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Modern Controls

report from the winter comissioning workshop at Caltech

ISC meeting, 26 March 2014

LIGO-G1400386

Winter commissioning workshop, February 2014 at Caltech

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(+ occasional visitors)

main focus: "modern controls"

Overview

- what is modern control?
- state space modeling
- “optimal” controllers
- and some more topics of the workshop…

Why modern control?

- so far we tweak controllers in frequency domain for best stability and best noise performance
- process might be unintuitive and sometimes requires a lot of trial and error

- often we have many error signals containing information about the system's state
- combining them in a good way is not trivial

→ want automated ways to find "optimal" controllers

What is modern control?

- typically done in state space
- start with good model of the system (usually linear)
- define goal and find best controller w.r.t. this goal
- final controller can be translated to transfer functions for implementation

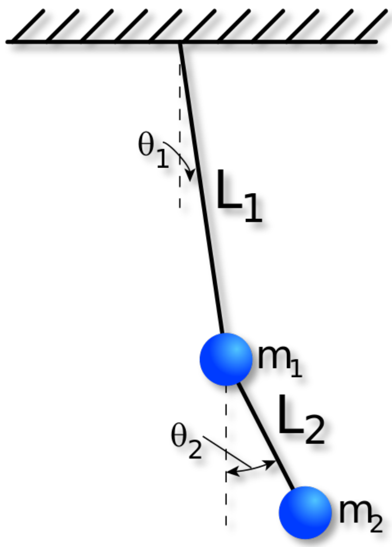
State space

- describe system by set of linear first-order differential equations:
  - system dynamics: $\dot{\vec{x}} = \mathbf{A}\vec{x} + \mathbf{B}\vec{u}$
  - observation: $\vec{y} = \mathbf{C}\vec{x} + \mathbf{D}\vec{u}$
  - *x*: internal states, *u*: external inputs, *y*: observables
- great tutorial by Gabriele: LIGO-G1400102

Observability

$$\dot{\vec{x}} = \mathbf{A}\vec{x} + \mathbf{B}\vec{u}$$
$$\vec{y} = \mathbf{C}\vec{x} + \mathbf{D}\vec{u}$$

- definition:
  "A system is observable if and only if the full state *x* at a given time can be reconstructed from a finite record of the measured outputs *y*."
- can be reformulated as algebraic criterion
- system can be observable even if matrix **C** has rank smaller than dim(*x*) (e.g. only one observable to observe multiple states)

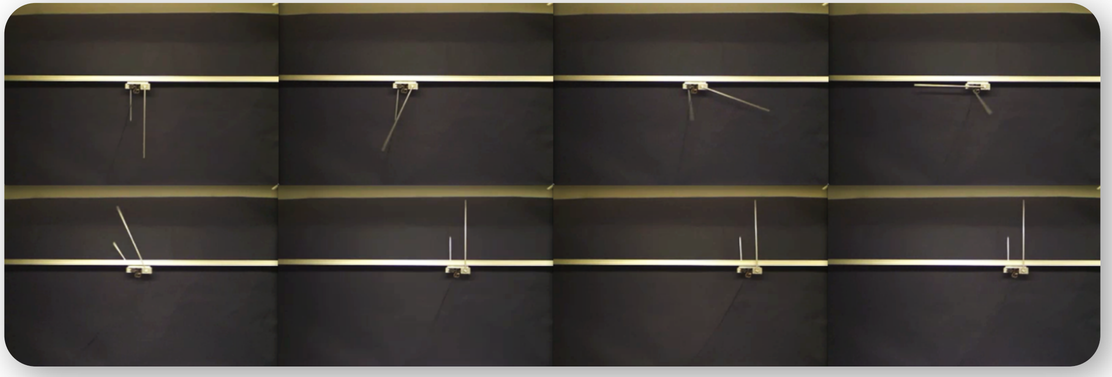


http://en.wikipedia.org/wiki/File:Double-Pendulum.svg

Controllability

$$\dot{\vec{x}} = \mathbf{A}\vec{x} + \mathbf{B}\vec{u}$$
$$\vec{y} = \mathbf{C}\vec{x} + \mathbf{D}\vec{u}$$

