Extract elastic strain energy and modal frequencies from ANSYS



• Loaded suspension fibres the majority of energy in gravity \Rightarrow dilute loss due to elastic energy

$$D = \frac{E_{\text{total}}}{E_{\text{elastic}}} \approx \frac{k_{\text{gravity}}}{k_{\text{fibre}}} \approx \frac{2L}{\xi} \sqrt{\frac{T}{YI}}$$

• ξ =2 (bending top/bottom)





Fibre Strength





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Fused Silica

• Fused silica is a remarkable material: high strength, low mechanical loss, can be welded and drawn into fibres

$$Y = \frac{\sigma}{\varepsilon}$$

• With 10kg on a 400 μ m diameter, 60 cm long fibre:

$$\sigma = 10 \times 9.81 / \pi \times (400 \times 10^{-6})^2 = 780 MPa$$

• and the extension is

$$\Delta L = \varepsilon \times L = \left[\frac{\sigma}{Y}\right] \times L = \left[\frac{780 \times 10^6}{72 \times 10^9}\right] \times 0.6 = 6mm$$



• The strain is 1% at this load !!!



Fused Silica

Photron 20000 fps Center	FASTCAM-APX R 1/20000 sec frame : -6316	Sm 512 x 256 -00:00:00.31580
6 6 5 5		

- Explosive fracture at 4 GPa (strain of 6%, extension of 11mm for a 20cm long fibre)
- aLIGO fibres operate with a safety factor of ≈ 7





Fibre Geometry

- The fibre geometry defines the dilution of the suspension and can be approximated analytically as $D \approx \frac{2L}{\xi \sqrt{T/YI}}$
- Fibres are not infinitely stiff attachments and thus the analytical dilution values are modified with FE analysis (D_{cvl}=75)





IGR

Fibre Profile

- Fibre stock is 3 mm diameter
 - Ease of welding and handling, stiff connection
- Short 800 μm diameter section
 - Bending takes place close to the attachment point (modal frequencies)
 - Reduces thermoelastic noise
- 400 μm diameter over the majority of the fibre length
 - Violin mode \approx 510Hz, vertical mode \approx 9Hz





Suspension Thermal Noise

Use the following loss terms to model the welds, ear horns and fibres

$$\phi_{\text{bulk}} = 1.2 \times 10^{-11} f^{0.77} \qquad \phi_{\text{TE}}(\omega) = \frac{YT}{\rho C} \left(\alpha - \sigma_o \frac{\beta}{Y}\right)^2 \left(\frac{\omega \tau}{1 + (\omega \tau)^2}\right)$$
$$\phi_{\text{surface}} \approx \frac{8h\phi_s}{d} \qquad \phi_{\text{weld}}(\omega) = 5.8 \times 10^{-7}$$



$$\phi_{total}(\omega) = \frac{1}{D} \left[\frac{E_1}{E_{elastic}} \phi_1(\omega) + \frac{E_2}{E_{elastic}} \phi_2(\omega) + \dots + \frac{E_n}{E_{elastic}} \phi_n(\omega) \right]$$

$$Ak T \left(\sum_{elastic} \omega^2 \phi_1(\omega) - \sum_{elastic} \phi_1(\omega) \right) = 0 \quad \text{for all } b$$

 $\phi_i(\omega) = (\phi_{\text{bulk i}}(\omega) + \phi_{\text{TE i}}(\omega) + \phi_{\text{surface i}}(\omega) + \phi_{\text{weld i}}(\omega))$

$$S_{x}(\omega) = \frac{4k_{B}T}{m\omega} \left(\frac{\omega_{o}^{2} \phi_{total}(\omega)}{\omega_{o}^{4} \phi_{total}^{2}(\omega) + (\omega_{o}^{2} - \omega^{2})^{2}} \right)$$

A.M. Gretarsson et al., Phys. Rev. A, 2000 G. Cagnoli and P.A. Willems, Phys. Rev. B, 2002 P.A. Willems, T020003-00 M.Barton et al., T080091-00-K A. Heptonstall et al., Phys. Lett. A, 354, 2006 A. Heptonstall et al., Class. Quant. Grav, 035013, 2010

