Detecting gravitational waves I: Basic principles

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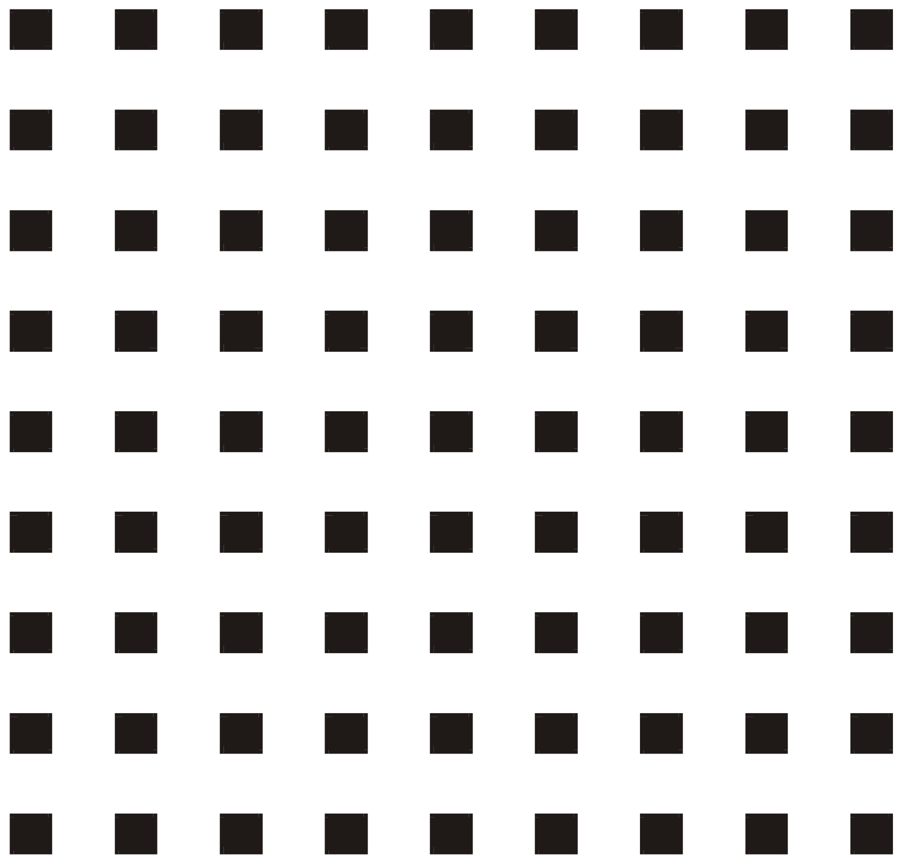
Outline

1. Gravitational waves and their role in the relativistic understanding of gravity
2. Understanding gravitational waves in terms of how they might be measured
3. Basic principles of interferometers

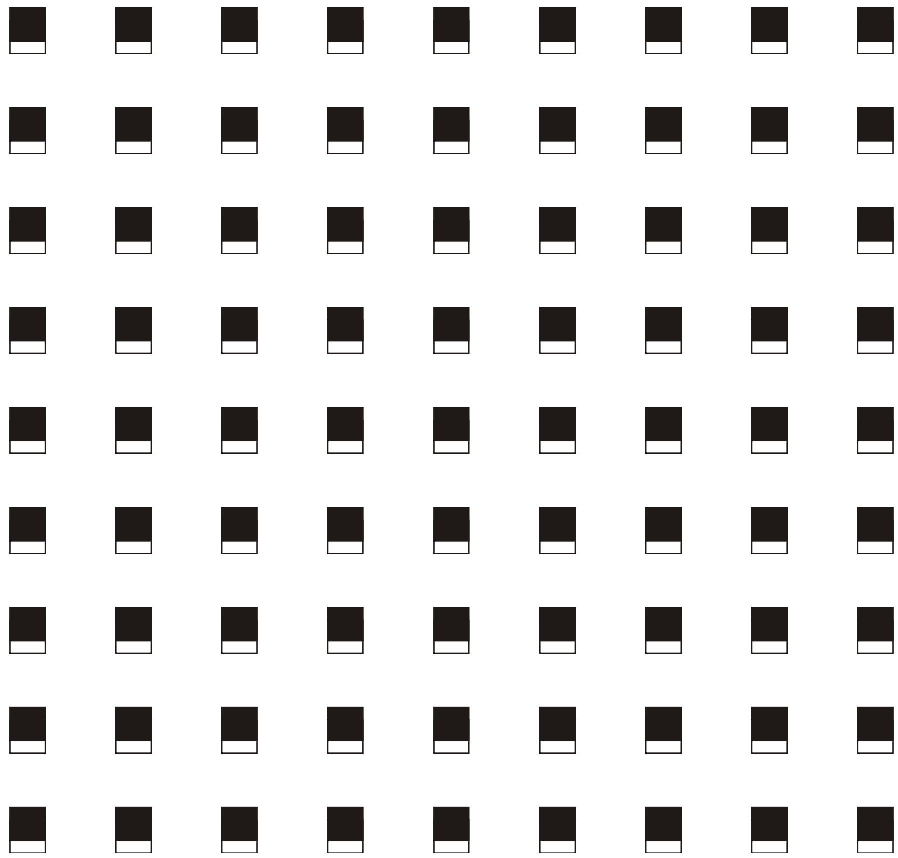
Gravitational waves must exist

- Relativity: no signals can travel faster than light.
- Newtonian gravity: shake a mass, and its gravitational field changes instantaneously throughout the universe: a super-relativistic "gravitational telegraph."
- Hence: Gravitational waves must exist, playing the same role for the gravitational field as EM waves do for the electric field.

