Basic controls tutorial

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Michèle Heurs

The goals of this tutorial are...

- ... to introduce some basic controls nomenclature
  - block diagrams
  - feedback vs. feed-forward control
  - open loop vs. closed loop
  - different types of plots
  - stability

- ... to provide a *hands-on* controls approach
  - transfer functions: definition and examples
  - characterisation tools: specifications, noise spectra, transfer functions, stability evaluation
  - practical skill set: prototyping, experimental testing, iteration,...

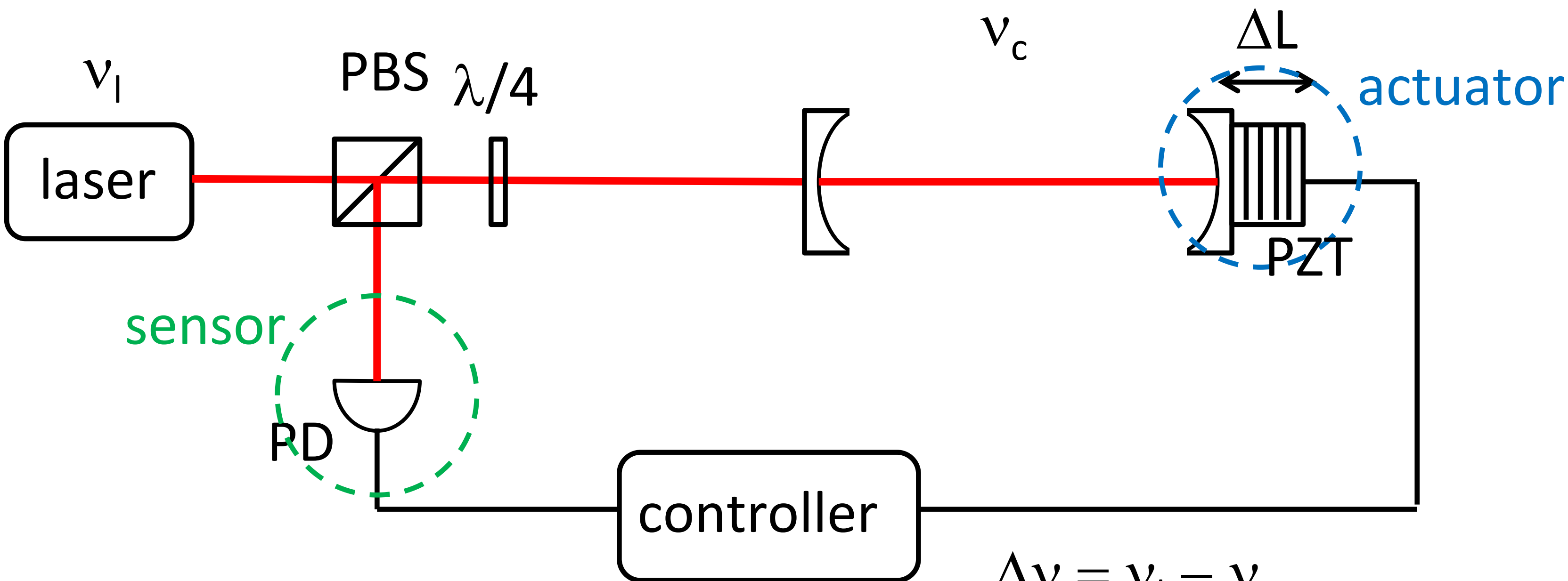
Examples of control in physics

- Astronomy: Tracking telescopes, adaptive optics, satellite ranging, ...
- Quantum Optics: diode laser temperature control, length control of optical resonators, MOTs, ...
- Gravitational Wave Detection: optics auto-alignment, interferometer control, recycling techniques,...
- ...

Example in GWD: optical resonator

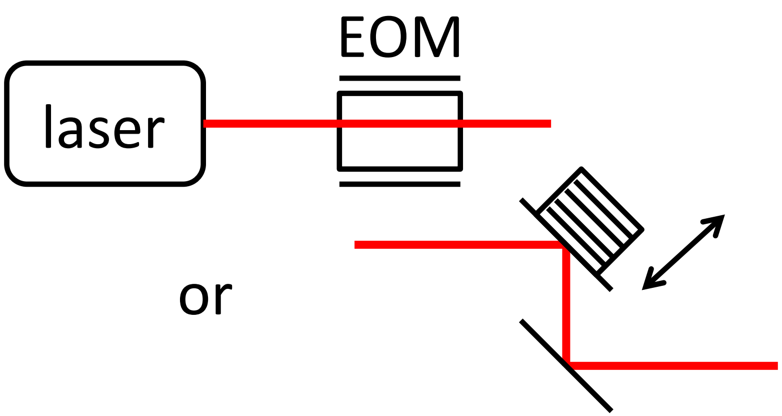
- Laser frequency stabilisation (for metrology)
  - Lock laser to cavity OR lock cavity to laser (depending on what is more stable!)
  - Consider cavity locking (e.g. for GWD)
    → have to keep cavity length „matching“ to laser output frequency
    What is needed?
    - *actuator* (e.g. PZT)
    - *sensor* (e.g. photodetector)
    - *feedback* (controller)