- definition:
  "A system is controllable if and only if every given state *x* can be reached in finite time by applying an input *u*."
- the dynamics of a controllable system can (in principle) be changed arbitrarily and the corresponding controller can be calculated algebraically



http://en.wikipedia.org/wiki/File:Smooth_nonlinear_trajectory_planning_control_on_a_dual_pendula_system.png

“Optimal” control

- what is the optimal controller/observer?
- need to formulate measure of goodness
  → cost function
- e.g.: want to keep errors and feedbacks small

LQR – linear quadratic regulator

- cost function: $$J = \int_0^\infty \left(\vec{x}^T \mathbf{Q} \vec{x} + \vec{u}^T \mathbf{R} \vec{u} \right) \mathrm{d}\, t$$
- quadratic in *x* and *u*, with weighting matrices **Q** and **R**
- can be solved analytically to find controller that minimizes cost *J* (Matlab functions available)
- the art is to find good weights
- resulting controller is always stable but might not be robust to uncertainties of the plant model

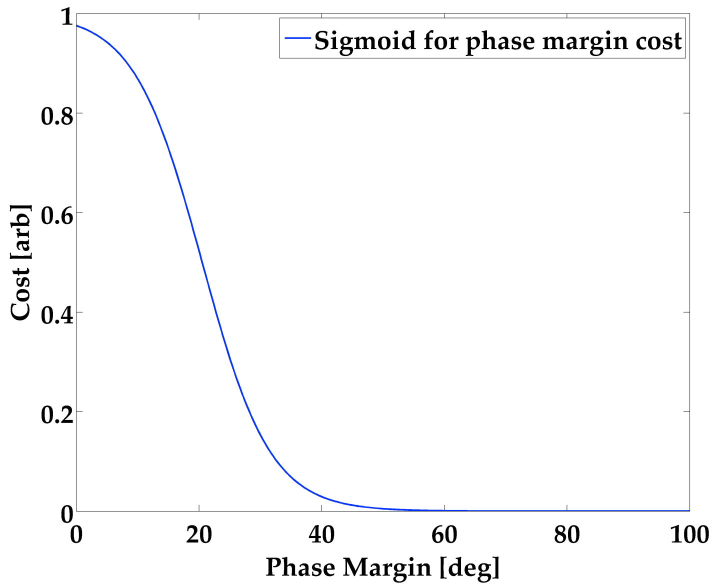
H-infinity control

- cost function: $$J = \left\| T_{\text{dist}} \right\|_\infty = \sup_\omega \left(T_{\text{dist}}(\mathrm{i}\omega) \right)$$
- T_{dist} is the transfer function from external disturbances to the error point (matrix for MIMO systems)
- iterative algorithms exist to solve this approximately
- system uncertainties can be included to find robust controller that is stable within the uncertainties

Generalized cost functions

- incorporate more straight-forward performance criteria into cost function:
  - noise suppression
  - impression of unwanted control noise
  - gain/phase margin

- optimize generic controller (e.g. described by poles + zeros) w.r.t. this cost function



Jenne Driggers, LIGO-G1400091

Optimization with generalized cost

- typically not analytically solvable
- might be high-dimensional parameter space with tricky cost distribution (e.g. local minima)
- possible approaches:
  - Newton's method
  - gradient descent
  - particle swarm optimization
  - ...
- good initial guess might help (e.g. existing controller)

Limitations

- assumes unchanging linear plant models
- can't do anything we couldn't already do with transfer functions
- model must be known very well, else not optimal or even unstable
- probably no canonical cost functions, so hand-tuning is still necessary