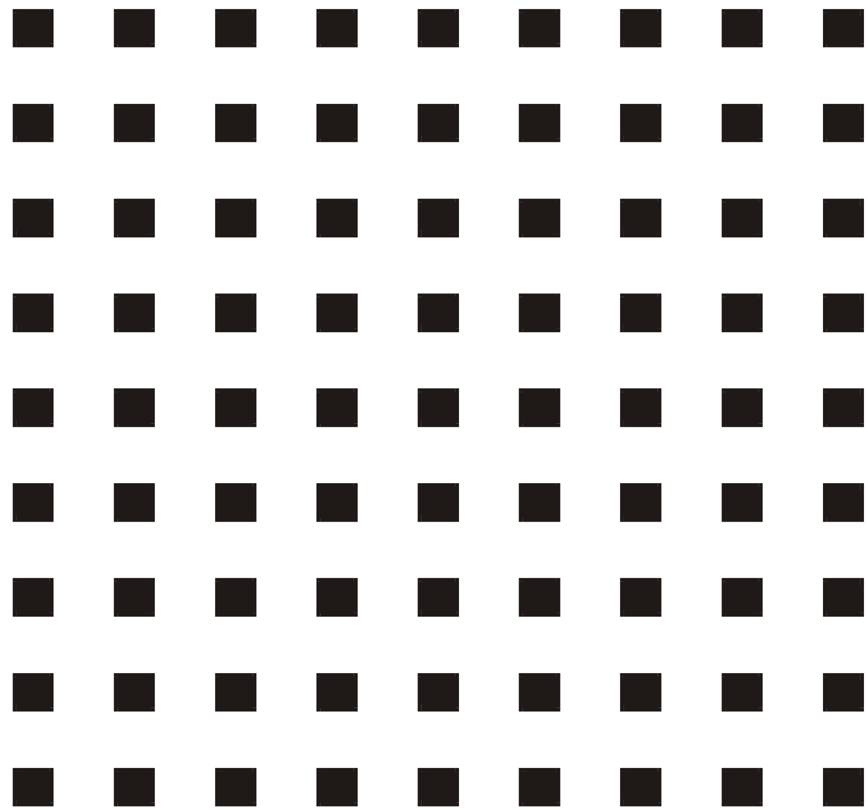
A set of charged particles



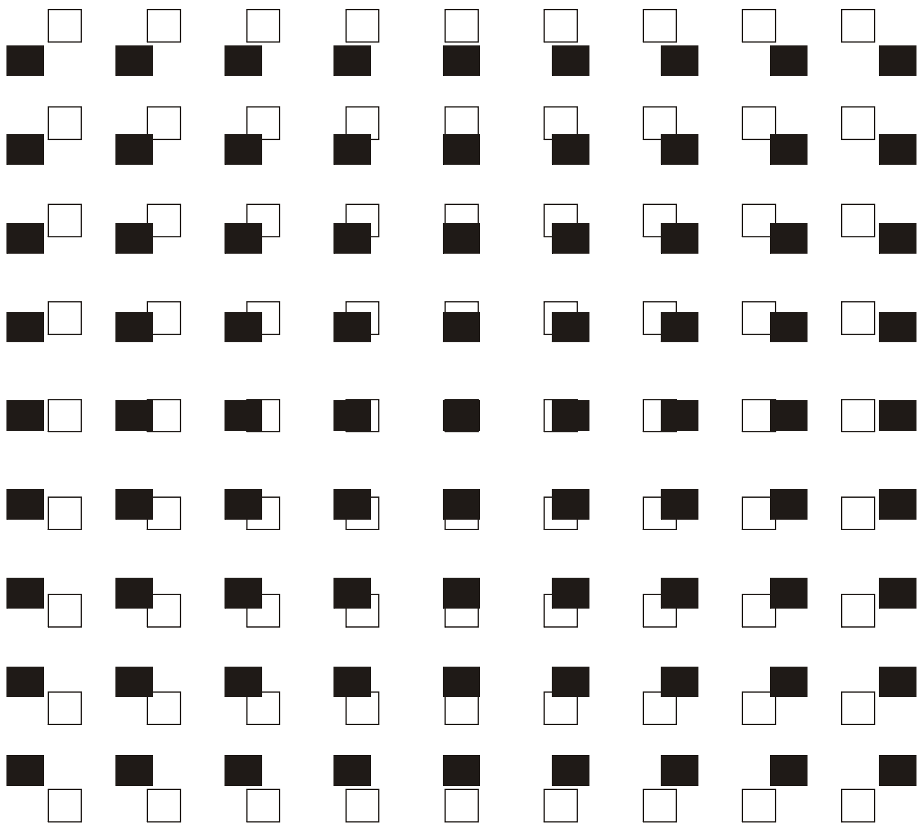
Electromagnetic wave moves charged test bodies



A set of freely-falling particles (with mass but no charge)

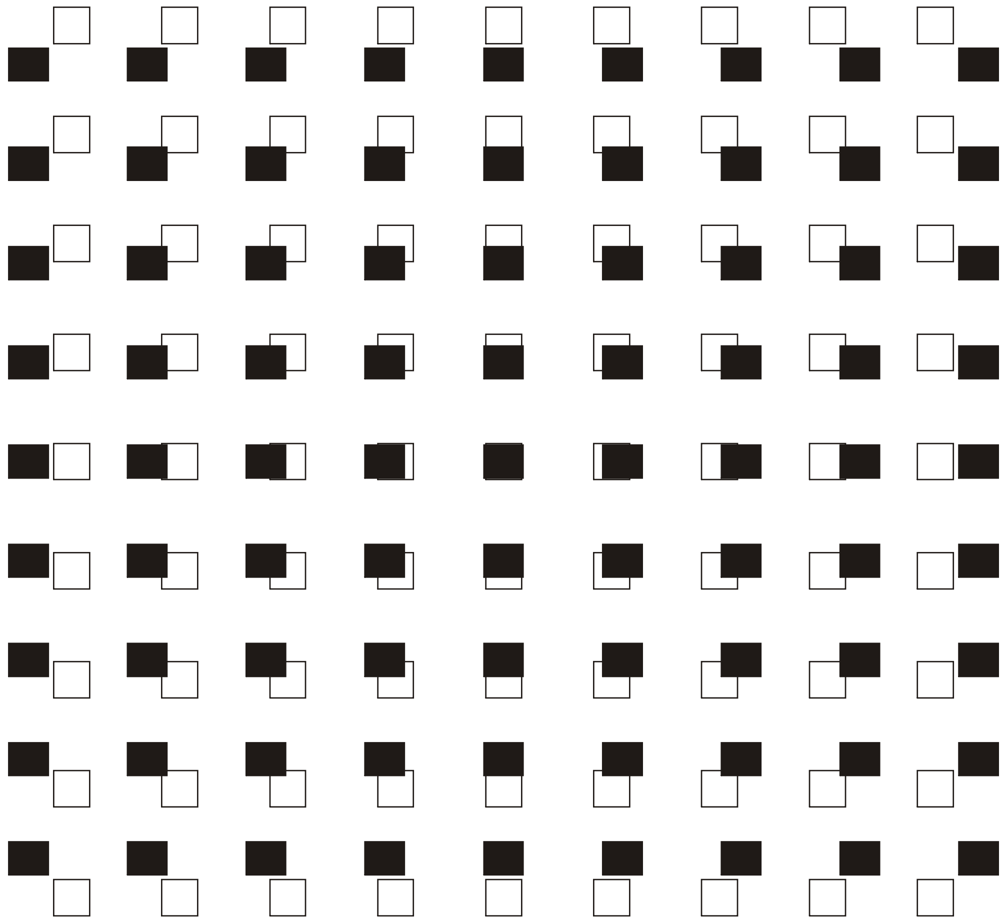


Gravity wave: distorts set of test masses in transverse directions



Gravitational wave:
a transverse quadrupolar strain

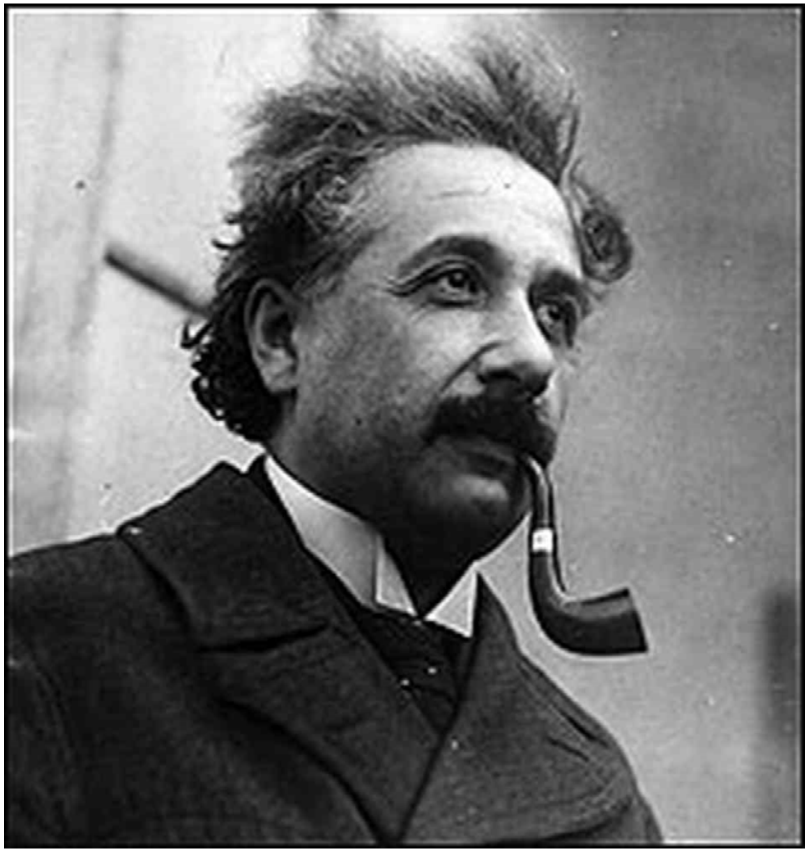
strain amplitude:
$h = 2\Delta L/L$



Einstein predicted gravitational waves in 1916 …

… but then doubted their existence for the rest of his life.

