Impedance sensing and control

David Rabeling
Nikhef, Amsterdam

AEI sensing and control meeting
14-12-2010

Overview

- Cavity impedance condition
- Impedance readout
- Experimental demo of IM control
- Applications of IM control
- Impedance control for (Ad)Virgo

Impedance matching of resonant optical systems

Impedance matching of resonant optical systems

- Impedance matching for optically resonant systems is a kin to that in electrical systems:
  - In electrical systems impedance matching provides optimum voltage, power, or current transfer.
  - In optical systems impedance matching provides the optimum electric field transfer.

Impedance matching of resonant optical systems

- Impedance matching for optically resonant systems is a kin to that in electrical systems:
  - In electrical systems impedance matching provides optimum voltage, power, or current transfer.
  - In optical systems impedance matching provides the optimum electric field transfer.
- Interrogation and control of the impedance matching condition offers an alternative active feedback control technique that has applications within GW interferometry, absorption spectroscopy, and quantum optics experiments.

The important parameters that active impedance matching optimises are:

The important parameters that active impedance matching optimises are:

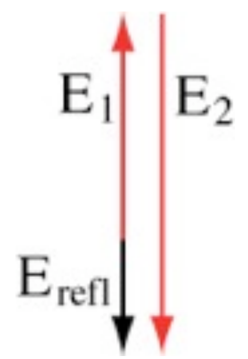
- optimum electric field transfer through the optical system.

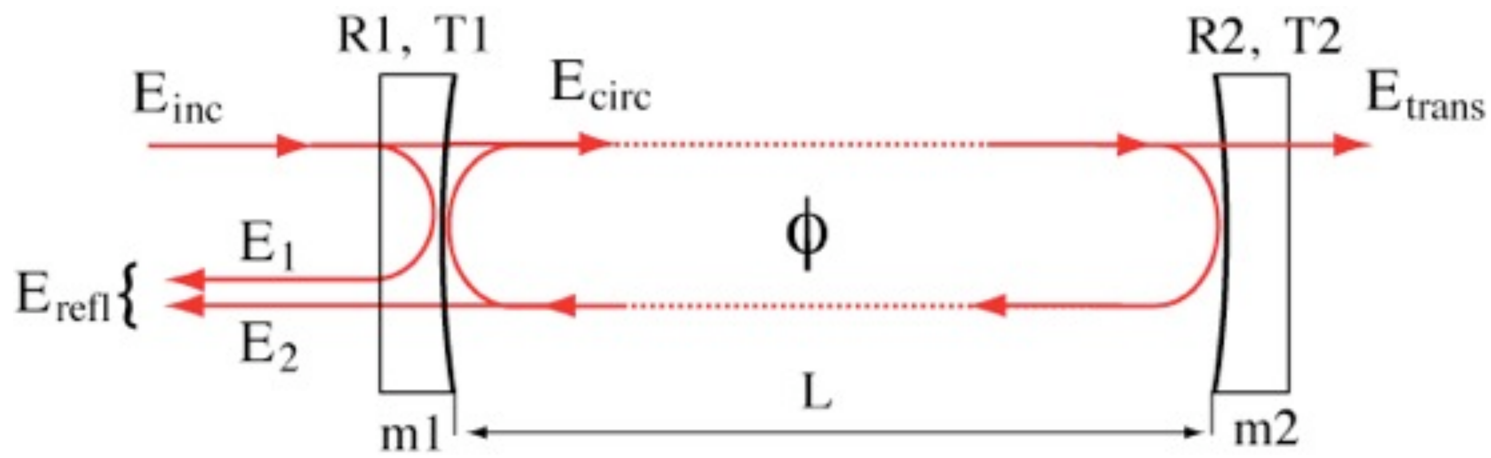
The important parameters that active impedance matching optimises are:

- optimum electric field transfer through the optical system.
- ensures that the reflected electric field is zero and the circulating field is maximised (assuming a back mirror who's reflectivity is dominated by loss).
- by optimising the circulating power, the technique also optimises the signal sensitivity of the system.

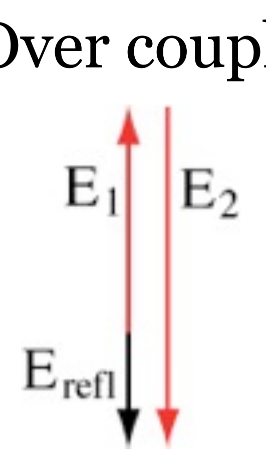
Cavity impedance

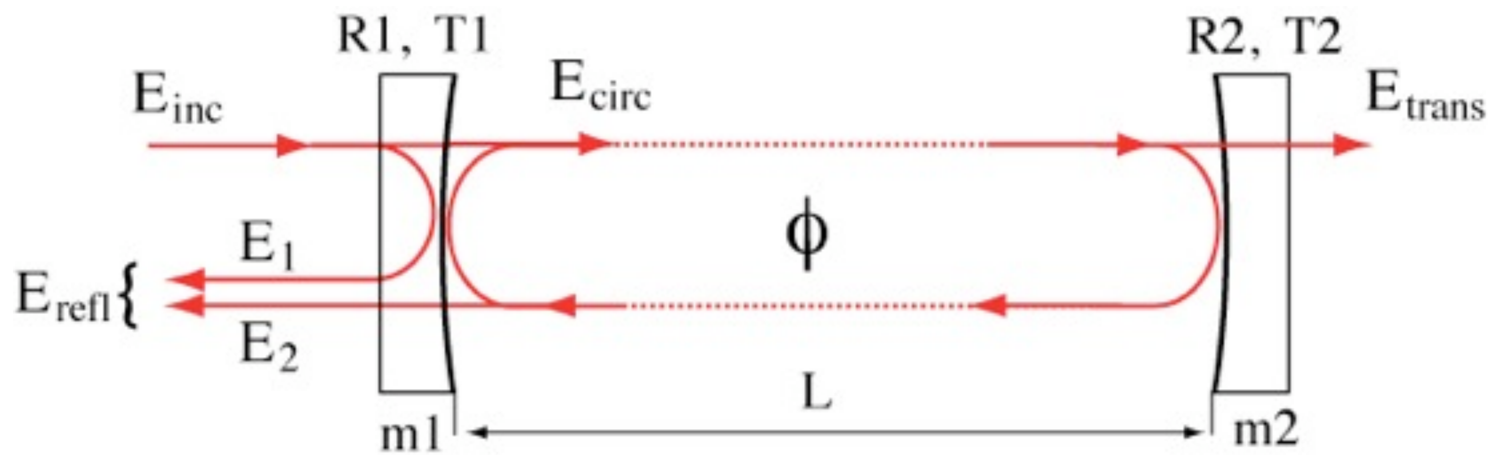
Over coupled





Cavity impedance

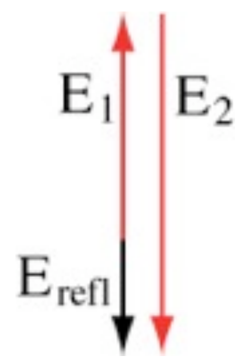


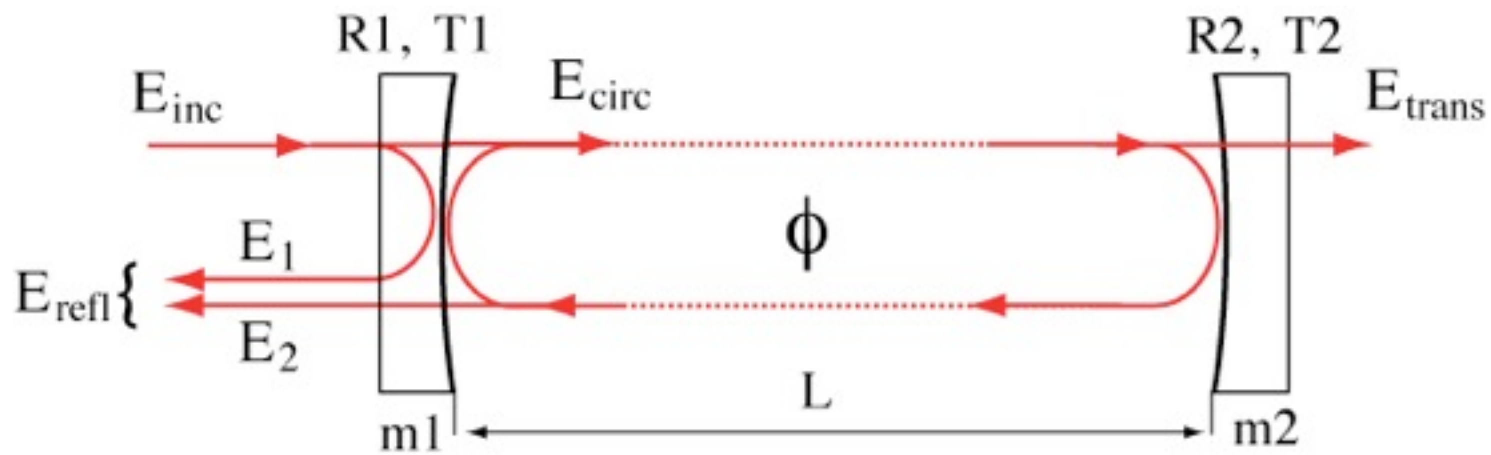


- The impedance condition of a cavity is often described by the reflected cavity electric field.