- Surface loss: dislocations, un-terminated dangling bonds and micro-cracks on the pristine silica surface
- Thermoelastic loss: heat flow across the fibre due to expansion/contraction leads to dissipation.
- Bulk loss: strained Si-O-Si bonds have two stable minima which can redistribute under thermal fluctuations. 39



Suspension Thermal Noise

- aLIGO utilises thermoelastic cancellation to meet the noise requirement of 10^{-19} m/ \sqrt{Hz} at 10Hz.
- Fused silica has a Young's modulus which increases with temperature





aLIGO Suspension Thermal Noise

• ANSYS predictions of violin mode quality factors are in good agreement (\approx 15%) with ringdown measurements at the LIGO Advanced Systems Test Interferometer (LASTI) at MIT where the first monolithic suspension was installed.



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Violin Ring-Down, 520.925 Hz, Q = 3.6e8 amplitude [arb] 6` 0 time [hours]





Ringdown Time to 1/e

• Violin frequency
$$c = \sqrt{\frac{T}{\mu}}$$
 with tension T and mass per unit length μ
 $T = mg/4 = 98.1N$
 $\mu = \pi \times r^2 \times \rho = \pi \times (200 \times 10^{-6})^2 \times 2202 = 0.000277 kg/m$
 $c = \sqrt{\frac{98.1}{0.000277}} = 595m/s$
 $f = \frac{c}{\lambda} = \frac{c}{2L} = \frac{595}{2 \times 0.6} = 496 Hz$

 $Q = \pi N_{1/e} \Rightarrow 3.6 \times 10^{\circ} = \pi N_{1/e}$ $N_{1/e} = 3.6 \times 10^{8} = 1.1 \times 10^{8}$ $\tau_{1/e} = \frac{1.1 \times 10^{8}}{f} = \frac{1.1 \times 10^{8}}{496} = 2.7 days$ $\tau_{1/e} = \frac{1.1 \times 10^{8}}{f} = \frac{1.1 \times 10^{8}}{496} = 2.7 days$



Useful Equations

r^2 –	$4k_BT\omega_0^2\phi$				
$\lambda_{thermal}$ —	$\overline{\omega m}\left(\left(\omega_0^2-\omega^2\right)^2+\left(\phi\omega_0^2\right)^2\right)$				

Thermal displacement noise

 $S_{x}(\omega) = \frac{4k_{\rm B}T}{\omega^{2}} \Re\left[\frac{1}{Z(\omega)}\right]$

Fluctuation Dissipation theorem

 $\frac{\Delta\omega}{\omega_0} = \phi = \frac{1}{Q}$

Quality factor



Dissipation dilution (in horizontal direction)

$$\phi_{\text{surface}} \approx \frac{8h\phi_s}{d}$$

 $\phi_{\rm TE}(\omega) = \frac{YT}{\rho C} \left(\alpha - \sigma_o \frac{\beta}{Y} \right)^2 \left(\frac{\omega \tau}{1 + (\omega \tau)^2} \right)$

 $\phi_{\text{weld}}(\omega) = 5.8 \times 10^{-7}$

$$\tau_{circular} = \frac{\rho C d^2}{4.32\pi\kappa}$$

Other useful equations

$$f_{pendulum} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f_{bounce} = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$$

m=mass on 1 fibre

$$f_{free_test_mass} = \sqrt{2} f_{bounce}$$

$$I = \pi R^4 / 2$$

 $Y = \frac{\sigma}{\varepsilon}$ $\Delta L = \left[\frac{\sigma}{Y}\right]L$

Dominant fibre loss terms (thermoelastic is minimised with appropriate fibre stress)



