

Searches for continuous gravitational waves

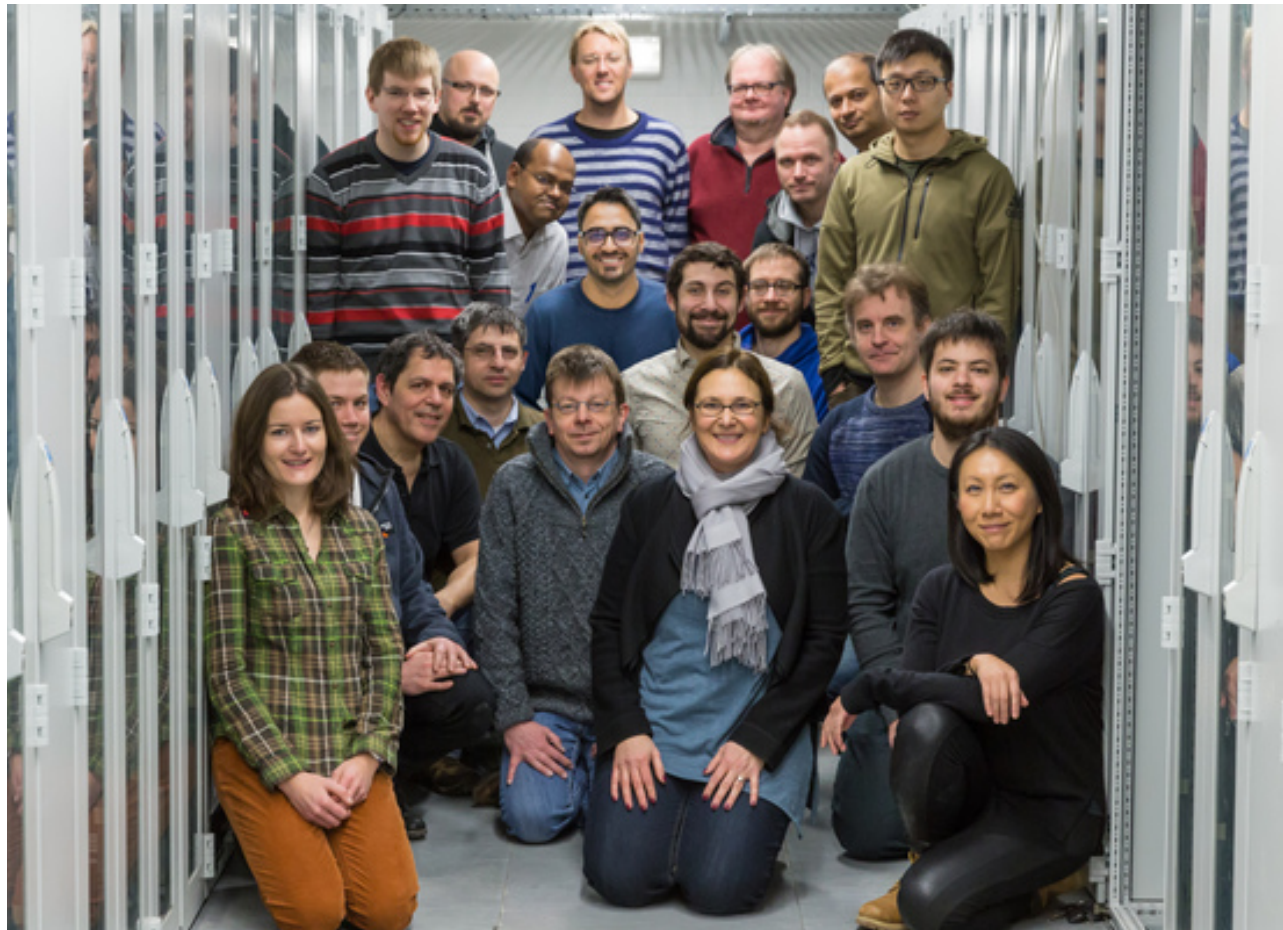


M. ALESSANDRA PAPA

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GOLM AND HANNOVER, GERMANY**