Cavity impedance

Over coupled

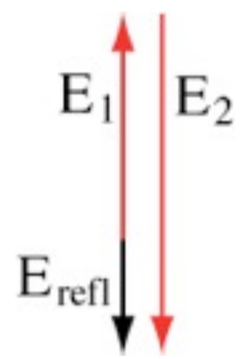


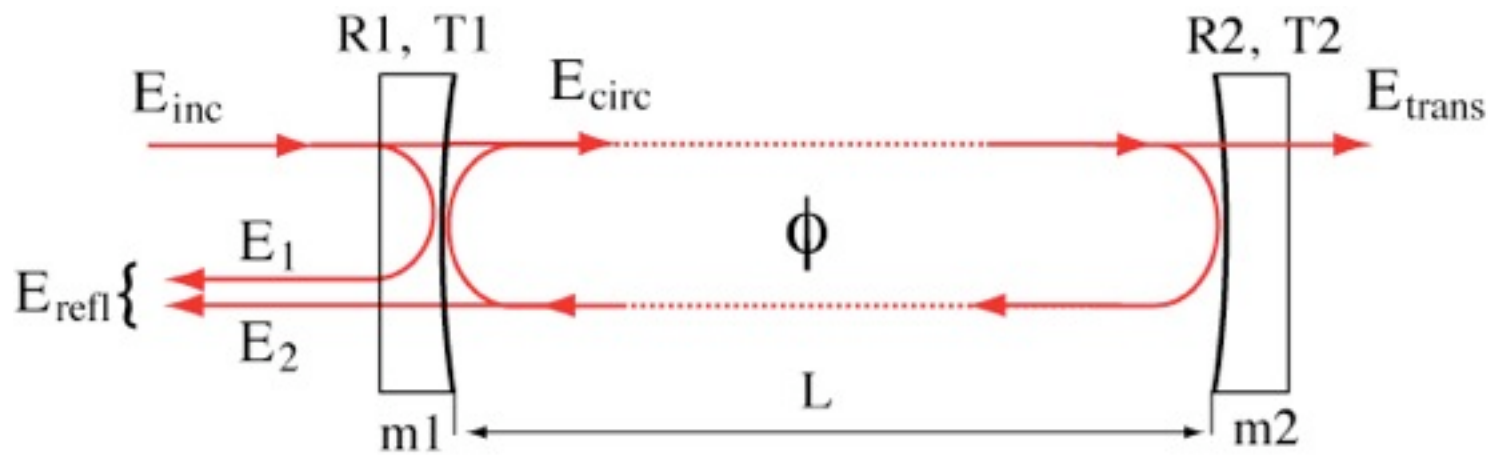


- The impedance condition of a cavity is often described by the reflected cavity electric field.
- The reflected cavity electric field is comprised of two terms:
  - The promptly reflected incident electric field E1
  - The circulating leakage electric field E2.

Cavity impedance

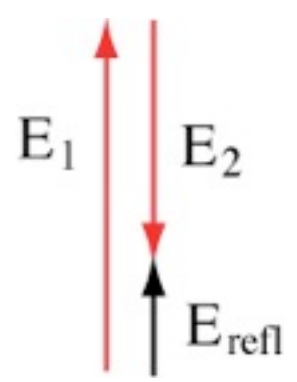
Over coupled

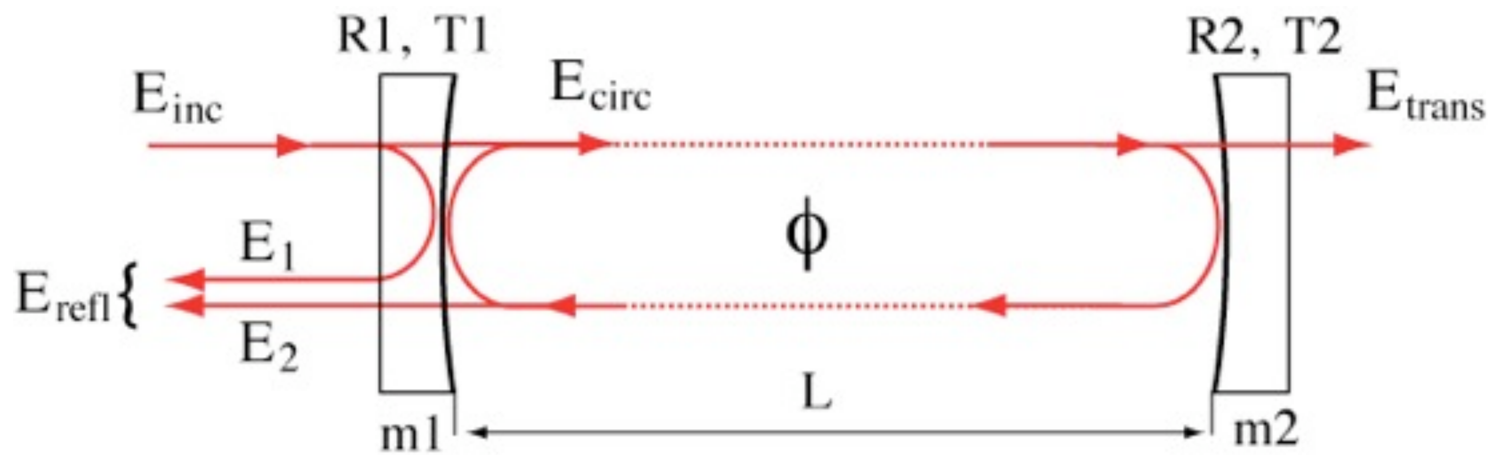




- The impedance condition of a cavity is often described by the reflected cavity electric field.
- The reflected cavity electric field is comprised of two terms:
  - The promptly reflected incident electric field E1
  - The circulating leakage electric field E2.
- This gives three impedance conditions, described by the mirror reflectivities; Over coupled, Under coupled, and impedance matched

Cavity impedance

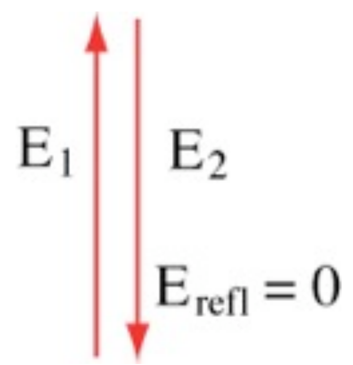


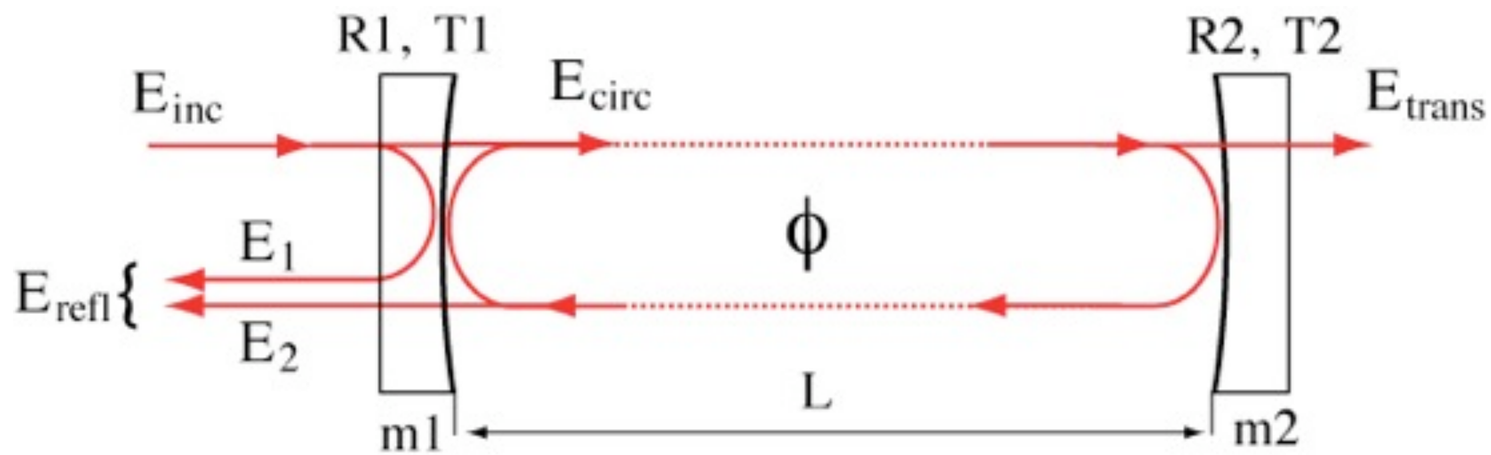


- The impedance condition of a cavity is often described by the reflected cavity electric field.
- The reflected cavity electric field is comprised of two terms:
  - The promptly reflected incident electric field E1
  - The circulating leakage electric field E2.
- This gives three impedance conditions, described by the mirror reflectivities; Over coupled, Under coupled, and impedance matched

Cavity impedance

Impedance matched!