„physics“ example: optical resonator

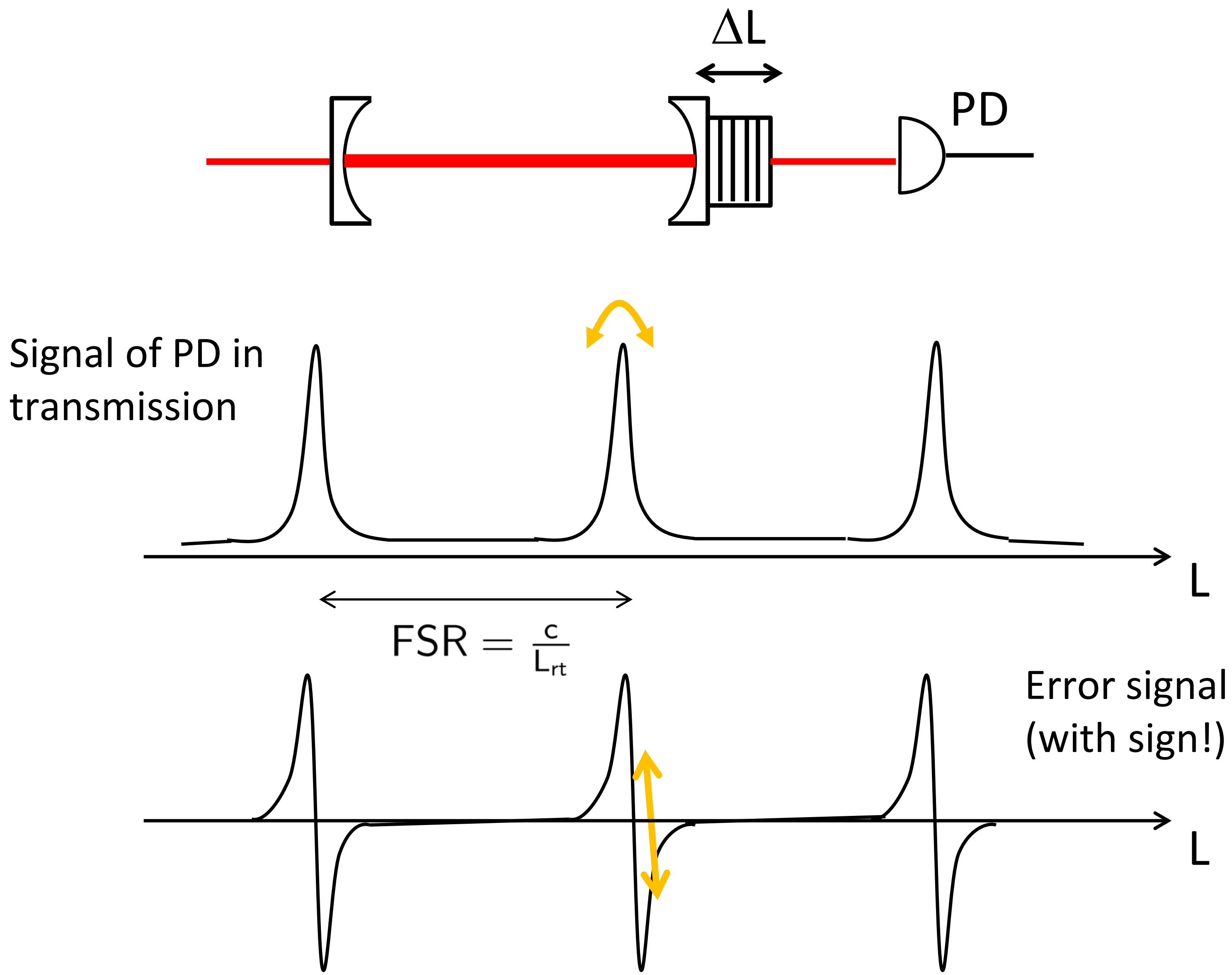


$$\Delta\nu = \nu_l - \nu_c$$

must be small!

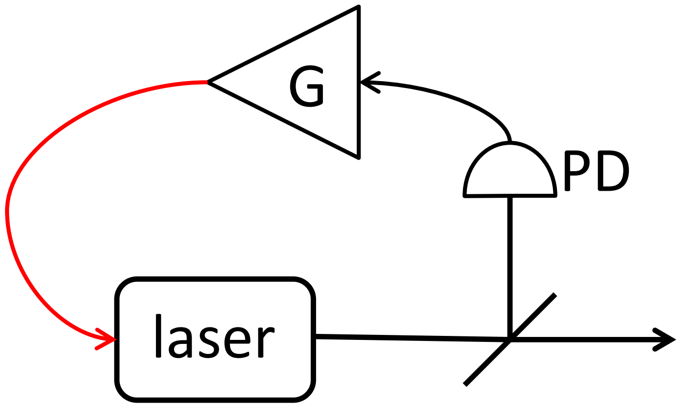


possibilities to imprint phase modulation sidebands on laser light


ΔL
PD
Signal of PD in transmission
L
FSR = c / L_rt
Error signal (with sign!)
L

Feedback vs. feed-forward
(on the example of laser intensity stabilisation)

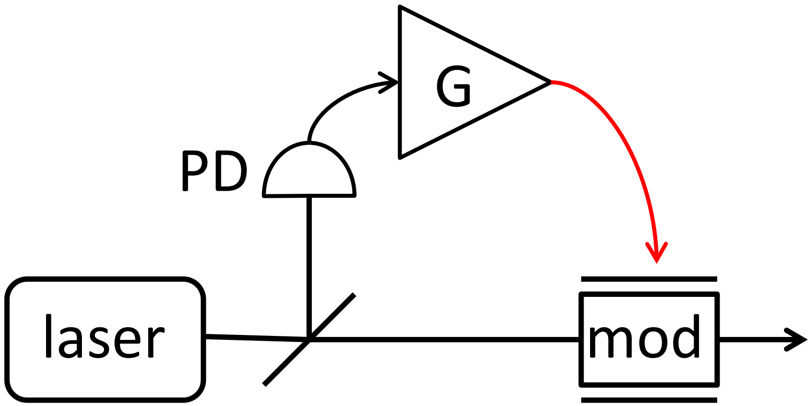
Often: feedback



e.g. laser intensity stabilisation,
laser frequency stabilisation,...

Effect of feedback is measured with detector in the loop

Sometimes: feed-forward!