The theory was so subtle, Einstein was never sure whether the waves were a coordinate effect only, with no physical reality.



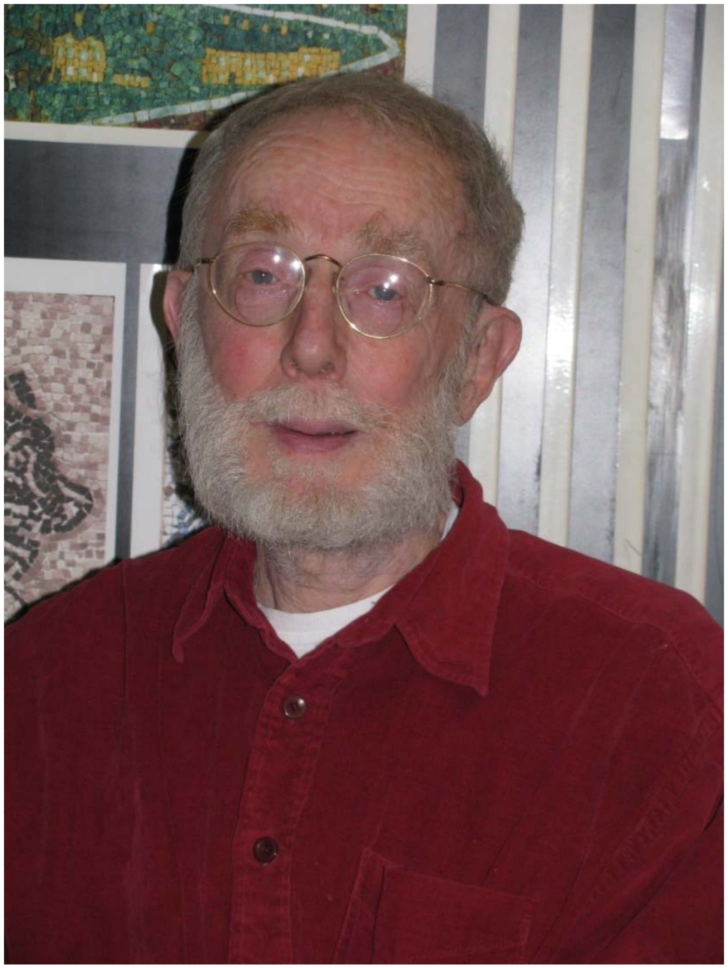
For decades, relativists doubted whether gravitational waves were real

In Eddington's early textbook on relativity, he quipped that some people thought that "gravitational waves travel at the speed of thought."

Think of this: Einstein proposed many experiments, including really hard ones, but never suggested a search for gravitational waves.

The controversy lasted four decades, until the Chapel Hill Conference in 1957.

Felix Pirani solved the problem of the reality of gravitational waves



Felix Pirani (shown here many years later) showed relativists that gravitational waves must have physical reality, because you could invent a (thought) experiment that could detect them.

Photo by Josh Goldberg

from the transcript of Pirani's talk at Chapel Hill, 1957

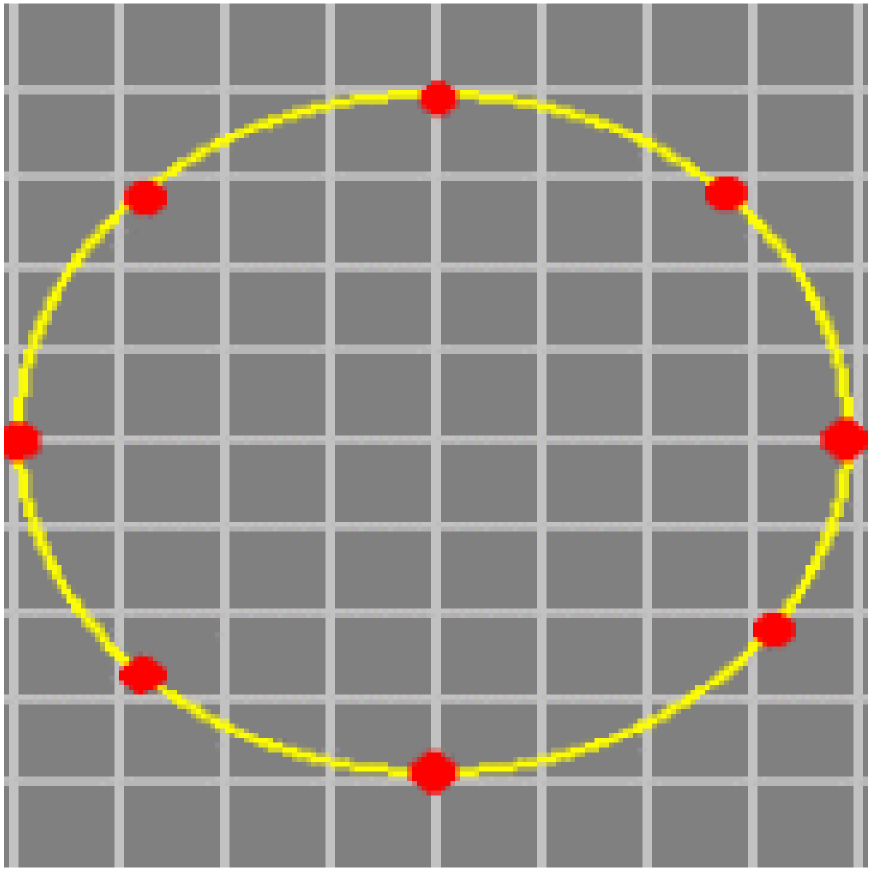
If now one introduces an orthonormal frame on ζ, v^μ being the timelike vector of the frame, and assumes that the frame is parallelly propagated along ζ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (1) becomes

$$\frac{d^2\eta^a}{d\tau^2} + R^a{}_{obo}\,\eta^b = 0 \qquad (a,b = 1,2,3) \tag{2}$$

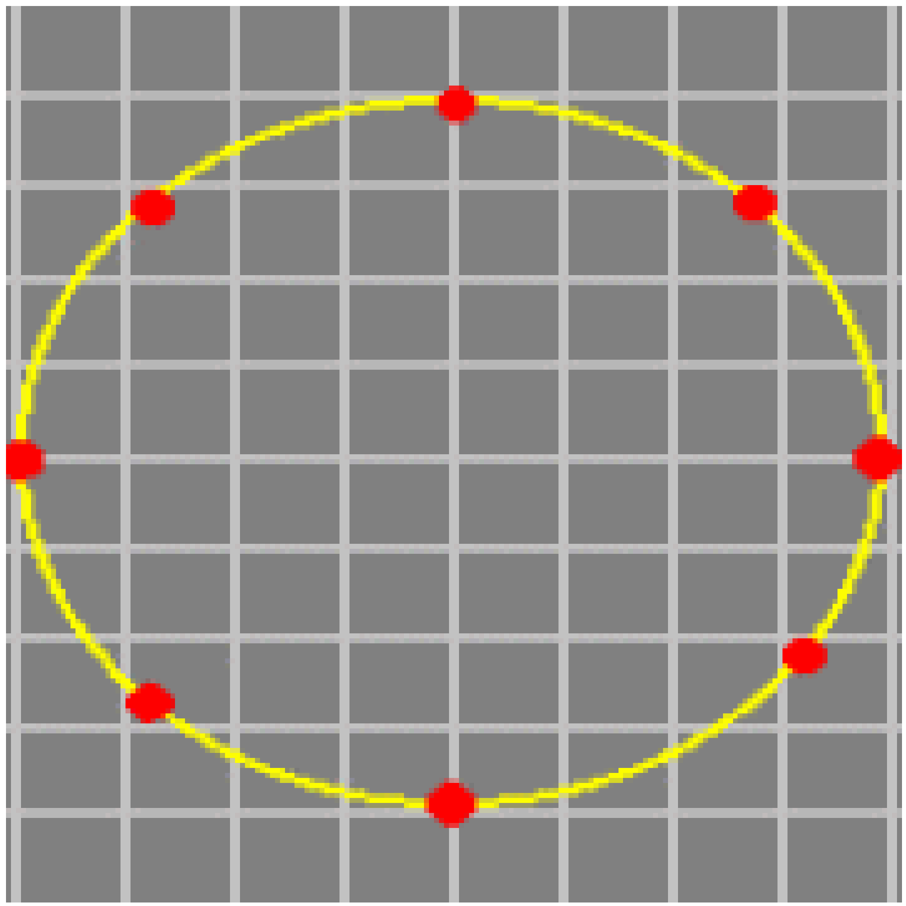
Here η^a are the physical components of the infinitesimal displacement and $R^a{}_{obo}$ some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.