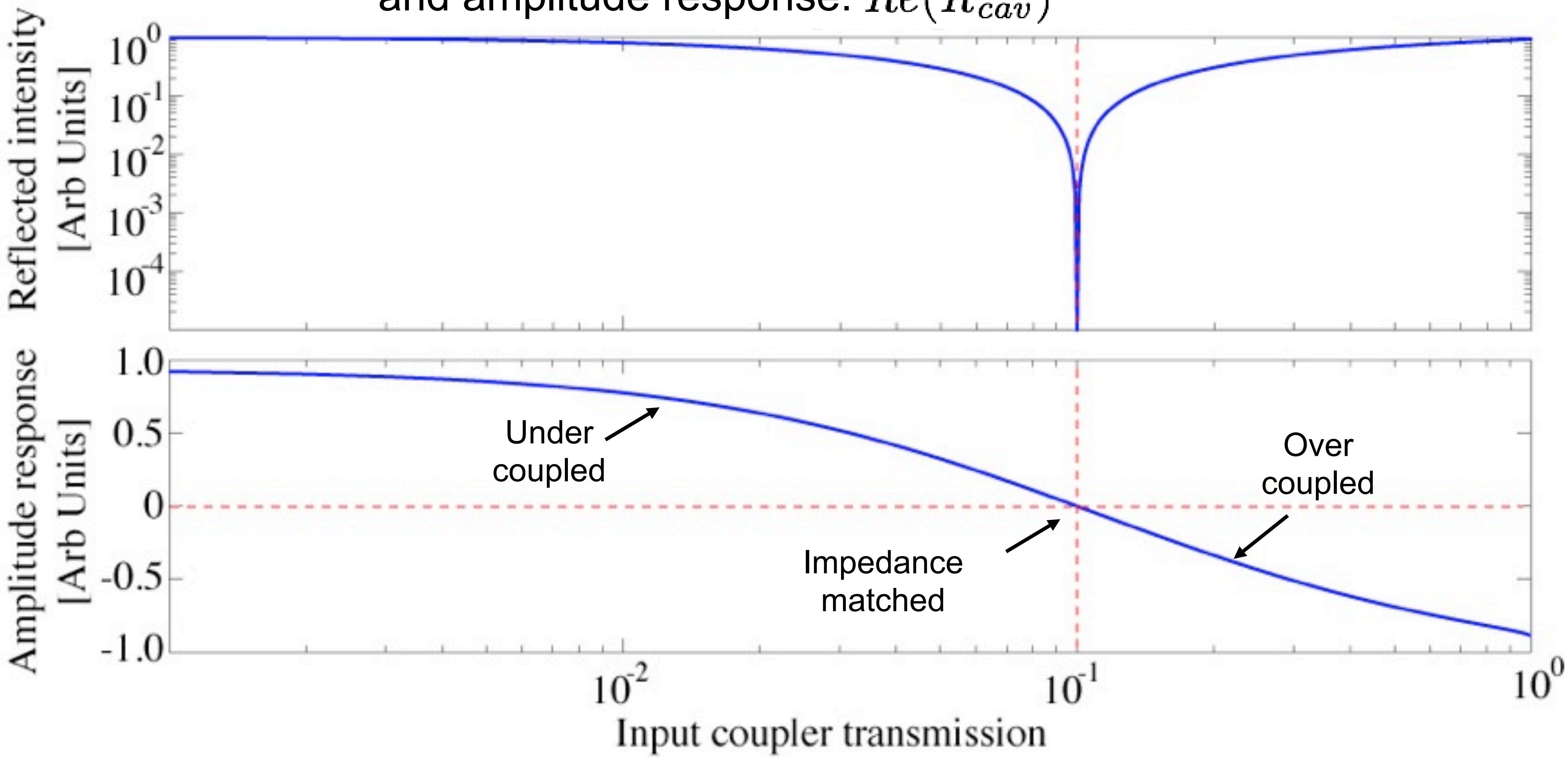


- The impedance condition of a cavity is often described by the reflected cavity electric field.
- The reflected cavity electric field is comprised of two terms:
  - The promptly reflected incident electric field E1
  - The circulating leakage electric field E2.
- This gives three impedance conditions, described by the mirror reflectivities; Over coupled, Under coupled, and impedance matched

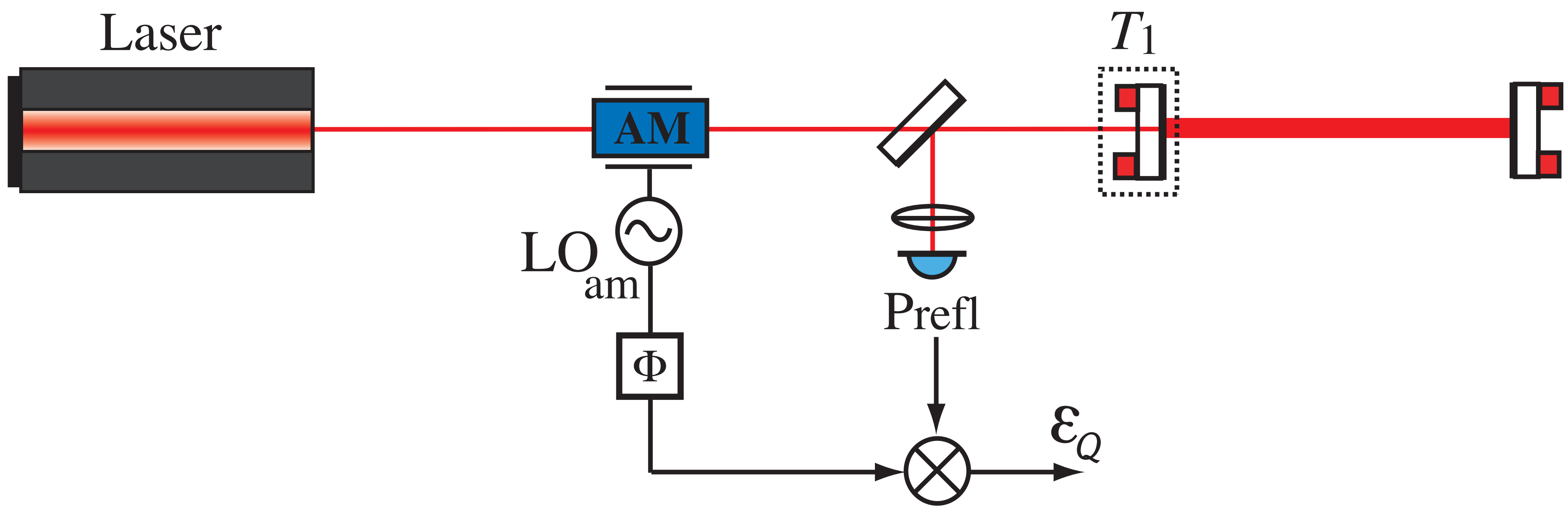
Cavity impedance

Complex reflection coefficient: $|R_{cav}|^2 = |E_{refl}/E_{inc}|^2$

and amplitude response: $Re(R_{cav})$



Impedance sensing



- By monitoring the beat signal between the carrier and a set of amplitude modulated sidebands which are well outside the coupled cavity linewidth, we can obtain a signal which is proportional to the reflected electric field amplitude:

$$E_{refl} \approx E_{inc}e^{i\omega_c t}\left[\mathcal{R}(\omega_c) + \frac{\beta}{2}\mathcal{R}(\omega_c+\omega_m)e^{i\omega_m t} + \frac{\beta}{2}\mathcal{R}(\omega_c-\omega_m)e^{-i\omega_m t}\right]$$

- Demodulation allows to extract a signal which is linearly dependent on the reflected amplitude response of the coupled cavity.

$$\mathcal{E}_Q = \sqrt{P_c P_s} \times Re[\mathcal{R}(\omega_c)\mathcal{R}^*(\omega_c+\omega_m) + \mathcal{R}^*(\omega_c)\mathcal{R}(\omega_c-\omega_m)].$$