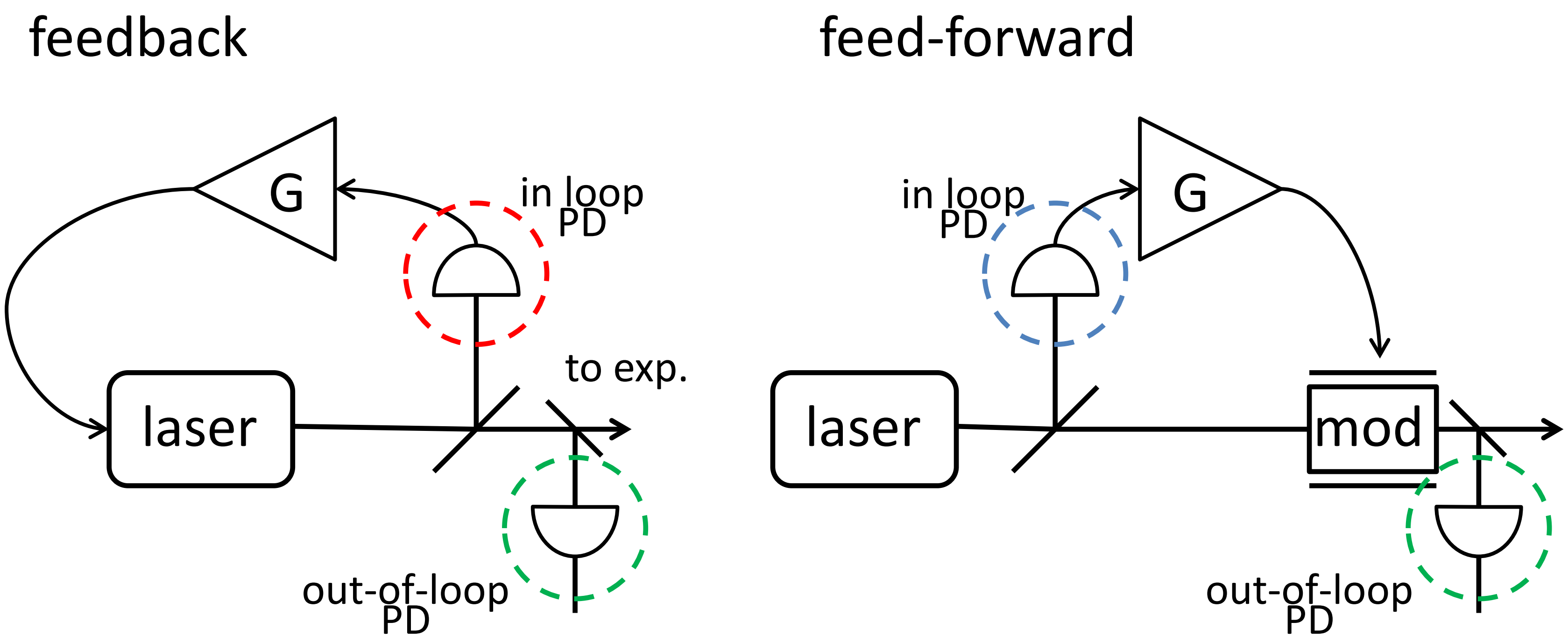


here: laser intensity stabilisation, too.
But also e.g. seismic vibration cancellation,...

Effect of feed-forward is NOT measured with detector in the loop

In loop vs. out of loop

(again, on the example of laser intensity stabilisation)



The in-loop result only tells you if your controller is working to specs. Only the out-of-loop result tells you if you‘ve been successful in suppressing the noise in the light going to the experiment! They can be (and often are) vastly different (with OOL suppression being a lot less than IL).

Some assumptions we will make

- frequency domain design methods only
- assume linearity (i.e. small signals only), but be aware of the dominant nonlinearities like range limits, saturation etc.

System of interest: the plant
(the thing we want to stabilise)

Ideal world: plant output = constant

but: external disturbances & intrinsic noise → fluctuations

- measure output with sensor
- error signal = reference level – plant output
  → goes into controller (compensator)
- controller output drives actuator
- Change reference signal → modify output: control input

- Goals: disturbance suppression and output control

Block diagram

each block is described by a DE for a(t)

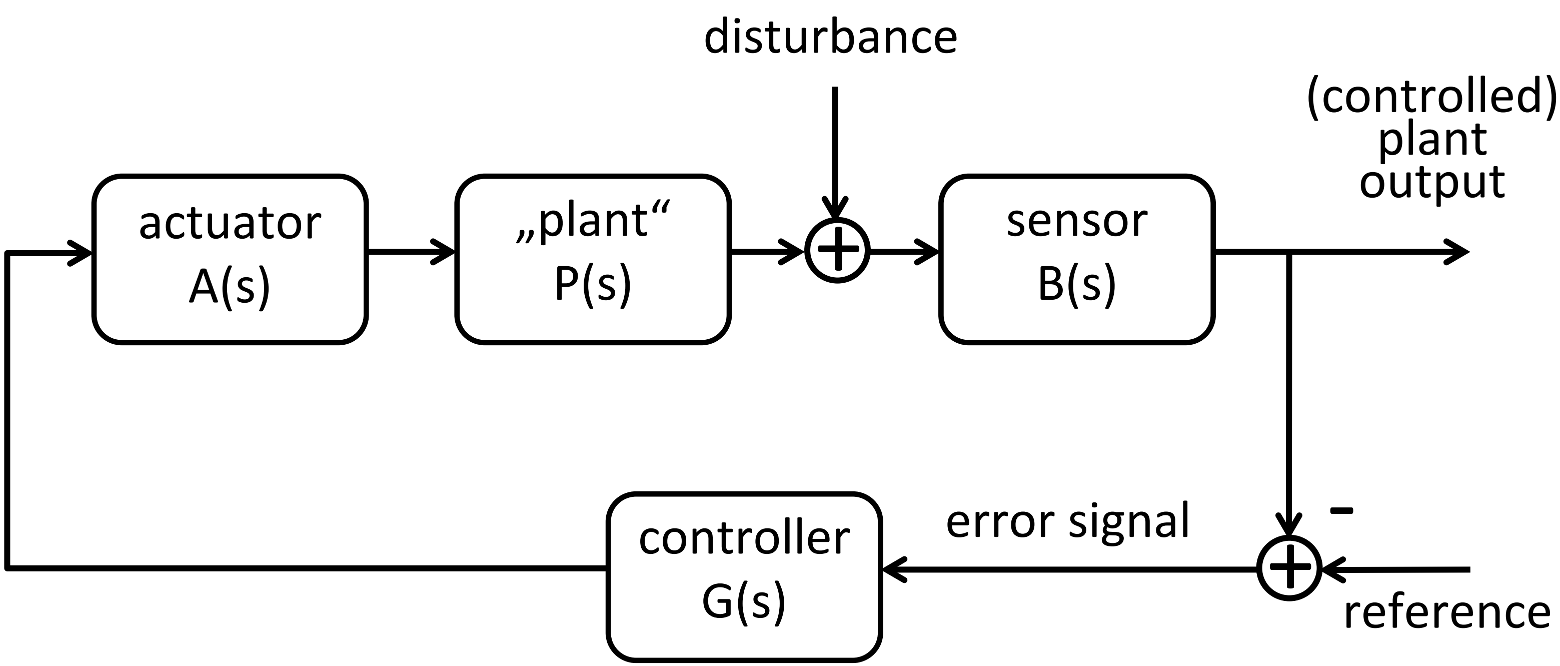
use Laplace transform A(s) with

oscillatory behaviour

damping

$s = \sigma + i\omega$

s: Laplace variable, σ: damping, $\omega = 2\pi f$: angular frequency, f: Fourier frequency