Pirani's set of neighboring freely-falling test masses



They respond in a measurable way to a gravitational wave



Hermann Bondi clarifies Pirani's point

Pirani's mentor Bondi arrived at Chapel Hill unsure about gravitational waves.

Listening to Pirani's talk, he asked whether you could connect two nearby masses with a dashpot, thus absorbing energy from the wave.

Energy absorption is the ultimate test of physical reality.

Pirani replied: "I have not put in an absorption term, but I have put in a 'spring'. You could invent a system with such a term quite easily."

Bondi is credited with the "sticky bead argument."

Proof by dialog that gravitational waves are real

BONDI: Can one construct in this way an absorber for gravitational energy by inserting a $\frac{d\eta}{d\tau}$ term, to learn what part of the Riemann tensor would be the energy-producing one, because it is that part that we want to isolate to study gravitational waves?

PIRANI: I have not put in an absorption term, but I have put in a "spring." You can invent a system with such a term quite easily.

Joe Weber at Chapel Hill

Joe Weber, co-inventor of the maser, was working with John Wheeler at Princeton on gravitational waves.

The two of them were at Chapel Hill, and listened well to Pirani's talk.

Joe Weber starts GW detection

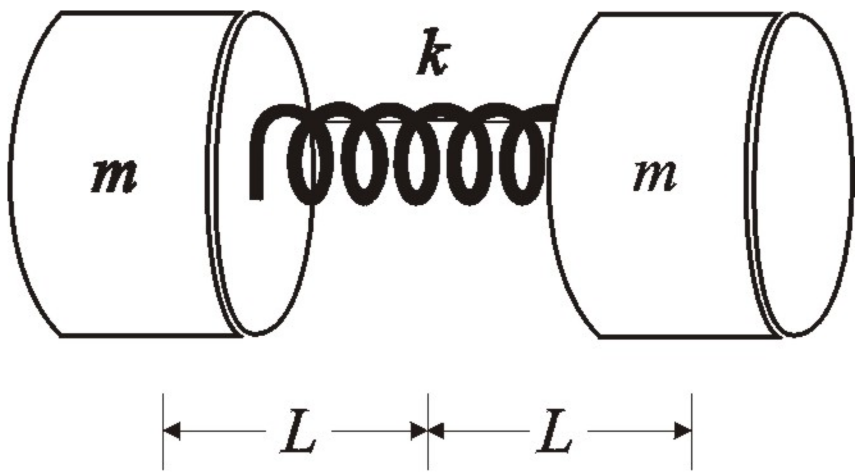
Weber and Wheeler recapped Pirani's argument in a paper written within weeks of the Chapel Hill conference.

He expanded on the experimental ideas in two Gravity Research Foundation essays (3^{rd} prize 1958, 1^{st} prize 1959), leading to his 1960 Phys. Rev. paper laying out the bar program.

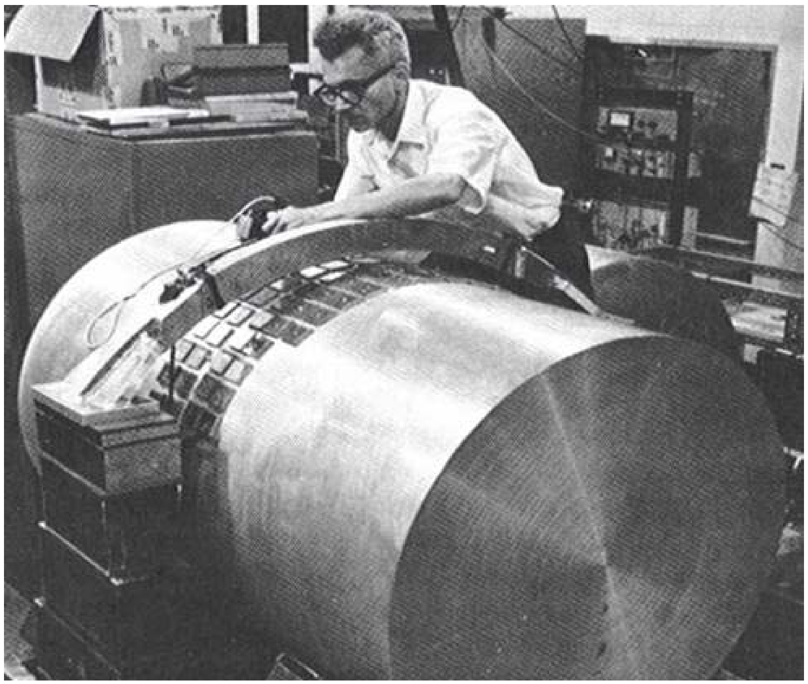
In other words: The search for gravitational waves started in January 1957 during Pirani's talk at Chapel Hill.

Resonant detector (or "Weber bar")

A massive (aluminum) cylinder. Vibrating in its gravest longitudinal mode, its two ends are like two test masses connected by a spring.

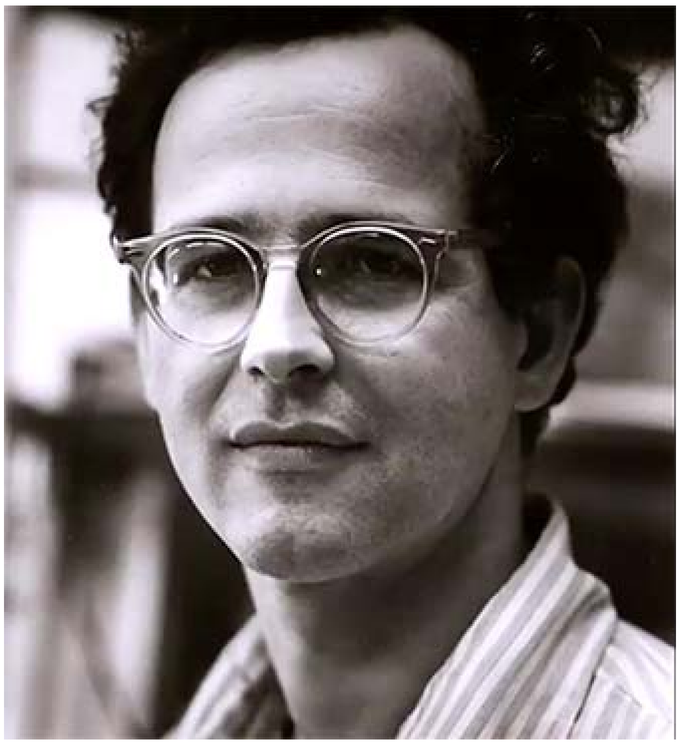


Weber's bar



Weber's gravitational wave detector was a cylinder of aluminum. Each end is like a test mass, while the center is like a spring. PZT's around the midline are Bondi's dashpots, absorbing energy to send to an electrical amplifier.