Impedance sensing

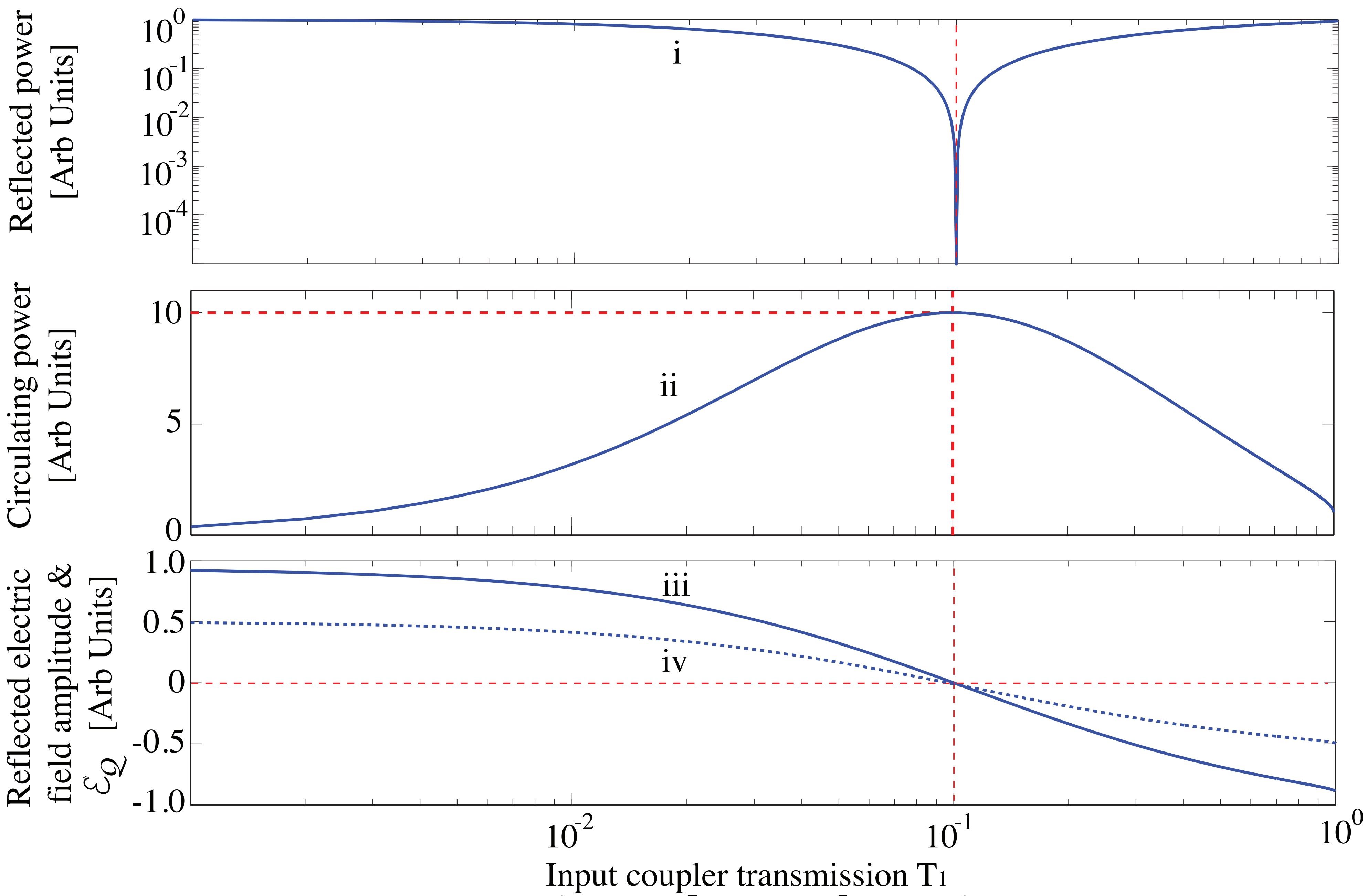
- The photo detector signal is:

$$\begin{aligned} P_{refl} \quad = \quad & P_c|\mathcal{R}(\omega_c)|^2 + P_s|\mathcal{R}(\omega_c+\omega_m)|^2 + P_s|\mathcal{R}(\omega_c-\omega_m)|^2 + \\ & 2\sqrt{P_cP_s} \times Re[\mathcal{R}(\omega_c)\mathcal{R}^*(\omega_c+\omega_m) + \mathcal{R}^*(\omega_c)\mathcal{R}(\omega_c-\omega_m)]\cos(\omega_m t) + \\ & 2\sqrt{P_cP_s} \times Im[\mathcal{R}(\omega_c)\mathcal{R}^*(\omega_c+\omega_m) + \mathcal{R}^*(\omega_c)\mathcal{R}(\omega_c-\omega_m)]\sin(\omega_m t) - \\ & 2P_s\cos(2\omega_m t). \qquad (6.2) \end{aligned}$$

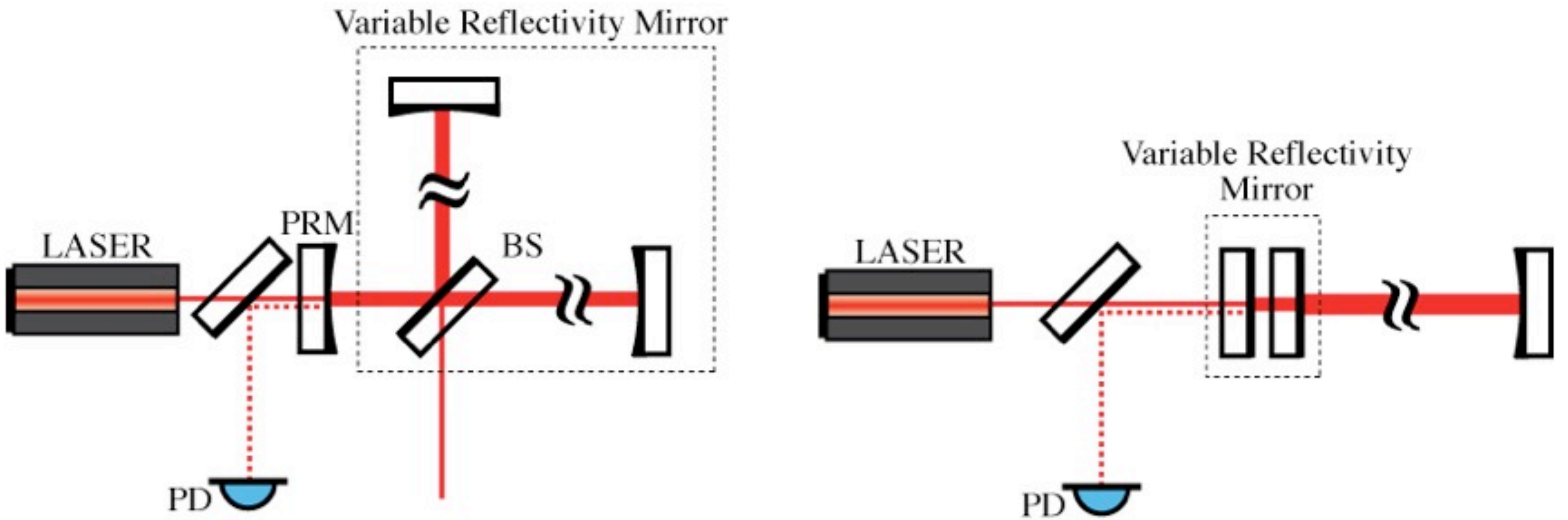
- Subsequent demodulation allows to extract a signal which is linearly dependent on the reflected amplitude response of the coupled cavity.

$$\mathcal{E}_{\mathcal{Q}} \quad = \quad \sqrt{P_cP_s} \times Re[\mathcal{R}(\omega_c)\mathcal{R}^*(\omega_c+\omega_m) + \mathcal{R}^*(\omega_c)\mathcal{R}(\omega_c-\omega_m)].$$