Laplace transform

- Definition of Laplace transform:
  there
  $$F(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}dt$$
  and back again
  $$f(t) = L^{-1}[F(s)] = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} F(s)e^{st}ds$$

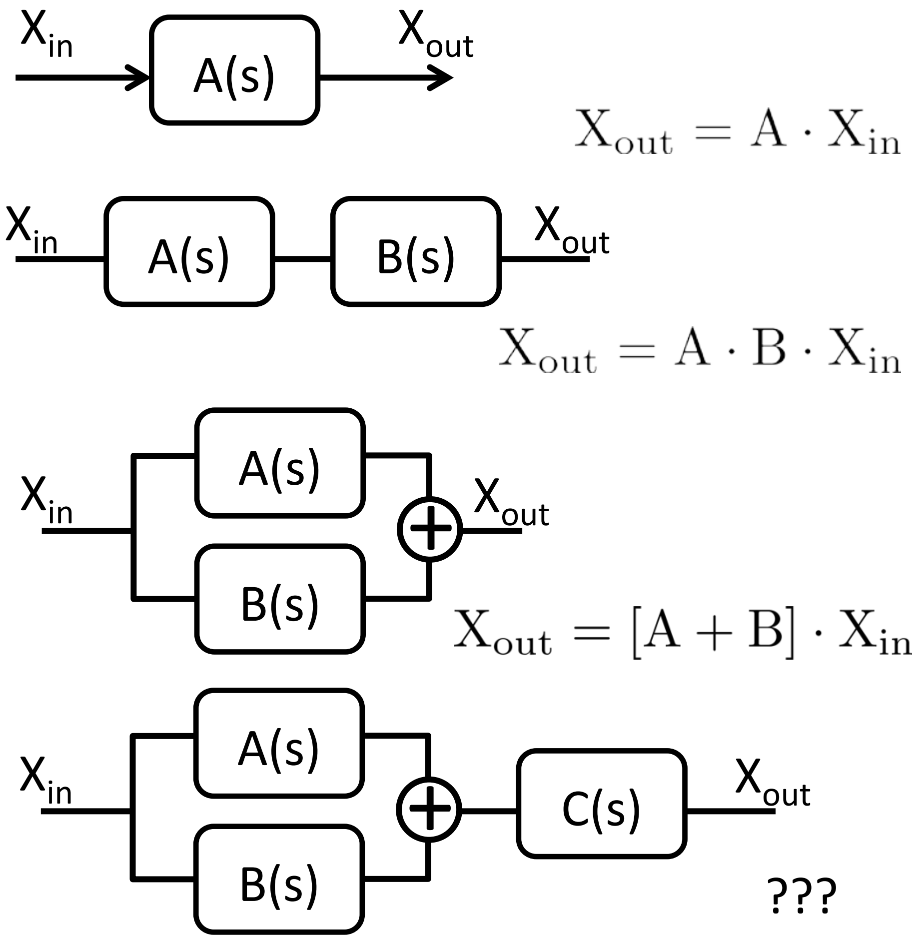
- Example:
  $$f(t) = e^{-at}$$
  $$F(s) = \int_0^\infty e^{-at}e^{-st}dt = \int_0^\infty e^{-(s+a)t}dt$$
  $$F(s) = -\frac{1}{s+a}e^{-(s+a)t}|_0^\infty$$
  $$\boxed{F(s) = \frac{1}{s+a}}$$