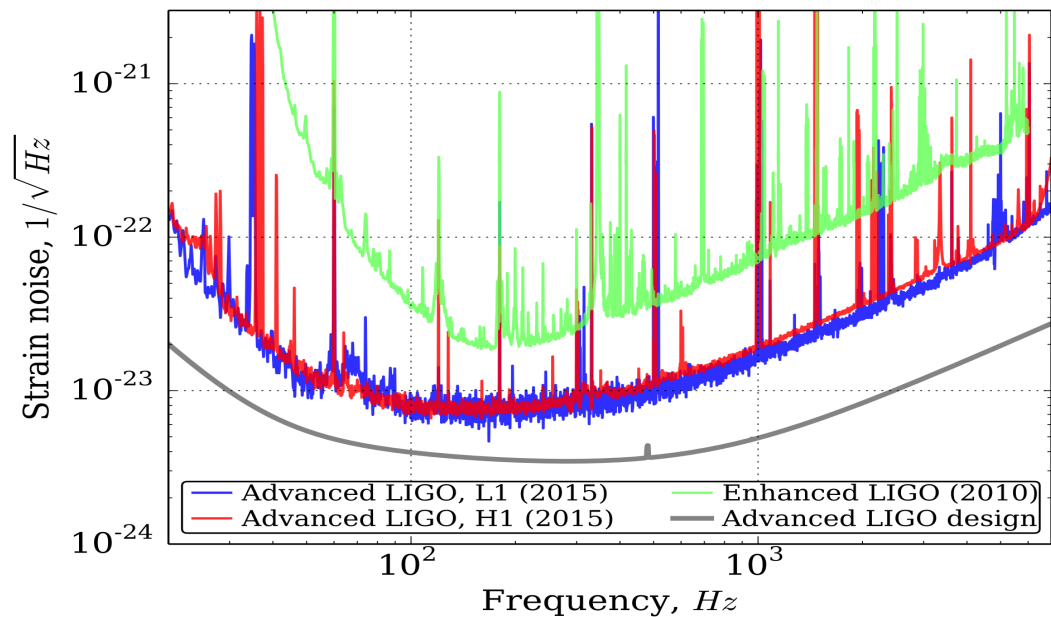
- PhD in Rome (Italy) ~ 20 years ago.
- Post doc in Potsdam
- Scientist, Faculty at UWM, Milwaukee (USA)
- Group Leader @ AEI, Max Planck Inst. for gravitational Physics (Germany):
www.aei.mpg.de
- Largest institute devoted to General Relativity and GW detection in the world.
- Have a family incl. 2 kids and 3 cats and other interests other than physics and science



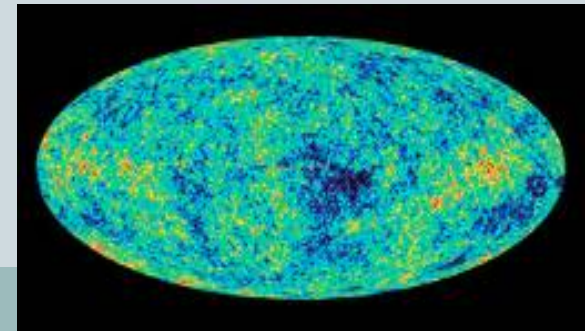
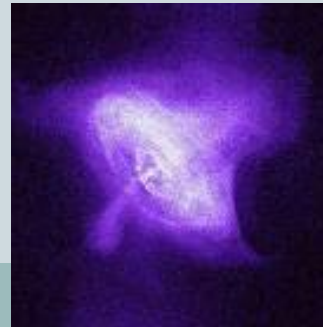
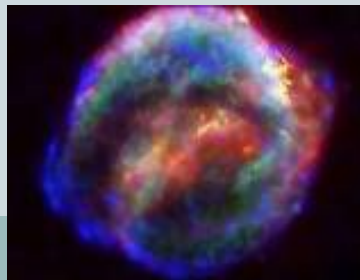
www.aei.mpg.de/24934/04_Continuous_Waves

What signals may be detectable ?

time scales of ms to s, compact objects, high accelerations:



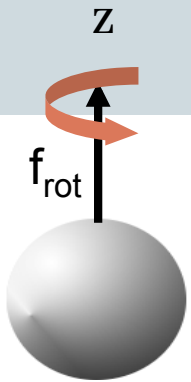
- from inspiraling compact objects
- bursts, typically arising from catastrophic events
- continuous quasi-periodic waves
- stochastic background of gravitational radiation



RECAP: signal from an isolated NS

4

- Nearly perfectly monochromatic signal at the source (there is a small spindown).
- Emitted by compact object as they rotate when they have a non-zero ellipticity ε
- Ellipticity values are highly uncertain and may reach values as high as 10^{-4} .