Impedance sensing

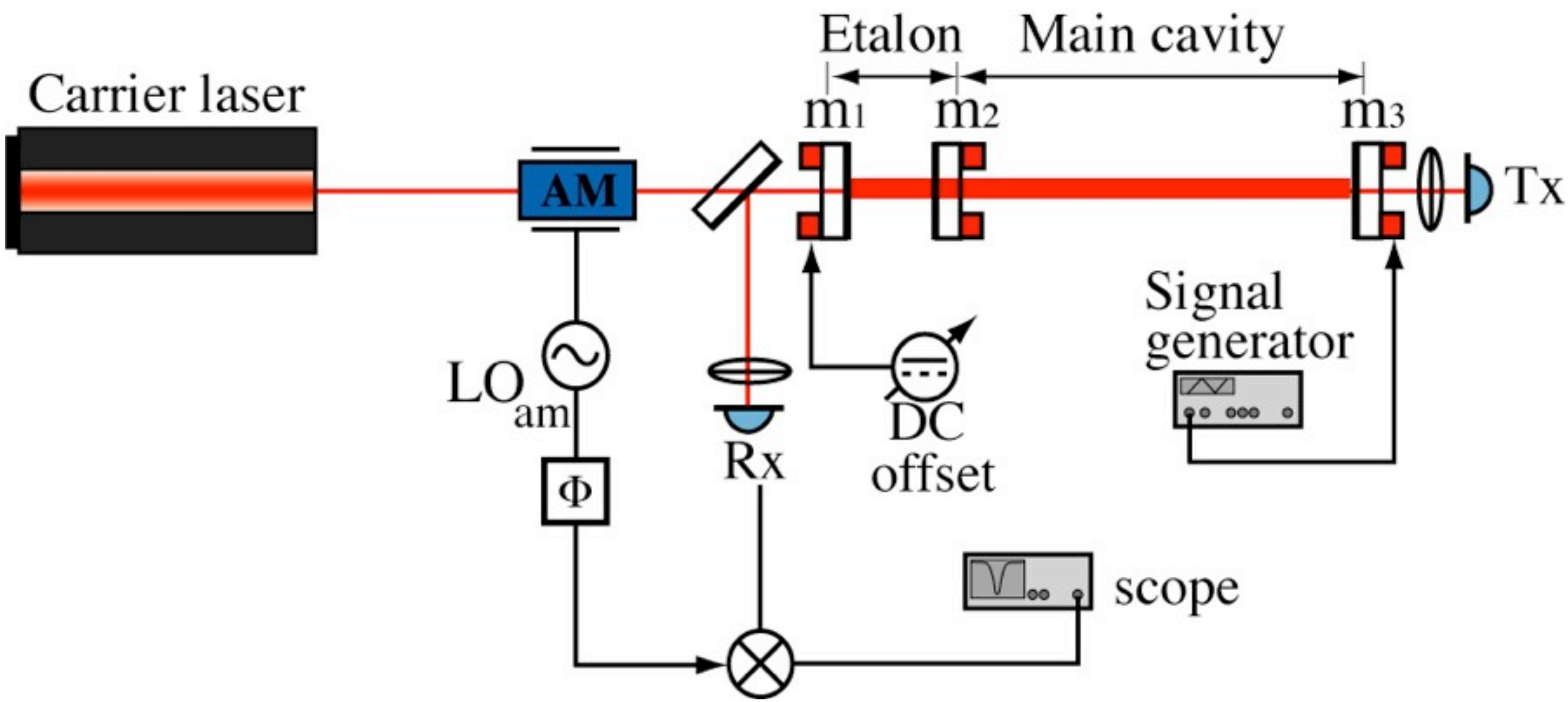


Changing the impedance

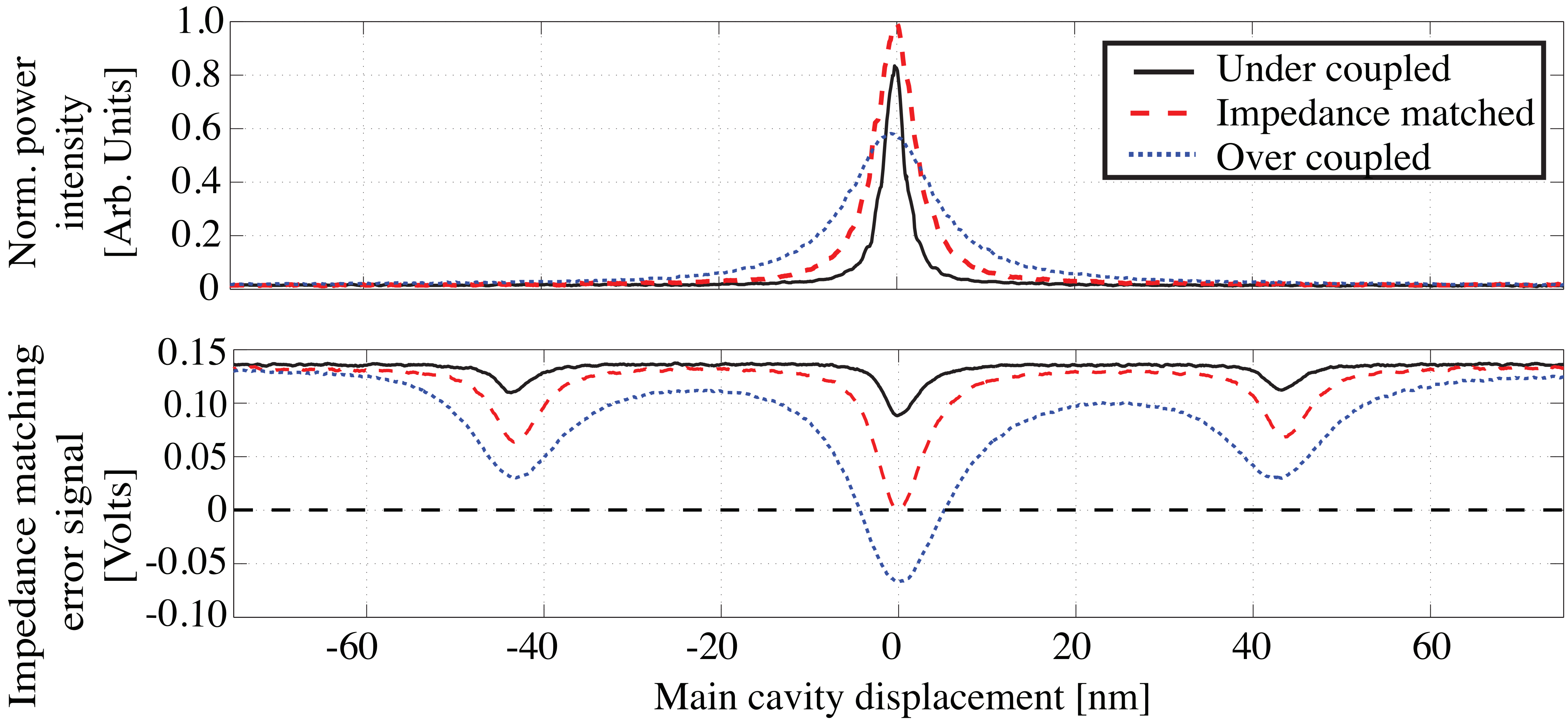


- We can use a Michelson as a variable reflectivity mirror.
- Or we can use a Fabry-Perot cavity as a variable reflectivity mirror.