Rainer Weiss, not at Chapel Hill



In 1957, Rai Weiss was a grad student of Jerrold Zacharias at MIT, working on atomic beams.

In the early '60's, he spent two years working with Bob Dicke at Princeton on gravity experiments.

Rainer Weiss reads Pirani

In 1964, Rai was back at MIT as a professor. He was assigned to teach general relativity. He didn't know it, so he had to learn it one day ahead of the students.

He asked, What's really measurable in general relativity? He found the answer in Pirani's papers presented at Chapel Hill in 1957.

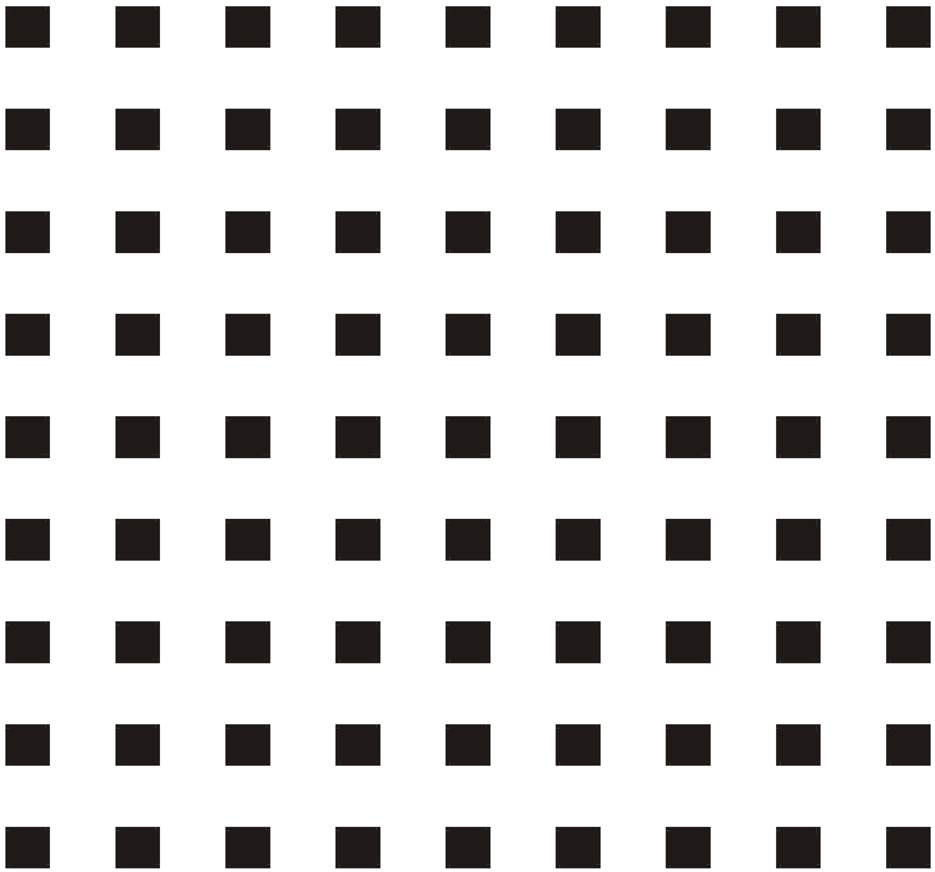
What Pirani actually proposed

In Pirani’s papers, he didn’t “put in” either a spring or a dashpot between the test masses. Instead, he said:

“It is assumed that an observer, by the use of light signals or otherwise, determine the coordinates of a neighboring particle in his local Cartesian coordinate system.”

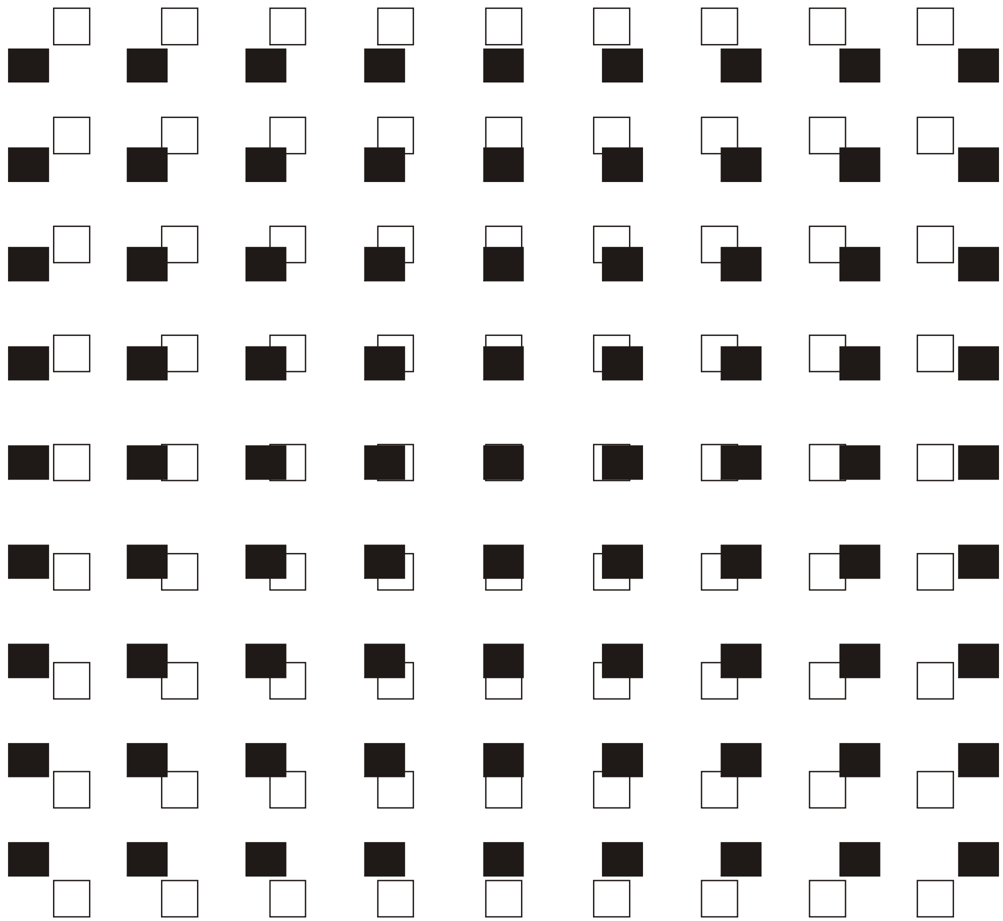
Zach’s lab at MIT was in the thick of the new field of lasers. Rai read Pirani, and knew that lasers could do the job.