Laplace transform

$$\mathrm{TF} = \frac{\mathrm{X_{out}}}{\mathrm{X_{in}}}$$

- Go to the (complex) frequency domain and use Laplace transforms

  → operations become simple multiplications

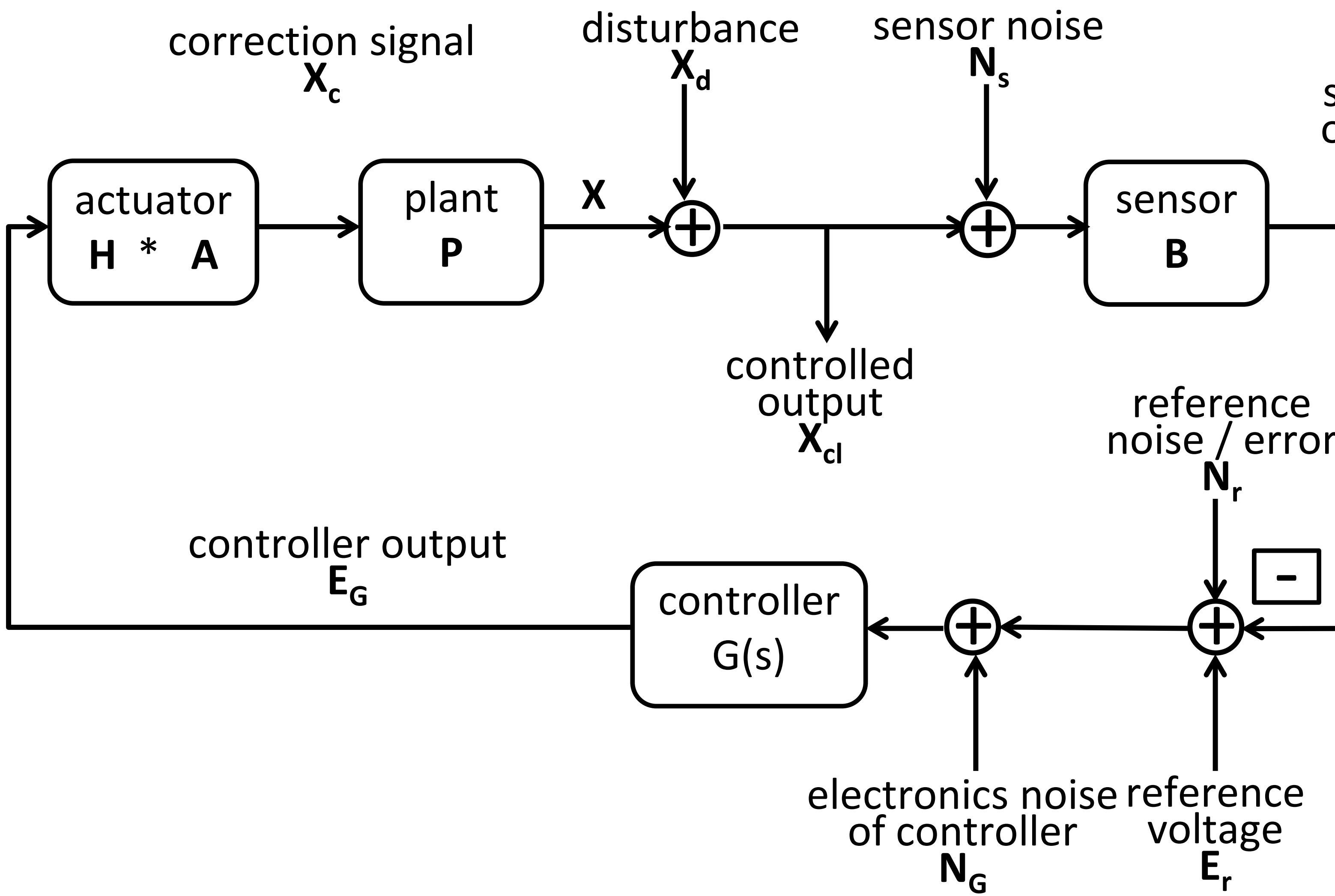


$$\mathrm{X_{out}} = \mathrm{A} \cdot \mathrm{X_{in}}$$

$$\mathrm{X_{out}} = \mathrm{A} \cdot \mathrm{B} \cdot \mathrm{X_{in}}$$

$$\mathrm{X_{out}} = [\mathrm{A} + \mathrm{B}] \cdot \mathrm{X_{in}}$$

Block diagram with inputs, outputs and noise



The maths to the above block diagram

I) $E_G = G \cdot (N_G + N_r + E_r - E_s)$
II) $X_c = A \cdot H \cdot E_G$
III) $X_{cl} = X_d + P \cdot X_c$

Eliminate E_s and E_G:
insert $E_s = B \cdot (N_s + X_{cl})$
in I

I′) $E_G = G \cdot (N_G + N_r + E_r - B(N_s + X_{cl}))$ in II

II′) $X_c = H \cdot A \cdot G \cdot (N_G + N_r + E_r - B(N_s + X_{cl}))$
$= HAG \cdot (N_G + N_r + E_r) - HAGB \cdot (N_s + X_{cl})$ in III

III′) $X_{cl} = P \cdot HAG(N_G + N_r + E_r - B(N_s + X_{cl})) + X_d$
$X_{cl}(1 + HAPGB) = HAPGB \cdot (1/B(N_G + N_r + E_r) - N_s) + X_d$

with HAPGB = L : $X_{cl} = \frac{L}{1+L} \frac{E_r + N_r + N_G}{B} - N_s \frac{L}{1+L} + X_d \frac{1}{1+L}$

open-loop transfer function: $L = HAPGB$

closed –loop transfer function: $\frac{L}{1+L} = \frac{B \cdot X_{cl}}{E_r}$ (disregarding sensor noise N_s)

For $|L| \gg 1$ (high gain limit): $X_{cl} \approx \frac{1}{B}(E_r + N_r + N_G) - N_s + \frac{X_d}{L}$

…and that means

- Closing the feedback control loop causes X_{cl} to track the reference scaled by the sensor gain (E_r/B) to accuracy $X_{fr}/|L|$+ noise in the system, and it and suppresses the free-running plant output to $X_{fr}/|L|$.
  This is very useful if large open loop gain L and low noise can be achieved.
- High performance = high open loop gain

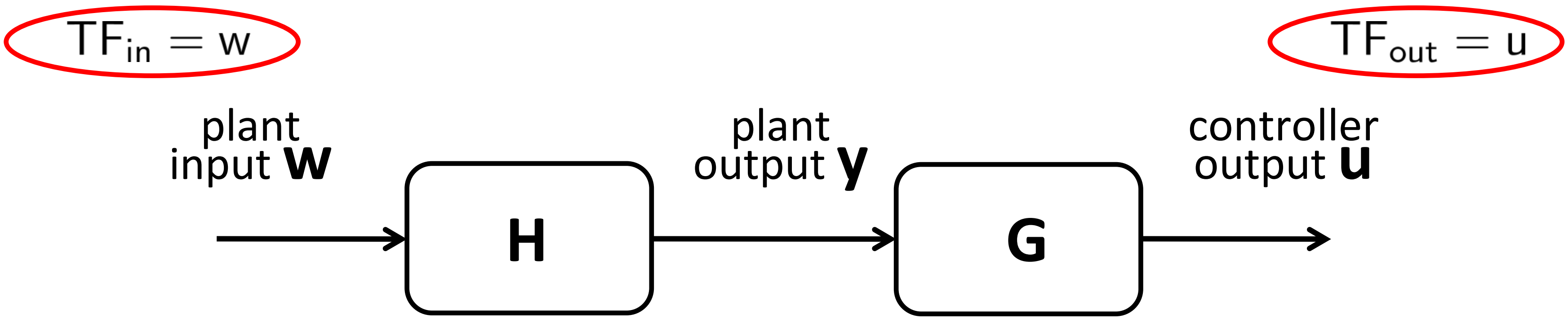
Simplified feedback control system (FCS) block diagram

- Disregard all noise contributions
- Assume high gain limit $|L| >> 1$

What we‘ll do now:

- calculation of open loop TF and closed loop TF
- calculation of
    - „disturbance transfer function“
    - „disturbance suppression function“

Open loop transfer function



$$
\begin{aligned}
u &= G \cdot y \\
\text{and} \quad y &= H \cdot w \\
\rightarrow u &= HGw
\end{aligned}
$$