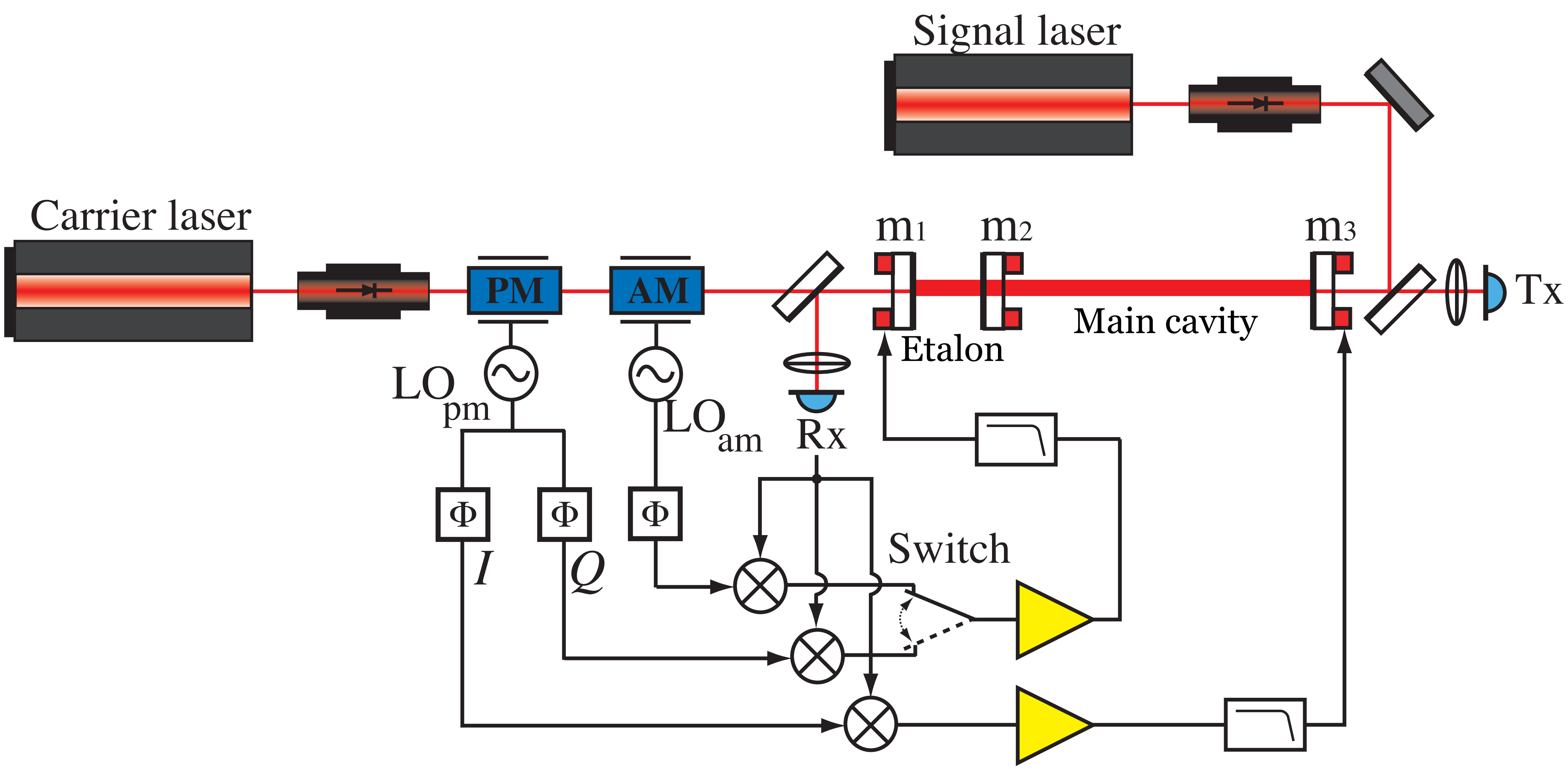
Experimental PD and error signals



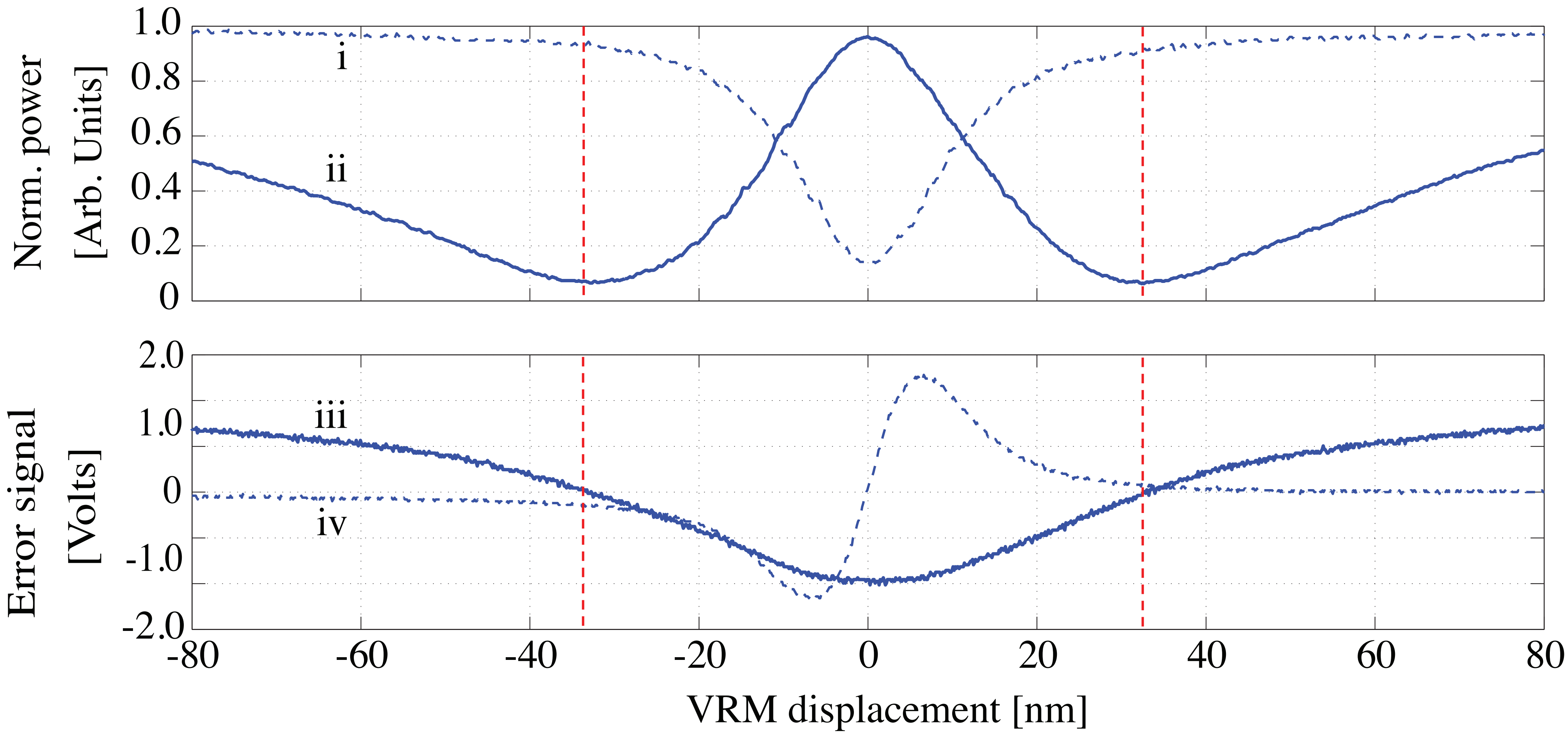
Experimental PD and error signals



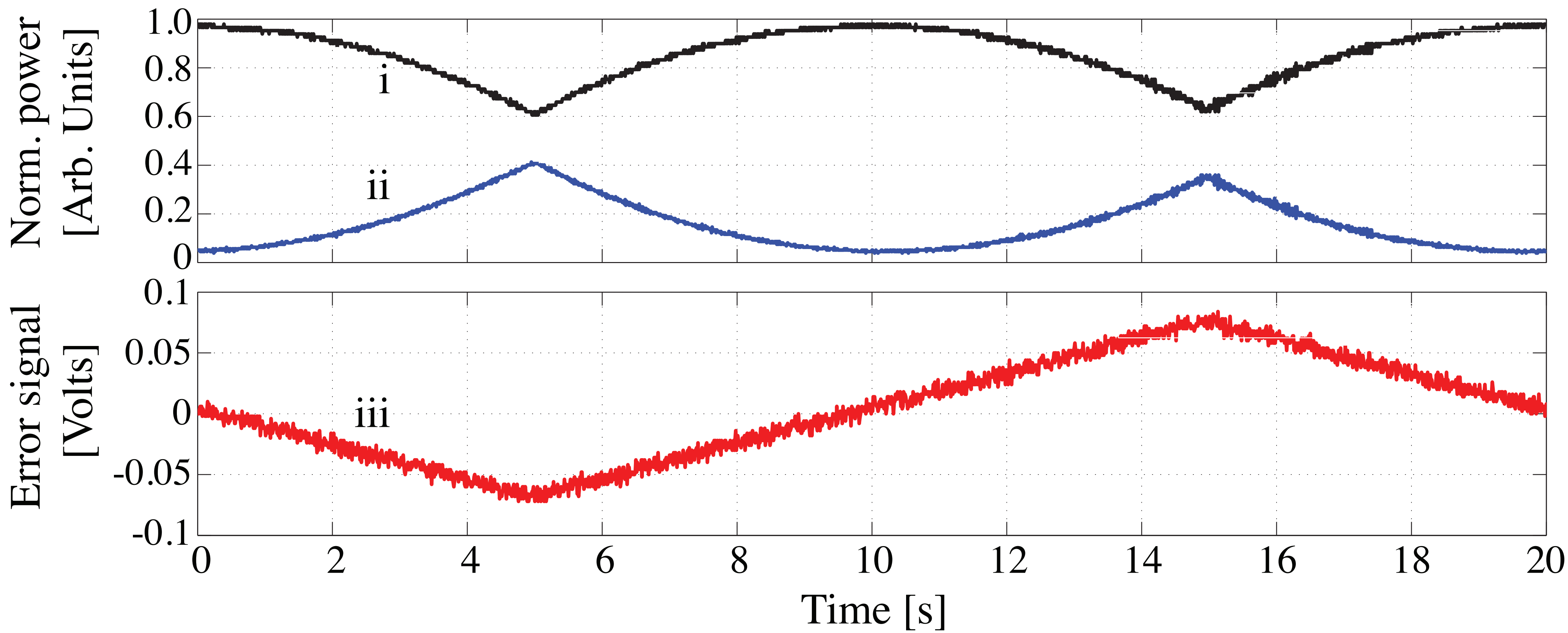
Impedance control



Impedance error signal



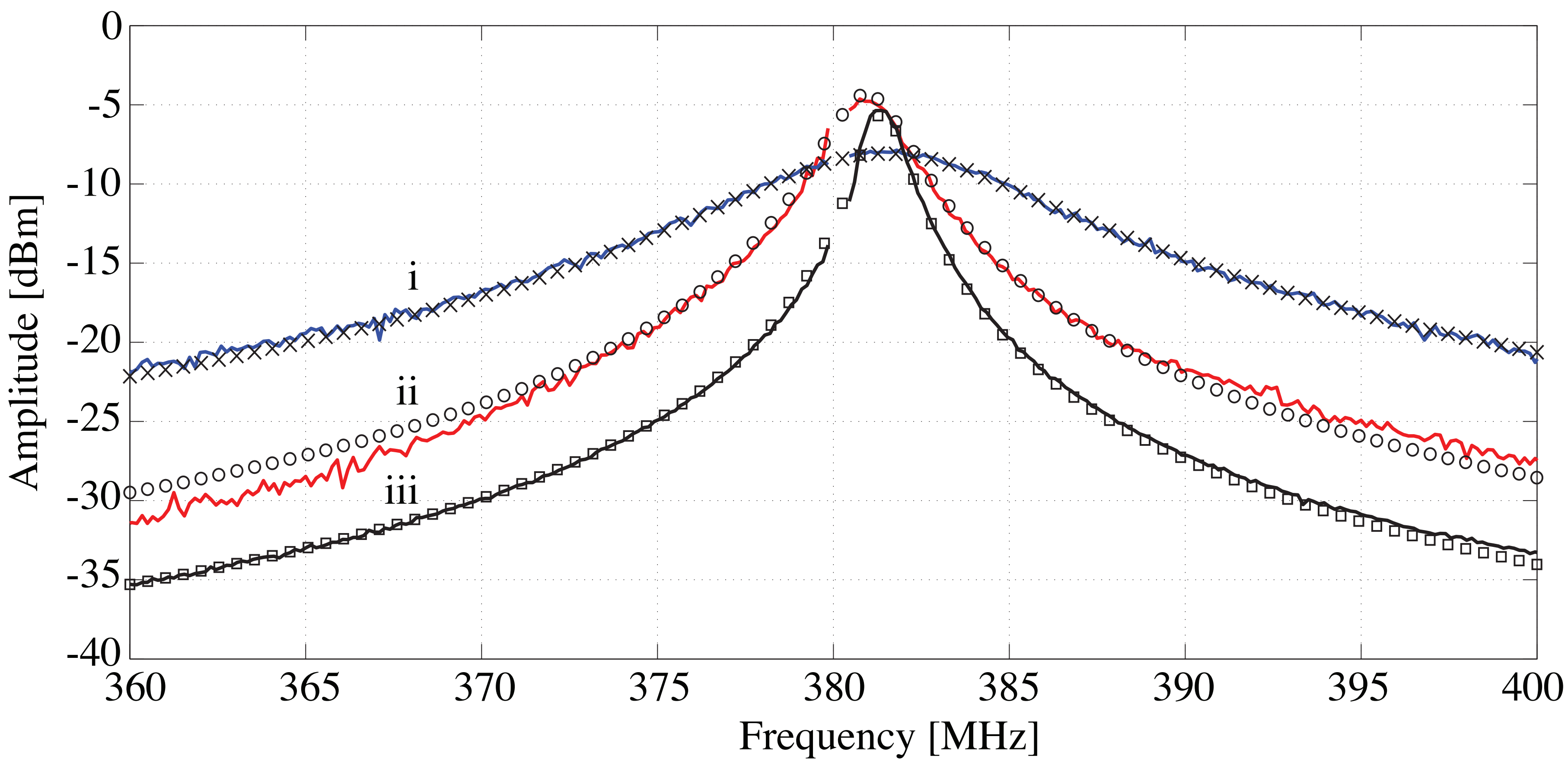
Impedance control