“More modern” controls

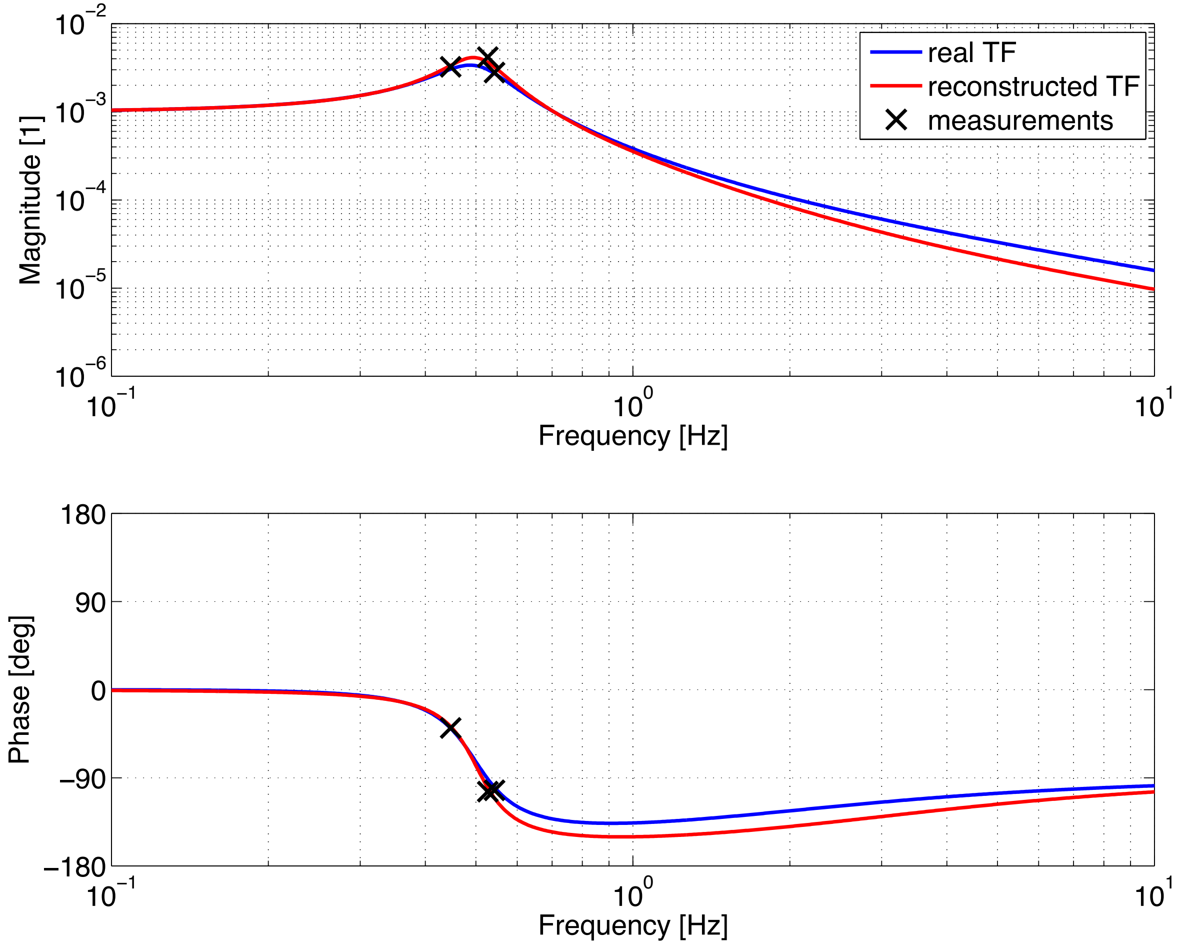
- Kalman filtering (time-domain optimal state estimator)
- Wiener filtering (optimal static noise cancellation with feedforward)
- adaptive filtering (gain control, adaptive feedforward)
- optimal transfer function measurements
- machine learning, e.g. for learning about an unknown plant (reinforcement learning, neural networks, ...)