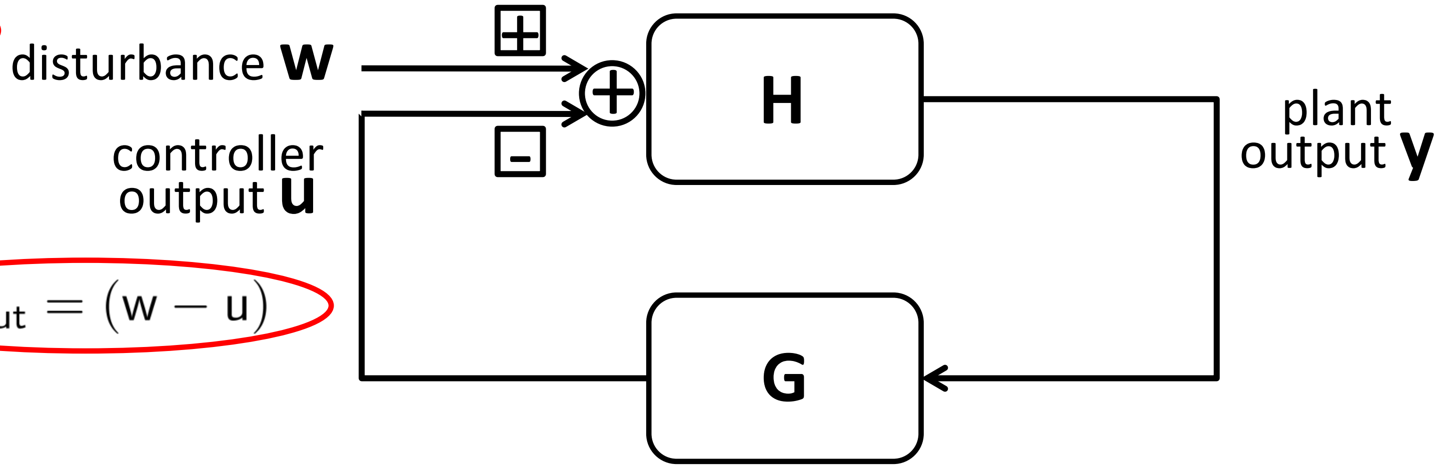
$$\frac{u}{w} = HG$$

open loop transfer function

Closed loop transfer function

$TF_{in} = w$

$TF_{out} = (w - u)$

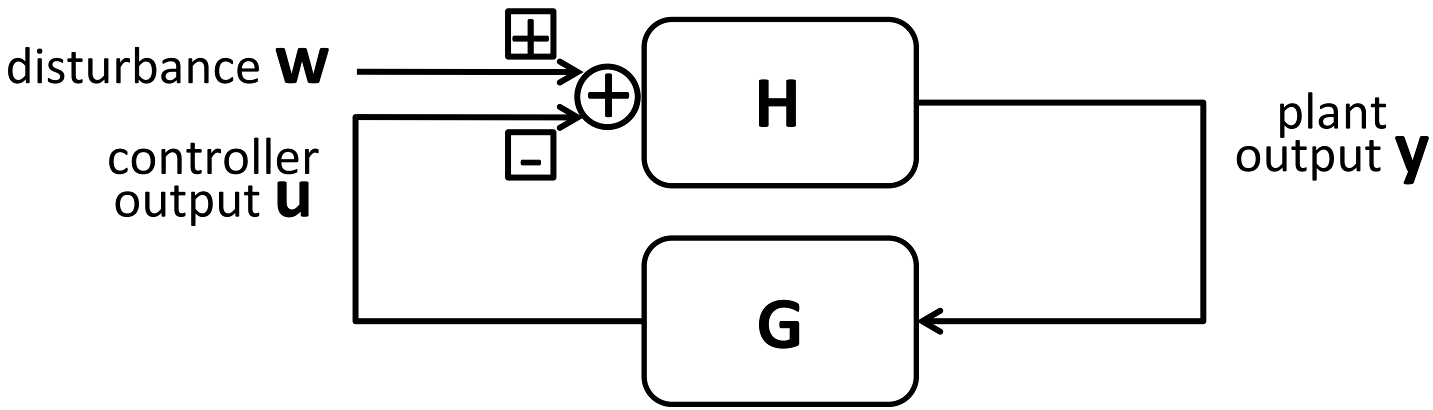


$$(I) \quad y = H \cdot (w - u) \quad \text{and} \quad (II) \quad u = G \cdot y$$

$$
\begin{aligned}
(I) \text{ in } (II) \quad \rightarrow u &= GH\,(w-u) \\
&= HGw - HGu \\
\rightarrow \frac{u}{w} &= \frac{HG}{1+HG} \\
\frac{w-u}{w} &= \frac{1}{1+HG}
\end{aligned}
$$

closed loop transfer function

Two different kinds of CL TFs!



disturbance transfer function:

$$TF_{in} = w \quad \text{and} \quad TF_{out1} = u$$

$$\begin{aligned}
(I)\quad y &= H \cdot (w - u) \\
(II)\quad u &= G \cdot y \\
(I)\text{ in }(II) \quad \rightarrow u &= GH\,(w - u) \\
&= HGw - HGu
\end{aligned}$$

$$\rightarrow \frac{u}{w} = TF_1 = \frac{HG}{1 + HG}$$

disturbance suppression function:

$$TF_{in} = w \quad \text{and} \quad TF_{out2} = (w - u)$$

$$TF_2 = 1 - TF_1 = 1 - \frac{HG}{1 + HG}$$

$$\rightarrow \frac{w - u}{w} = TF_2 = \frac{1}{1 + HG}$$

Displaying transfer functions

Multiple options: Bode plot, Nyquist plot, Pole-zero-plot, ...