A set of freely-falling test particles



Gravitational wave: a transverse quadrupolar strain

strain amplitude:
$h = 2\Delta L/L$



Gravitational waveform lets you read out source dynamics

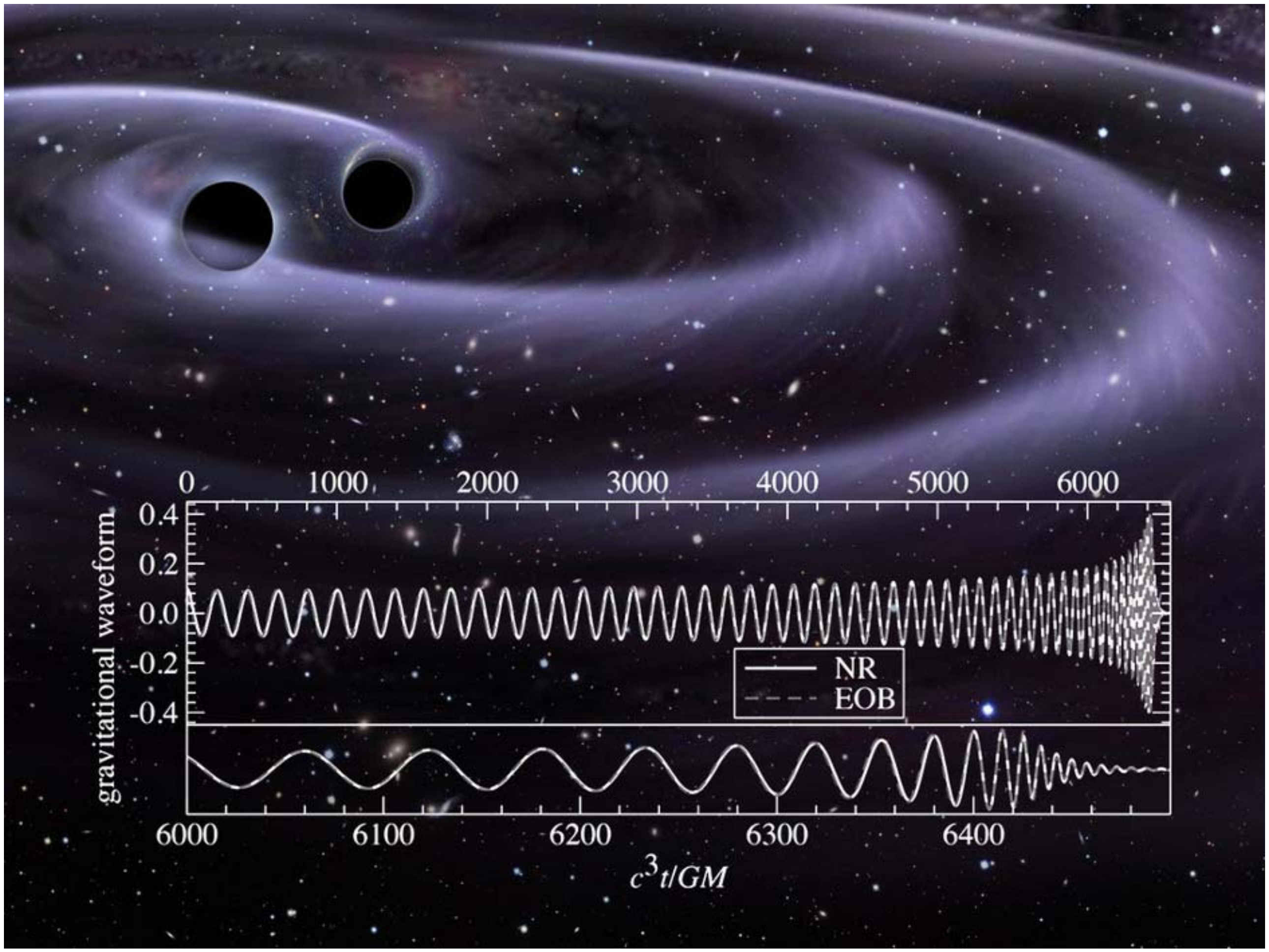
The evolution of the mass distribution can be read out from the gravitational waveform:

$$h_{\mu\nu}(t) = \frac{1}{R}\frac{2G}{c^4}\ddot{I}_{\mu\nu}(t - R/c)$$

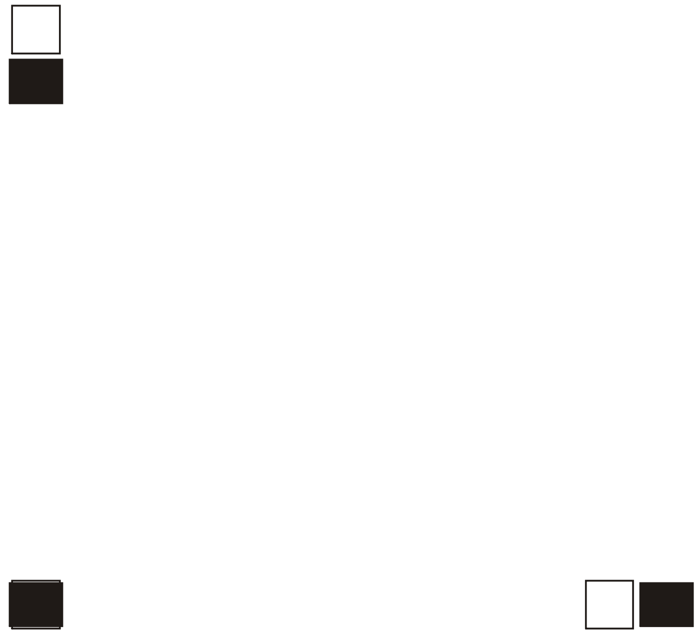
Coherent relativistic motion of large masses can be directly observed from the waveform!

$$I_{\mu\nu} \equiv \int dV \left(x_\mu x_\nu - \delta_{\mu\nu} r^2/3\right)\rho(r).$$