Summary

- want to simplify/automate controller design
- modern control theory is based on state space models
- good plant models must be known
- controllers can be optimized w.r.t. some cost function
- cost functions still need to be designed
- adaption / learning might help

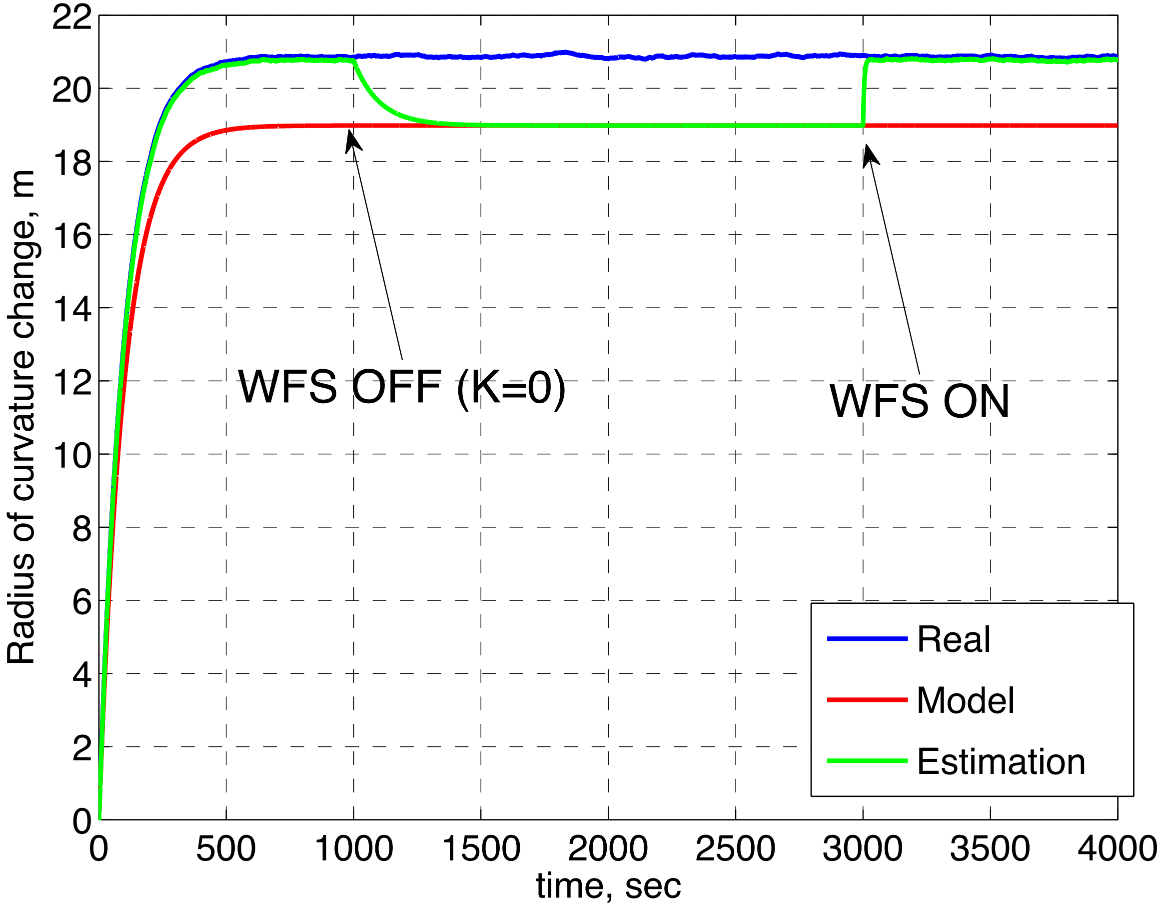
Backup slides:

Optimal TF measurement

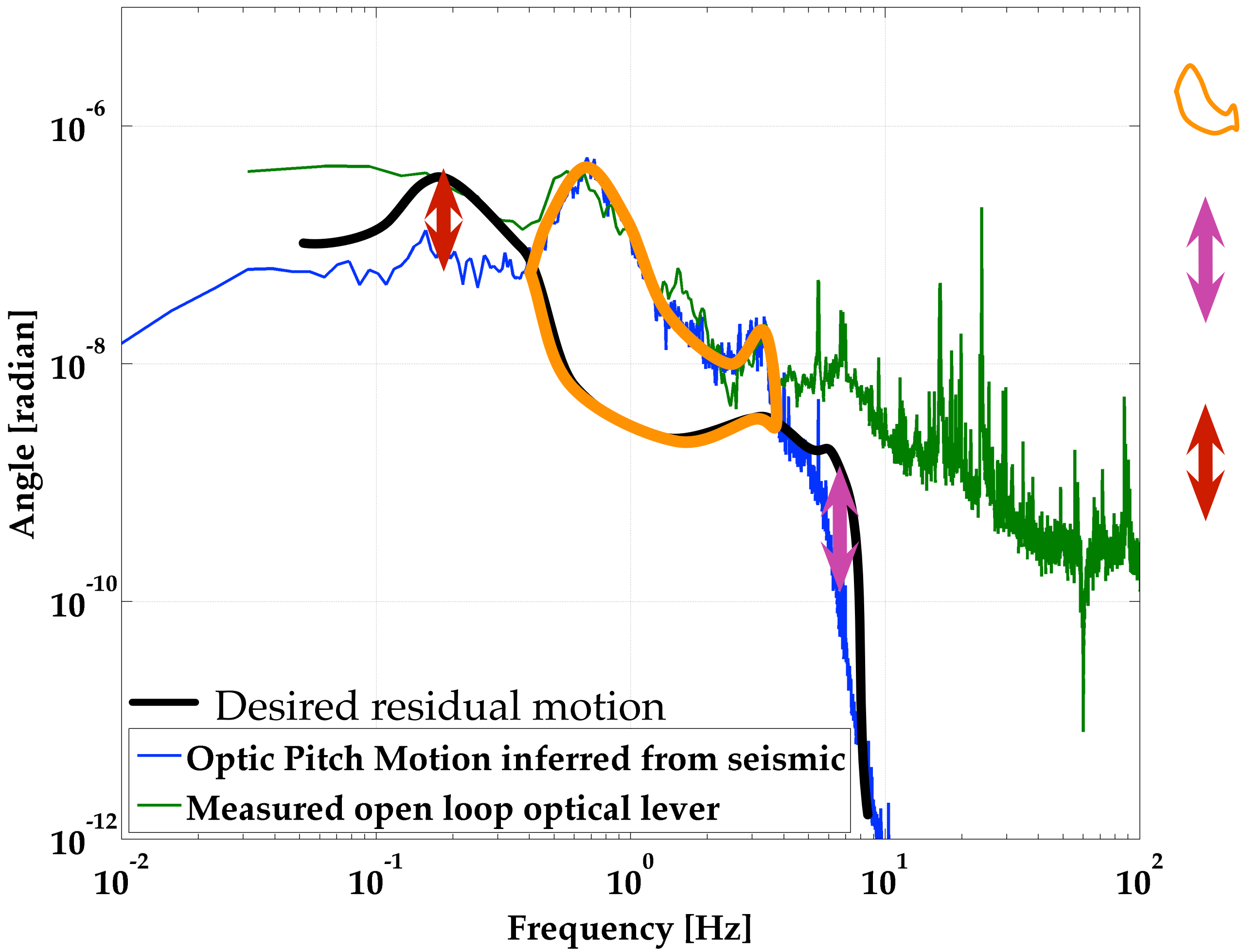


Kalman filter

Lock loss



Generalized cost function



Maximize area

Make ratio ~1

Make ratio ~1

Phase margin > 30 degrees

Gain margin $\gtrsim$ 15 dB

Wiener filtering