We‘ll focus on *Bode plots* here, to display the frequency response of linear time-invariant systems

A Bode plot (usually) consists of two graphs:

Bode plot

- Bode *magnitude plot* = frequency response gain (in dB)

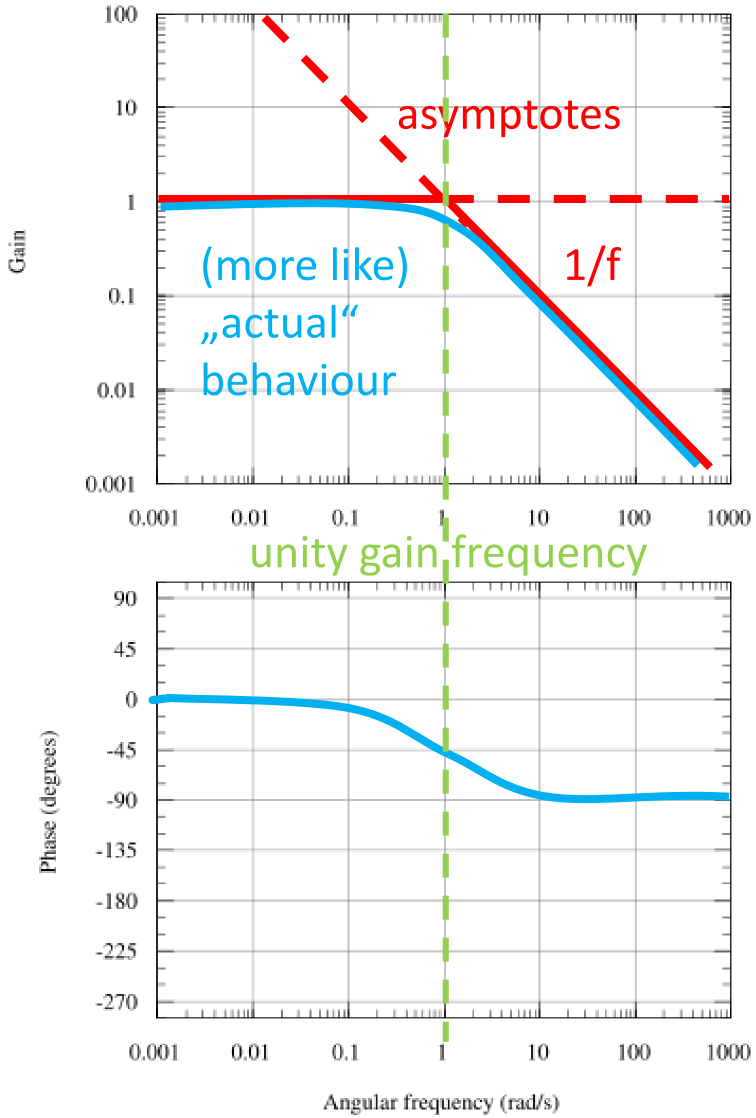
$$A = -20 \cdot \log|H(i\omega)|$$

- Bode *phase plot*= frequency response phase shift (in deg)

$$\phi = -\arctan(\frac{\omega}{\omega_c})$$

The two are connected by dispersion relations!
(Kramers-Kronig)

Bode plot of a lowpass



$$H(i\omega) = \frac{1}{1 + i\frac{\omega}{\omega_c}}$$

Magnitude (in dB):

$$A = -20 \cdot \log_{10}|H(i\omega)|$$

Unity gain:

$$|H(s)| \cdot |G(s)| = 1$$

Phase (in deg):

$$\phi = -\arctan(\frac{\omega}{\omega_c})$$

Stability

- *Definition*: A system is stable if it settles following a disturbance.

- *For intuitive understanding*: look at behaviour of feedback signal at
  - frequencies well below UG
  - frequencies around UG
  - frequencies far above UG

Stability

- Instability leads to oscillation, possibly saturation!
- Dynamics of systems can be described by DEs:

$$a_0 \frac{d^n x(t)}{dt^n} + a_1 \frac{d^{n-1} x(t)}{dt^{n-1}} + \ldots + a_n x(t) = f(t)$$

•$x(t)$: parameter of interest
•a_i : coefficients (constant)
•$f(t)$: driving force (inhom. DE)

General behaviour of x(t): solve DE with f(t)=0

$$x_n(t) = \sum_{i=1}^{n} c_i e^{r_i t}$$

r_i : roots of characteristic equation associated with DE

$$P_n(r) = \sum_{k=0}^{n} a_k r^k = 0$$

$P_n(r)$: characteristic polynomial of DE

- System is *stable* if x(t) is bounded for $t \rightarrow \infty$

Poles and zeros (briefly)

Pole

- low-pass filter behaviour
- for real poles: cut-off freq. of system is position of pole
- for complex poles: cut-off freq. of system is length of vector (absolute value)
- *roll-off*: 20dB/decade per pole
- poles cause *phase lag*
  (-45° @ position of pole,
  -90° @ frequencies far above)

Zero

- high-pass filter behaviour
- for real zeros: cut-off freq. of system is position of zero
- for complex zeros: cut-off freq. of system is again length of vector (absolute value)
- *roll-up*: 20dB/decade per zero
- zeros cause *phase lead*
  (+45° @ position of zero,
  +90° @ frequencies far above)

Stability and transfer functions

- TFs are usually fractions of polynomials $\frac{P_m(s)}{P_n(s)}$
  - zeros of numerator = zeros of TF
  - zeros of denominator: poles of TF

# of poles must be larger than # of zeros!

(i.e. TF $\rightarrow$ 0 for iω large)

Loop gain and stability

- |required noise reduction| ~ gain in loop (including the plant!)