Pattern of oscillations from black hole inspiral and merger

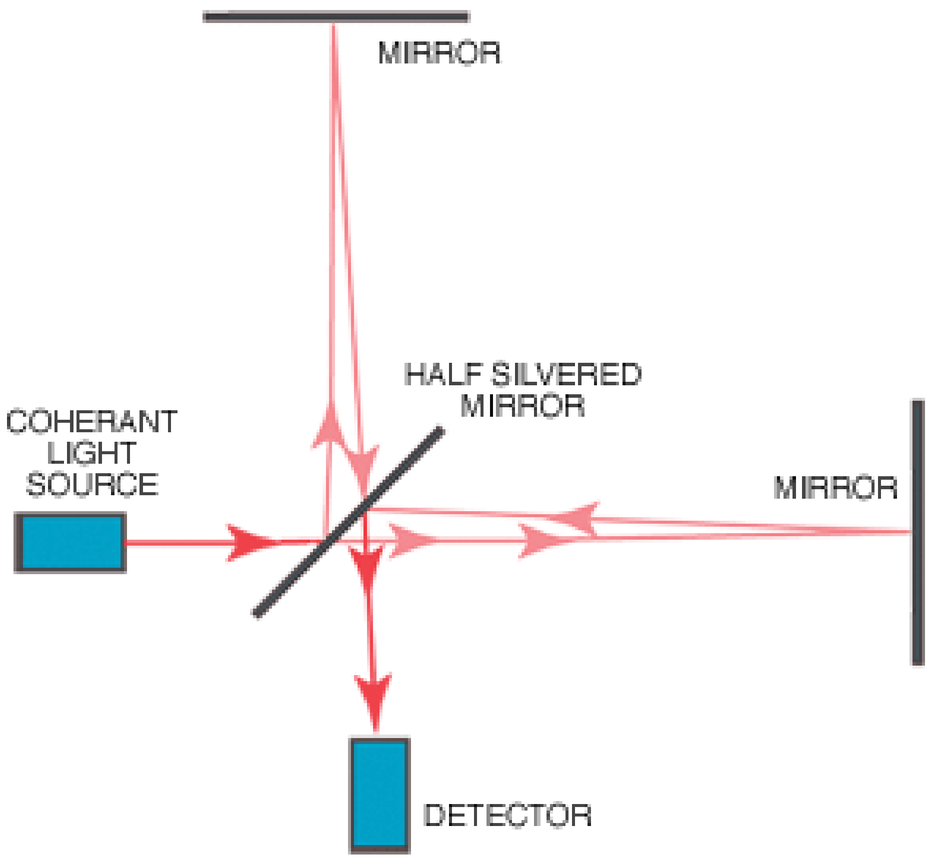


Three test masses



Sensing relative motions of distant free masses

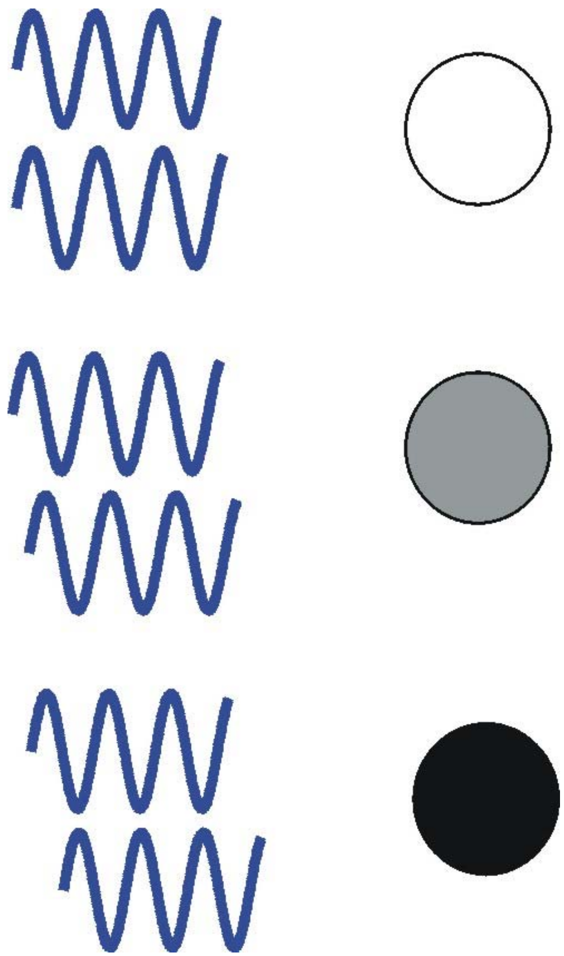
Michelson interferometer



A length-difference-to-brightness transducer

Wave from x arm.

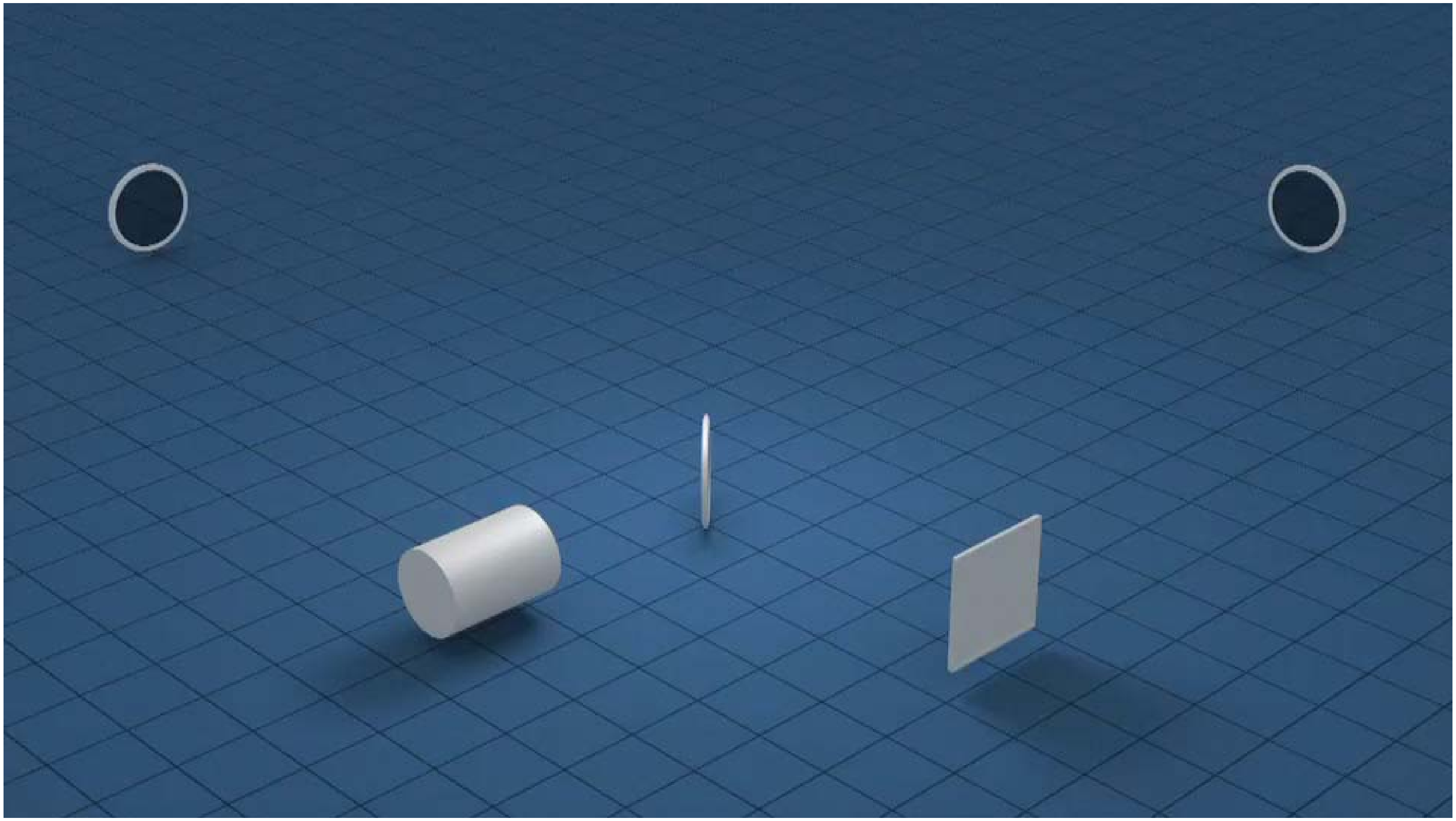
Wave from y arm.



Light exiting from beam splitter.

As relative arm lengths change, interference causes change in brightness at output.

“measur[ing] the relative acceleration of pairs of particles” with a Michelson interferometer



Rai Weiss envisions LIGO in 1972

(V. GRAVITATION RESEARCH)

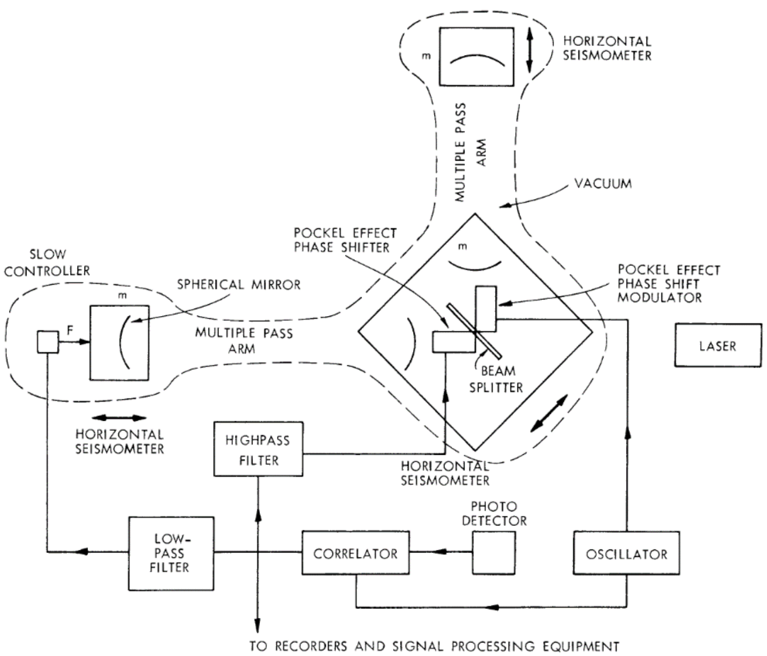


Fig. V-20. Proposed antenna.