**Bumpy
Neutron Star**

- $f_{\text{gw}} = 2 f_{\text{rot}}$ and the GW amplitude at the detector at a distance d from source is :

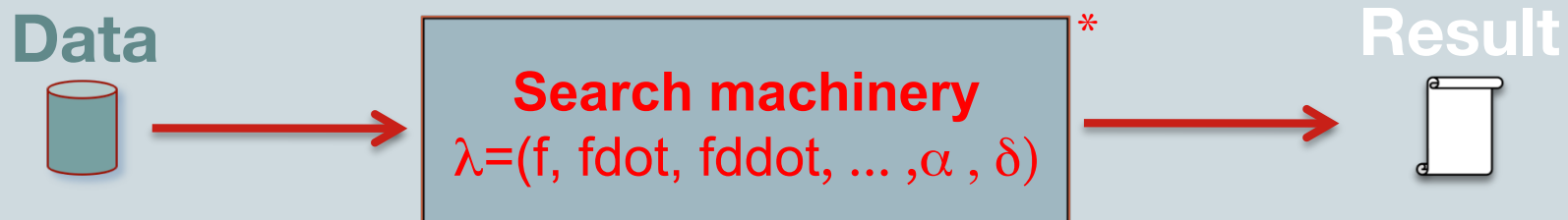
$$\varepsilon = \frac{|I_{xx} - I_{yy}|}{I_{zz}}$$

$$h_0 = \frac{4\pi^2 G I_{zz} \varepsilon f_{\text{gw}}^2}{c^4 D}$$

Searching for a CW signal



- A CW waveform is typically defined by $\mathbf{a}=(f, \dot{f}, \ddot{f}, \dots, \alpha, \delta, \cos \iota, \psi, \phi_0, h_0)$
- Searching for a signal like this means



* Usually we do not need to explicitly search over $\cos \iota, \psi, \phi_0, h_0$

Result of a search



Result



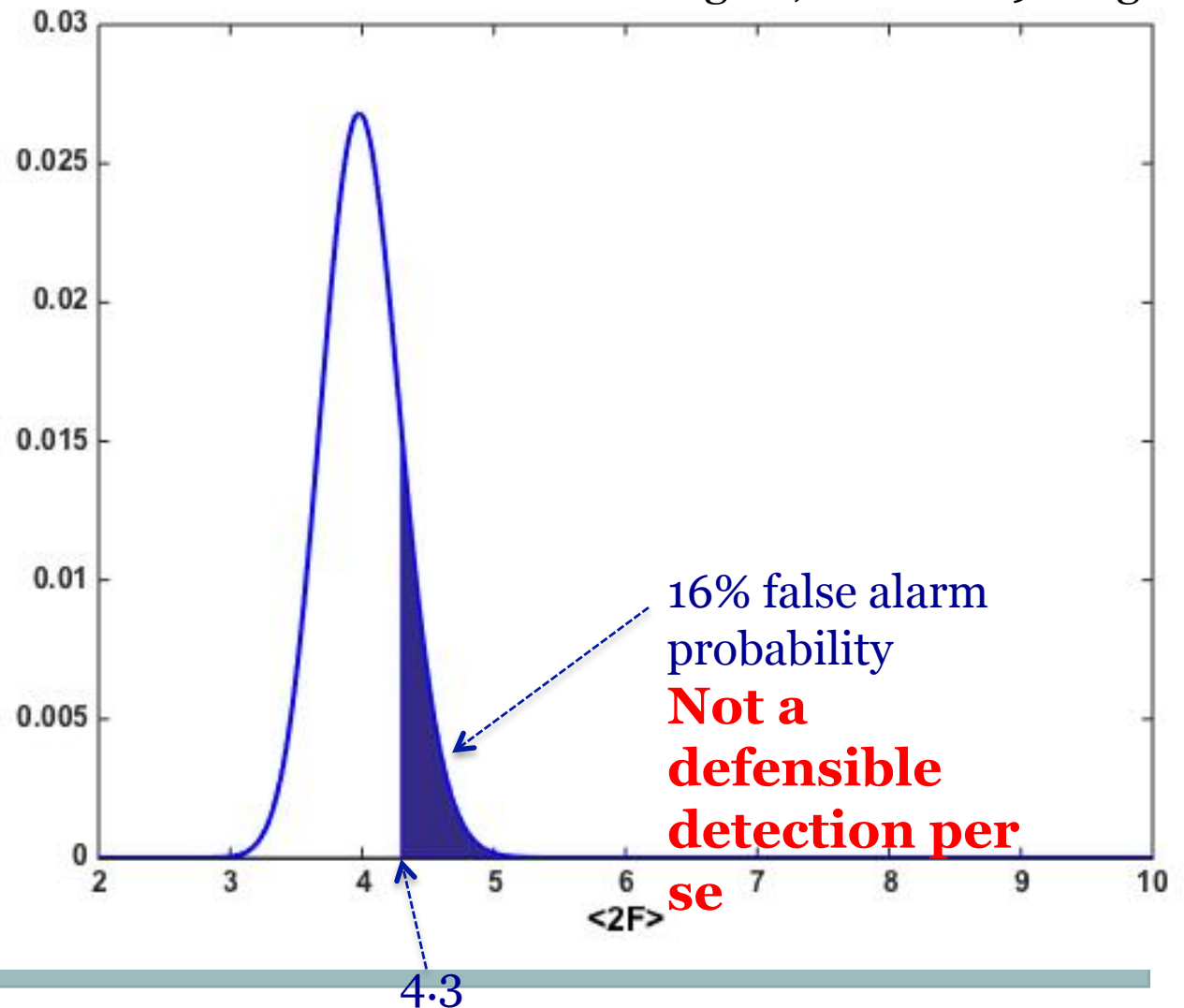
- This is detection statistic, a score, that tells us how likely it is that there is a signal like the one we are looking for.
- When the data contains noise, the detection statistic value is a realization of a random variable
- Based on the value of the random variable we can decide whether the data contains a signal or not