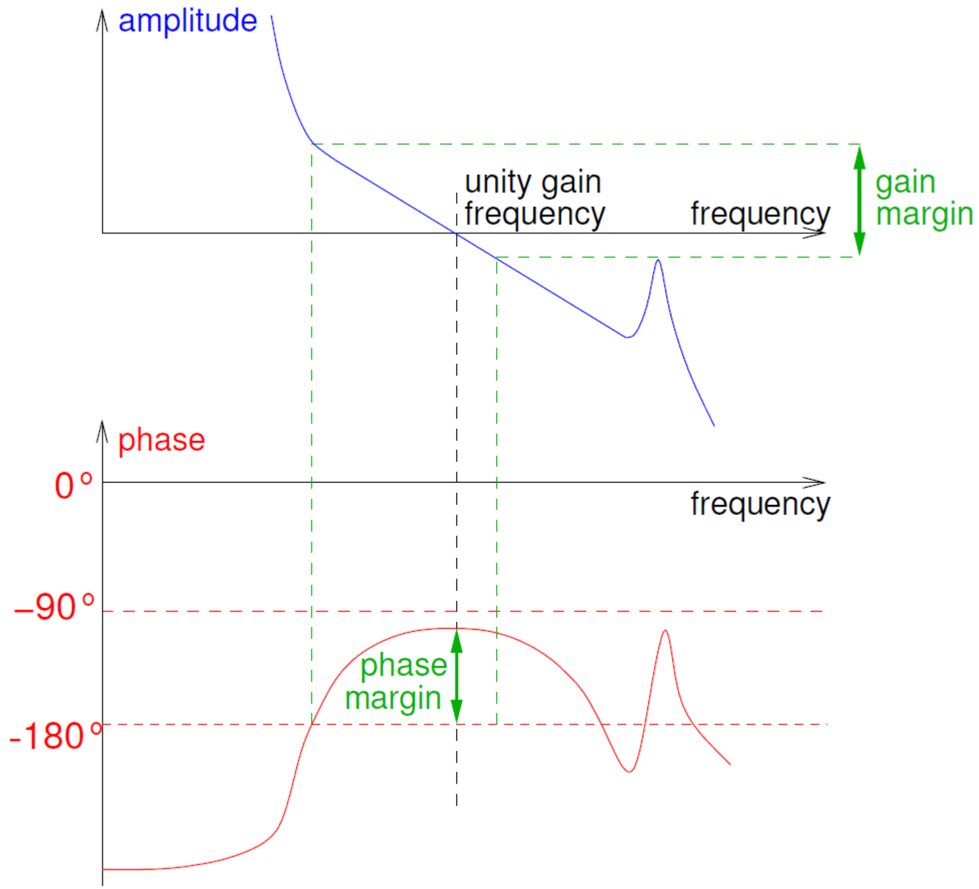
- *unity gain frequency* ω_{UG} : $\left| G\left(\omega_{UG}\right) \cdot H\left(\omega_{UG}\right) \right| = 1$

- *phase at UG:* $\phi\left(GH\right)|_{UG} > -180^{\circ}$

  (otherwise system oscillates – positive feedback)

- *Phase margin:* "stay away from -180°"

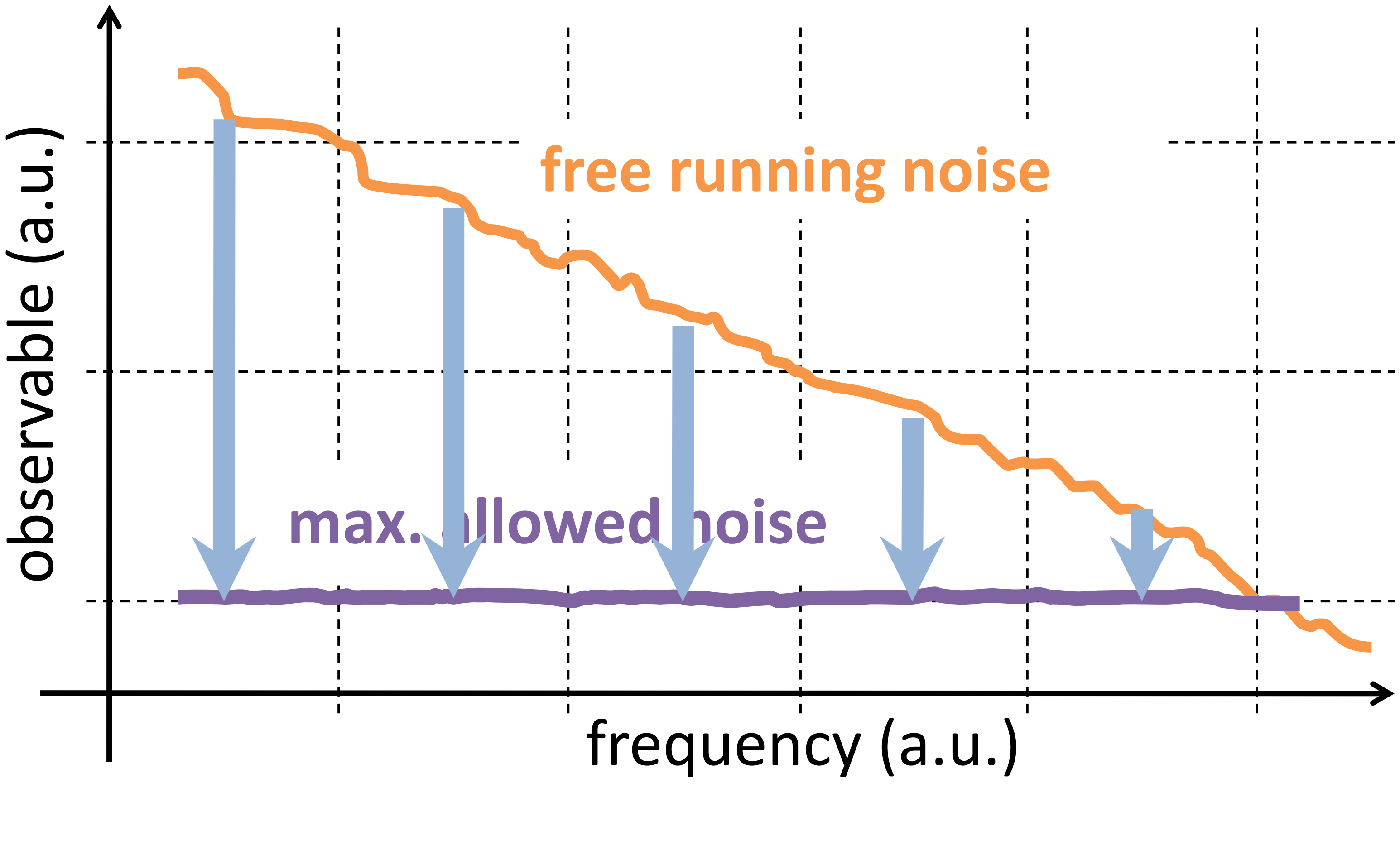
Phase and gain margin



Stabilisation: How do we do it?

- What is the stabilisation task?
  - What system?
  - Which sensors and actuators?
  - Free-running noise? => measure noise spectrum
  - Specifications to be achieved? => plot in noise spectrum
  - TF of plant? => measure (OL if possible)

  => the „difference“ (in dB) is what we need to provide by our controller!


free running noise
max. allowed noise
observable (a.u.)
frequency (a.u.)