Weiss thought about Weber's claimed detections. True or not, he saw how to do many orders of magnitude better, by implementing Pirani's free-test-masses-measured-by-lasers as a Michelson interferometer. Arms could be kilometers long. Lasers could measure sub-nuclear distances. $\Delta L/L \sim 10^{-21}$ could be achieved.

The greatest unpublished paper in 20th century experimental physics?

Rai never published this paper. It appeared in a Quarterly Progress Report for MIT's Research Lab of Electronics:

https://dspace.mit.edu/handle/1721.1/56271

It lays out a plausible design for a kilometer-scale interferometric detector. Most importantly, it gives a *tour de force* analysis of almost every noise source that needs to be taken into account.

LIGO was born right here.

Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

A wave propagating in the *z*-direction can be described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Two free parameters implies two polarizations

Here is Rai Weiss's calculation, as he learned to do it from Pirani

- Rai knew that an interferometer compares the light travel time through one arm with the light travel time through the other arm.
- For light moving along the x axis, we are interested in the interval between points with non-zero *dx* and *dt*, but with *dy* = *dz* = 0:

$$ds^2 = 0 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2$$

Solving for variation in light travel time: start with *x* arm

$$ds^2 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2 = 0$$

h(t) can have any time dependence, but for now assume that *h(t)* is constant during light's travel through ifo.

Rearrange, take square root, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11}\right) dx$$

then integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$

Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

"Transverse-traceless (TT) gauge"

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

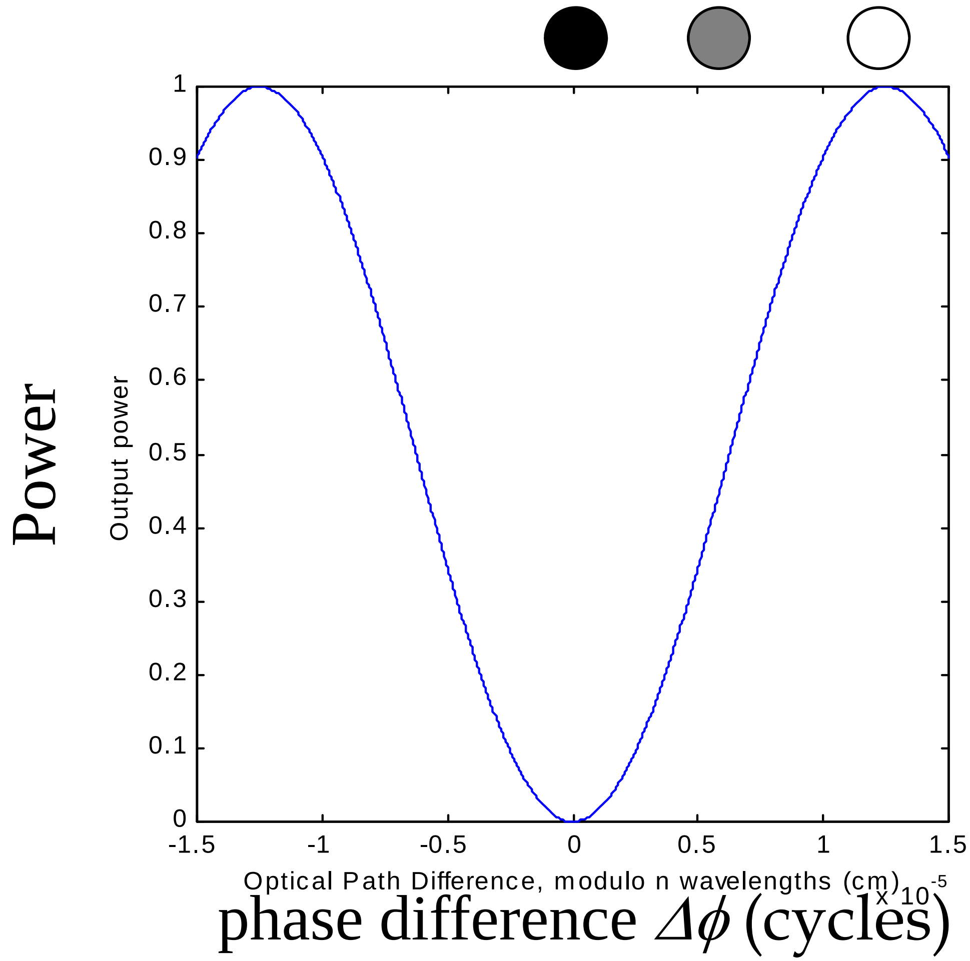
y-arm ($h_{22} = -h_{11} = -h$):

$$\Delta t_y = -hL / c$$

Difference between x and y round-trip times:

$$\Delta \tau = 2hL / c$$

Interferometer output vs. arm length difference



Interpretation

A gravitational wave's effect on one-way travel time:

$$\Delta t = \frac{h}{2}\frac{L}{c}$$

Just as if the arm length is changed by a fraction

$$\frac{\Delta L}{L} = \frac{h}{2}$$

In the TT gauge, we say that the masses didn't move (they mark coordinates), but that the separation between them changed.

Metric of the space between them changed.

Comparison with rigid ruler, force picture

We can also interpret the same physics in a different picture, using different coordinates. Here, define coordinates with rigid rods, not free masses.

With respect to a rigid rod, masses <u>do</u> move apart. In this picture, it is as if the gravitational wave exerts equal and opposite forces on the two masses.

This is the best way to understand Weber's bars.

Extra material

People often wonder about the “rubber ruler puzzle”

If a gravitational wave stretches space, doesn’t it also stretch the light traveling in that space?

If so, the “ruler” is being stretched by the same amount as the system being measured.

And if so, how can a gravitational wave be observed using light?

How can interferometers possibly work?

A related case: the expanding universe

In cosmology, one typically uses *co-moving coordinates,* marked by freely-falling test masses (i.e., galaxies).

As the universe expands

- galaxies get farther apart
- light traveling through the universe is stretched (cosmological redshift)

Do galaxies move? Depends who you ask …

Light in an interferometer arm

Imagine many freely-falling masses along arms of interferometer.

Test case: imagine that a *step function* gravitational wave, with amplitude h and + polarization, encounters interferometer.

Along x arm, test masses suddenly farther apart by $(1+h/2)$.

Wavefronts near each test mass stay near the mass. (No preferred frames in GR!)

If the arms are stretched, then the light is stretched.

The arms of an interferometer are lengthened by a gravitational wave.

The wavelength of the light in an interferometer is also lengthened by a gravitational wave, by the same factor.

OK, so how can interferometers work?

The argument given above proves that there is no *instantaneous* response to a gravitational wave.

But, we don't just care about the instantaneous response. We watch the entire history of the interferometer output.

The time-dependent response

The x arm was lengthened by the gravitational wave.

Light travels at *c*. So, light will start to arrive late, as it has to traverse longer distance than it did before the wave arrived.

Delay builds up until all light present at wave's arrival is flushed out. Then delay stays constant at $\Delta\tau = h(2NL/c)$.

Consider the DC response …

New light produced by the laser (after gravitational wave has passed by) isn’t affected by the gravitational wave.

(Its wavelength is determined by the length of a rigid resonant cavity.)

So if we wait to measure using all “new light”, it must reveal the changed arm lengths.

We never (or never should have) said that we were using light as a ruler.

Pirani taught us to use the travel time of light signals between free masses to sense the passage of a gravitational wave.

That is what Rai Weiss did from the beginning.

In the end, there is no puzzle: Interferometers *can* work.