Frequency response data

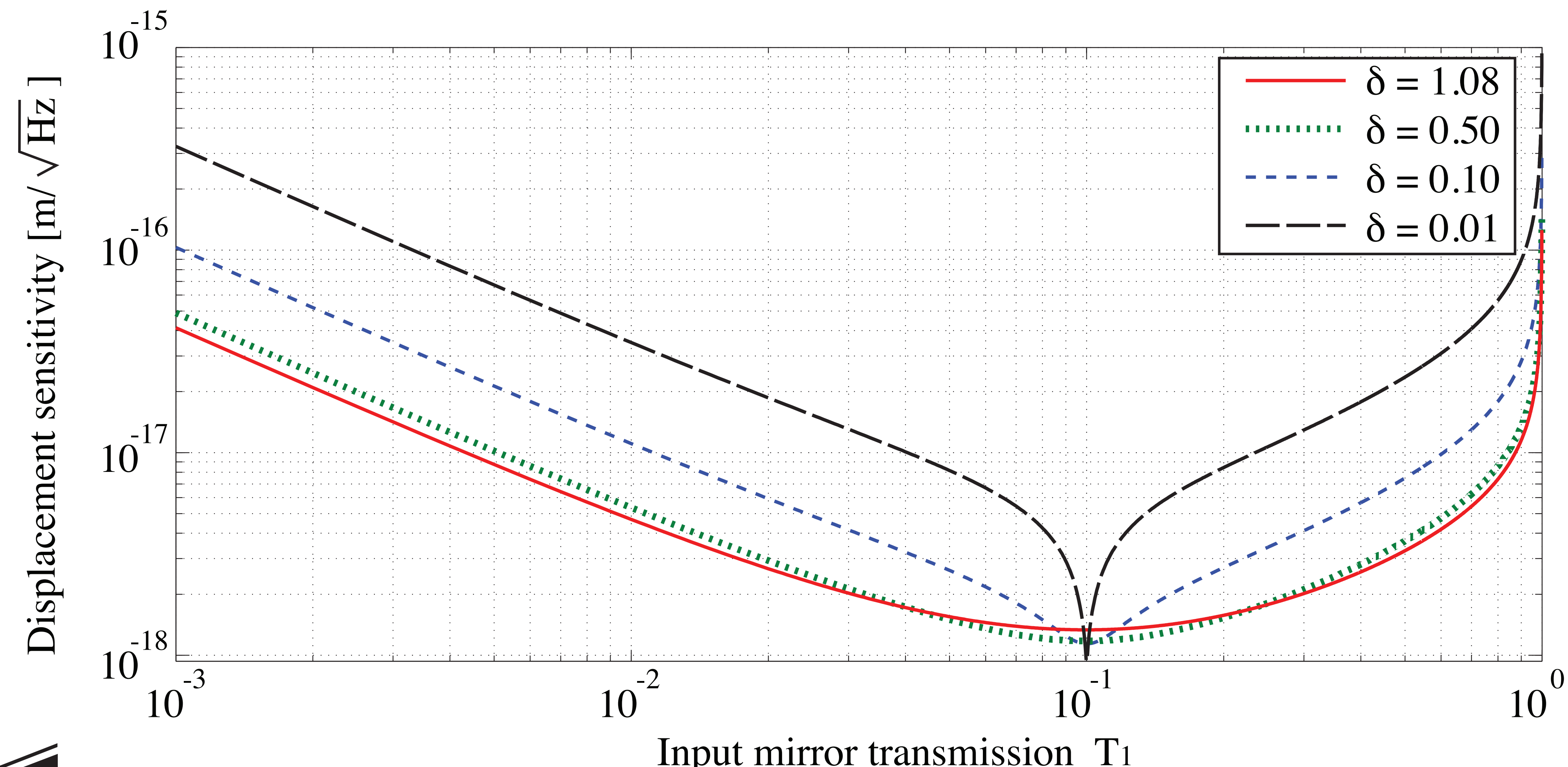
Best fit parameters of the coupled cavity signal response data.

Response	Coupling	VRM reflectivity [%]	Coupled cavity bandwidth [MHz]	Finesse
(i)	Over coupled	0.869	8.8	43.2
(ii)	Impedance matched	0.968	2.27	167.4
(iii)	Under coupled	0.983	1.33	286.2