How do we decide that we have detected something ?



- detection statistic ($\langle 2F \rangle$) is like a score measuring likelihood of having a detection
- imagine we were testing only 1 point in parameter space (1 wave-shape):
 $\langle 2F^* \rangle = 4.3$ from the data
- standard hypothesis testing:

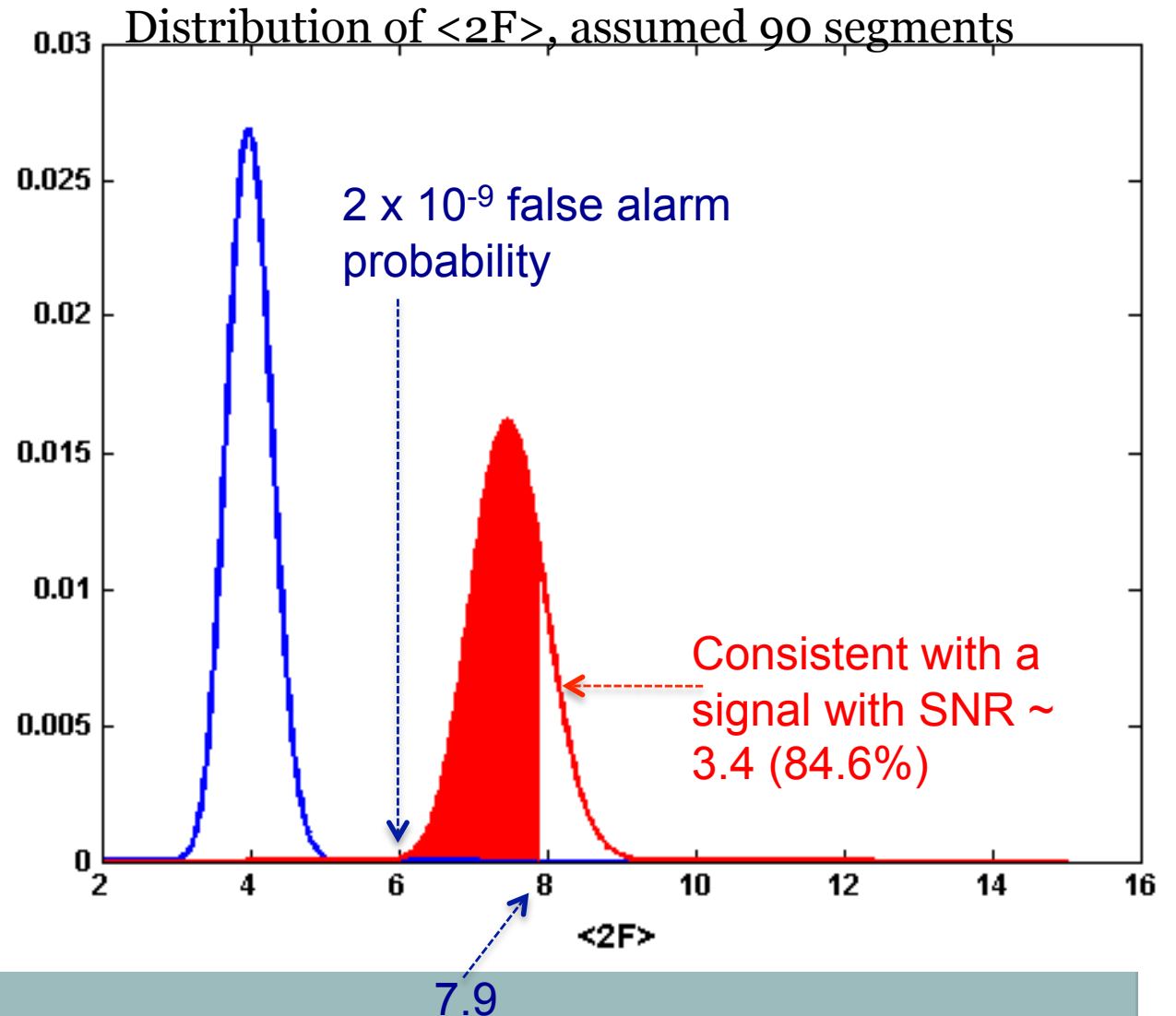
Distribution of $\langle 2F \rangle$ if no-signal, assumed 90 segments



This is what a confident detection would look like



- detection statistic ($\langle 2F \rangle$) is like a score measuring the likelihood of having a detection
- imagine we were testing only 1 point in parameter space (1 wave-shape):
 $\langle 2F^* \rangle = 7.9$ from the data
- standard hypothesis testing:



The trials factor

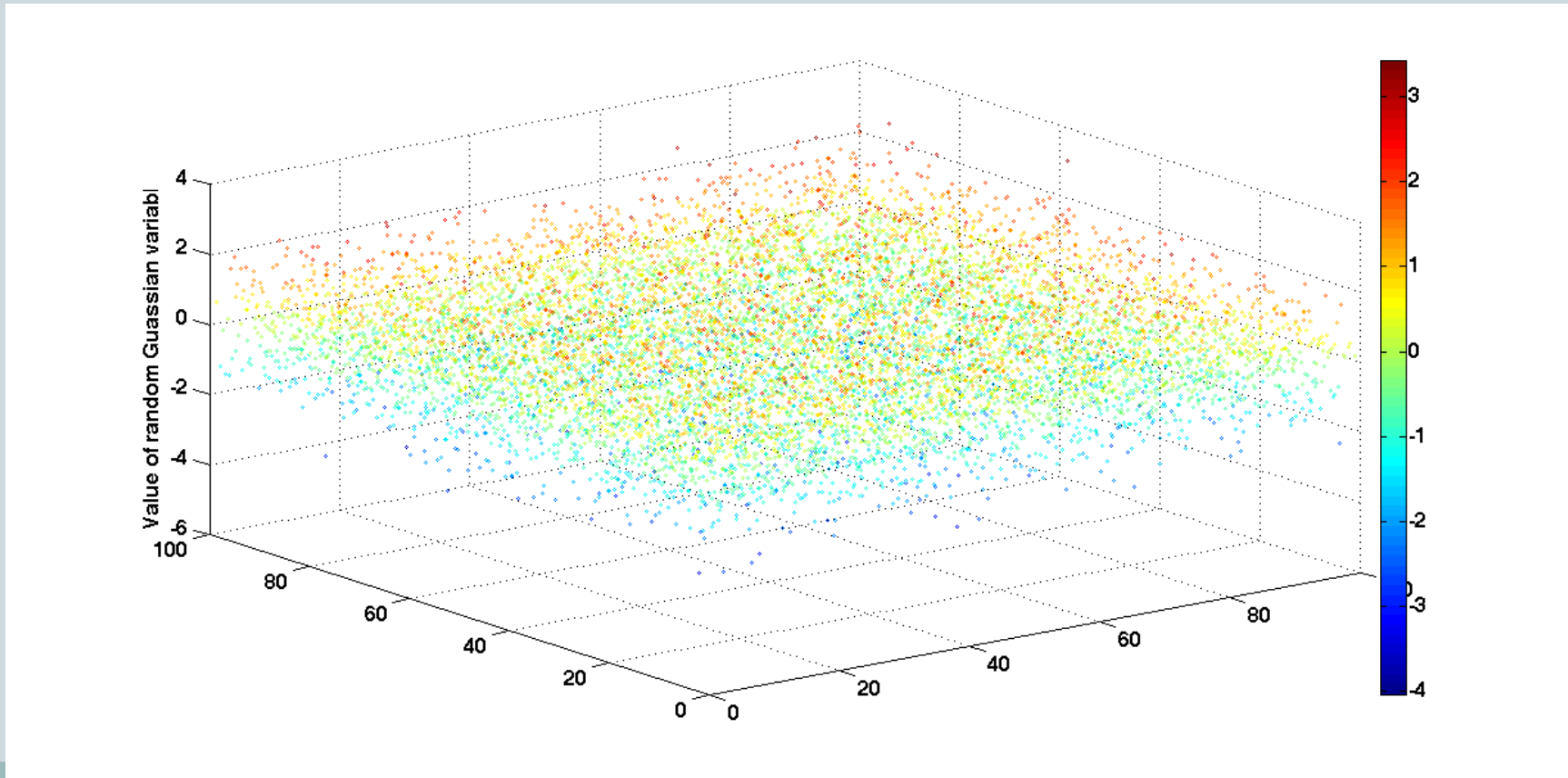


The trials factor



- A CW waveform is typically defined by $a=(f, \dot{f}, \ddot{f}, \dots, \alpha, \delta, \cos \iota, \psi, \phi_0, h_0)$
- In many instances we look for signals with unknown parameters → template banks
- We get many values of the detection statistic, one per searched waveform
- If we're lucky we find *one* signal, so the bulk of our results are due to noise

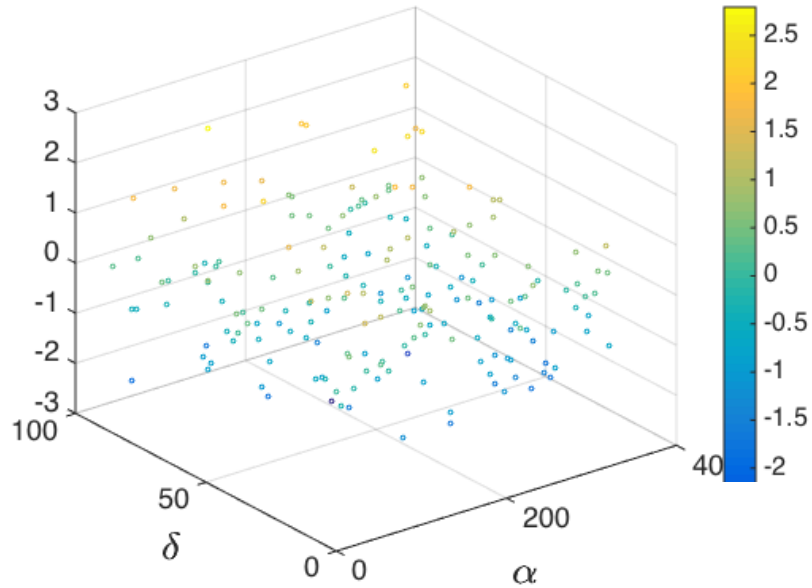
Is there a signal here ?