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A Simple Model

• Using the previous equations

А	В	С	D	E	F	G	Н		I.	J	K
mass	length	rthin	rend	fibre_area_thin	fibre_area_end	stress_thin	stress_end		Т	Tension	l I
40	0.6	2.00E-04	4.10E-04	1.26E-07	5.28E-07	7.81E+08	1.86E+08		300	98.1	2.21935E-14
С	К	alpha	beta	density	Y	h_phis	tau		1/tau		
740	1.4	3.00E-07	1.50E-04	2202	7.30E+10	6.00E-12	0.014416343	3	69.36572		
f_pendulum	f_bounce	f_violin	dilution	stretch	speed						
0.64354513	8.800875277	496.1804725	1.48E+02	0.006416342	595.416567						
w	f	pendulum	vertical	total		phi_surface	phi_thermoelas	stic		total loss	diluted loss
0.62831853	0.1	2.62724E-16	2.28E-19	2.62724E-16		2.40E-07	8.12487E-10)		2.41E-07	1.63E-09
0.69115038	0.11	2.51849E-16	2.17E-19	2.51849E-16		2.40E-07	8.93721E-10)		2.41E-07	1.63E-09
0.75398224	0.12	2.42555E-16	2.08E-19	2.42555E-16		2.40E-07	9.74		10		
0.81681409	0.13	2.34545E-16	2.00E-19	2.34545E-16		2.40E-07	1.05€	16-	13		
0.87964594	0.14	2.27598E-16	1.93E-19	2.27599E-16		2.40E-07	1.137	1E-1	14 —		i
0.9424778	0.15	2.21546E-16	1.86E-19	2.21547E-16		2.40E-07	1.218	10	15		A I
1.00530965	0.16	2.16259E-16	1.80E-19	2.16259E-16		2.40E-07	1.299	10-	15		
1.0681415	0.17	2.11635E-16	1.75E-19	2.11635E-16		2.40E-07	1.381 🚽	1E-1	16 📃		- \
1.13097336	0.18	2.07593E-16	1.70E-19	2.07593E-16		2.40E-07	1.462 E	4 -	17		
1.19380521	0.19	2.04068E-16	1.66E-19	2.04068E-16		2.40E-07	1.54	10-	1/		
1.25663706	0.2	2.01008E-16	1.61E-19	2.01008E-16		2.40E-07	1.624 .0	1E-1	18 —		
1.31946891	0.21	1.9837E-16	1.58E-19	1.9837E-16		2.40E-07	1.705	10	10		
1.38230077	0.22	1.96121E-16	1.54E-19	1.96121E-16		2.40E-07	1.786	10-	19		
1.44513262	0.23	1.94232E-16	1.51E-19	1.94232E-16		2.40E-07	1.868 🖥	1E-2	20 —		
1.50796447	0.24	1.92681E-16	1.47E-19	1.92681E-16		2.40E-07	1.949 Ĕ	1 - 1	D 4		
1.57079633	0.25	1.91451E-16	1.45E-19	1.91451E-16		2.40E-07	2.030	10-4	21		
1.63362818	0.26	1.90528E-16	1.42E-19	1.90528E-16		2.40E-07	2.111	1E-2	22		
1.69646003	0.27	1.89902E-16	1.39E-19	1.89902E-16		2.40E-07	2.192 S	1 - 1	22		
1.75929189	0.28	1.89566E-16	1.37E-19	1.89566E-16		2.40E-07	2.273 🗖	10-4	23		
1 82212374	0.29	1 89518F-16	1 34F-19	1 89518F-16		2 40F-07	2 354	1F_1	24		
• • Sneet	I / Sheet2 / S										





A Simple Model

• Using the previous equations



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aLIGO Sensitivity







Thin Fibre Work

- Light suspensions (1g-100g) are being developed around the world for radiation pressure/SQL experiments (ICRR, MIT, Glasgow, AEI Hannover, Italy)
- Glasgow has developed triple suspensions for AEI











Welding





Heavy Suspensions

- We have already built up 2 single fibre 40kg tests (using 1200MPa fibre stress)
- Latest system been hanging since Dec 9th 2016
- 4 fibre (160kg) planned for late 2017









Cryogenic Suspensions



- At 120K, use radiative cooling to remove heat
- At T<50K, need to use conduction through fibres



- Cryogenic suspensions operating at 120K/20K offer significant gains in sensitivity
- Silicon has a zero in its thermal expansion at these temperatures





Crystal growth machine (Glasgow)

KAGRA 23kg sapphire payload (R. Kumar)



- Suspensions are a key component to provide:
 - Seismic isolation of the test masses
 - Low thermal noise operation
- aLIGO fused silica installation is a mature technology. This represents a well engineered robust design which is optimised for low thermal noise performance
- Good understanding of loss terms => high Q factors for violin modes
- Thin fibre work underway for suspending 100g/1g optics
- Further lowering thermal noise requires lower mechanical loss and/or low temperature operation
 - Heavy test masses and/or higher fibre stress
 - Cryogenic operation