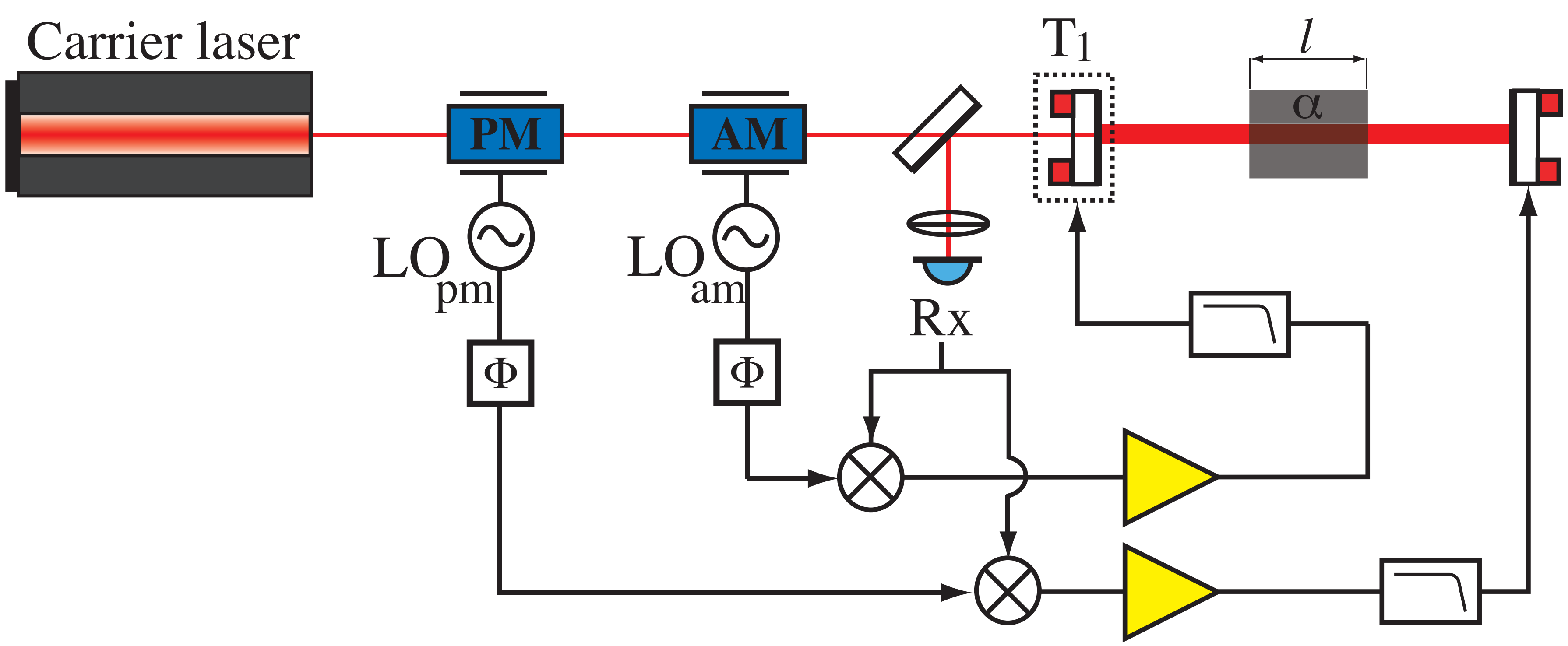
PDH sensitivity VS impedance

$$\mathcal{SN}_{(\mathcal{I})} = \frac{1}{8}\sqrt{\frac{hc\lambda}{2\mathcal{F}^2}}\sqrt{\frac{P_c|\mathcal{R}(\omega_c)|^2+3P_s}{P_cP_s}} \quad \left[\frac{\text{m}}{\sqrt{\text{Hz}}}\right]$$

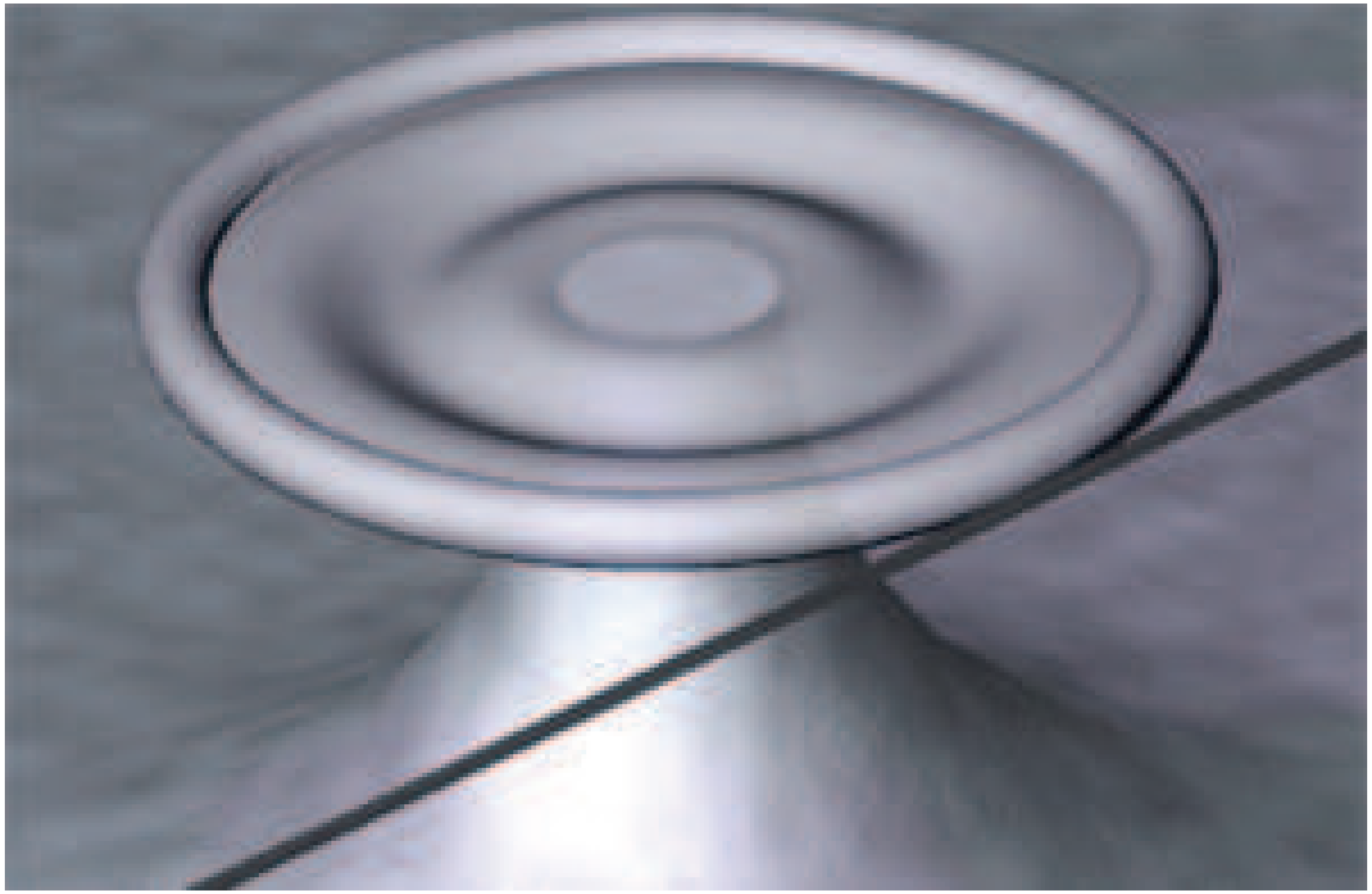


Absorption spectroscopy

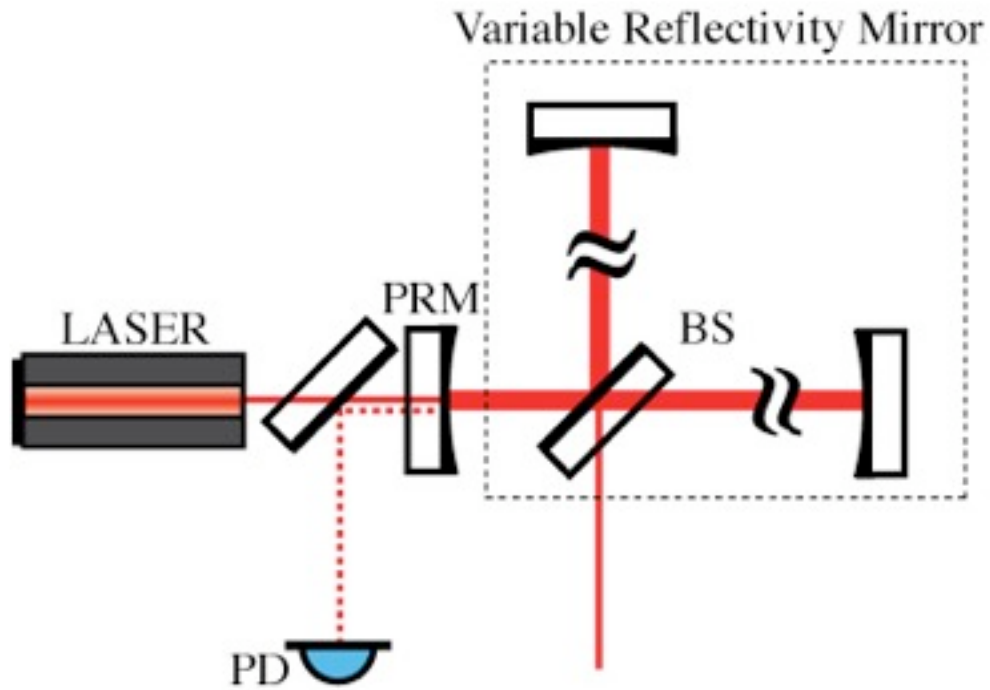
$$(\alpha l)_{\mathcal{SN}} = \frac{\pi}{2\mathcal{F}}\sqrt{\frac{2h\nu}{P_c}} \qquad \left[\frac{1}{\sqrt{\mathrm{Hz}}}\right]$$



Metrology for evanescent coupling in micro-ring cavity



GW readout...?



SNL sensitivity

$$\mathcal{SN}_t = \frac{1}{2\mathcal{F}t_1}\sqrt{\frac{h\lambda c}{P_c}} \approx \frac{1}{2\pi\sqrt{\mathcal{N}}}\sqrt{\frac{h\lambda c}{P_c}} \quad \left[\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}\right]$$

$$\mathcal{SN}_{(\mathcal{Q})} = \frac{\sqrt{3}}{8\mathcal{F}}\sqrt{\frac{h\lambda c}{2P_c}} = \frac{\sqrt{3}}{4\pi\mathcal{N}}\sqrt{\frac{h\lambda c}{2P_c}} \quad \left[\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}\right]$$

Mich. op. cond.

$$\phi_m = 0$$

$$\phi_m = \arccos(t_1)^*$$

$*$(assuming lossless mirrors)