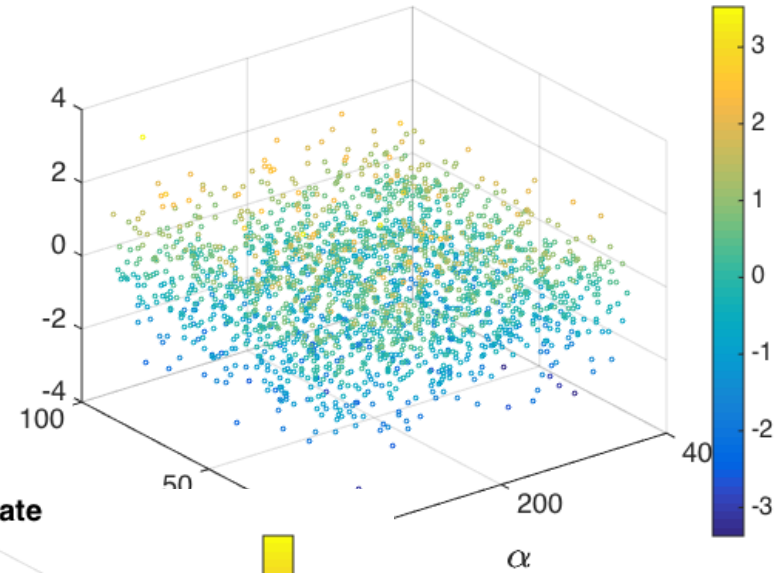
The more you search, the more you find, just by accident



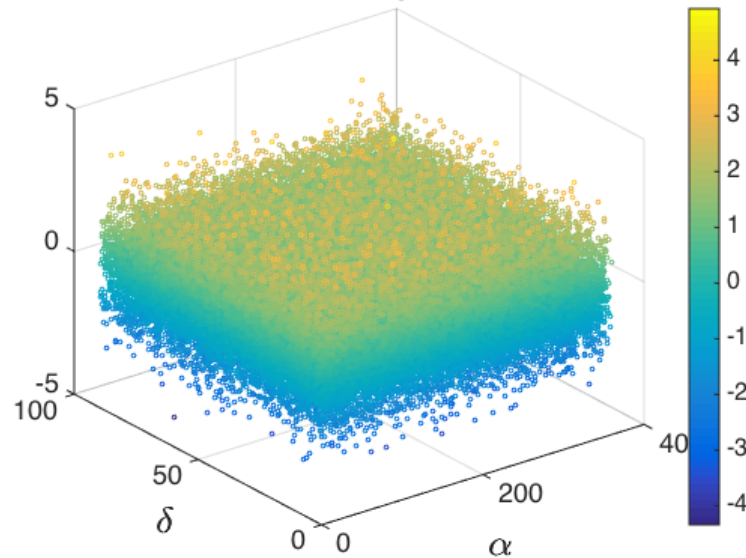
200 templates



2000 template

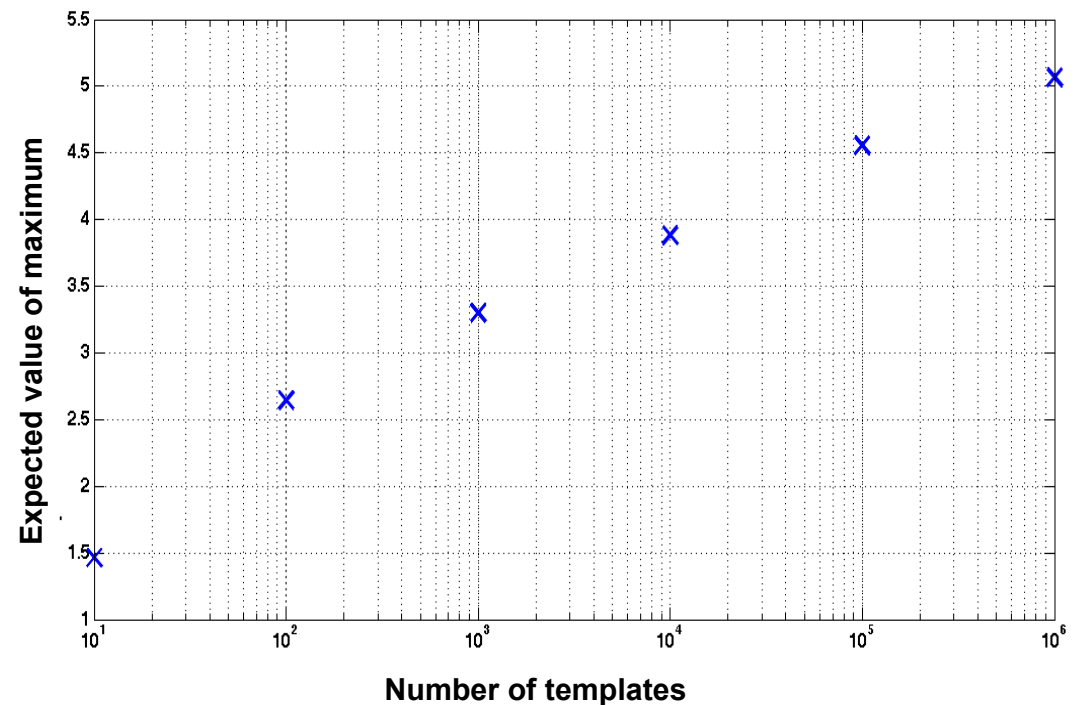


80000 template



This is the (concept of the) trials factor

- As the number of search templates increases :
 - the chances of an accidental high value of the detection statistic, increase
 - If we want the same significance, the minimum detectable signal has to increase → we loose sensitivity



Upper limits



Upper limits



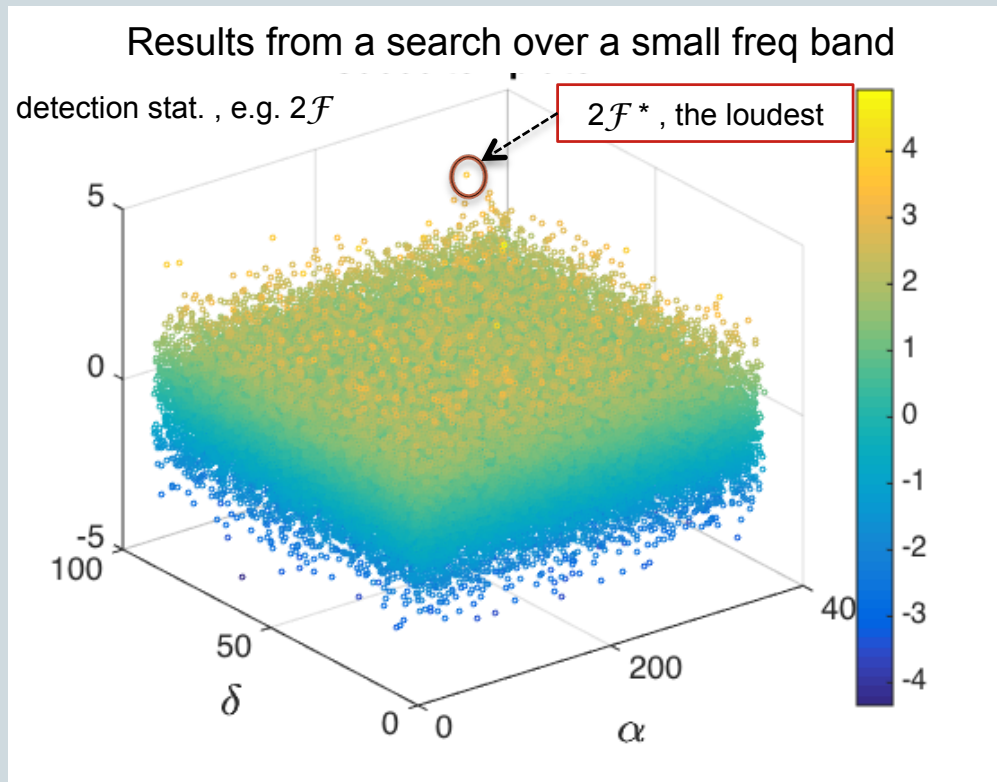
- Since the first operation of LIGO and Virgo searches for CW signals have been carried but no detection has been made so far
- Can these null results tell us anything ?

Upper limits



- Since the first operation of LIGO and Virgo searches for CW signals have been carried but no detection has been made so far
- Can these null results tell us anything ?
 - YES. They tells about what is not there.

GW amplitude upper limits and the loudest event



- None of these det stats is significant enough with respect to random fluctuations. In particular the highest (loudest) one, $2\mathcal{F}^*$, isn't.
- We pretend that the loudest is due to a signal and ask:
 - What is the smallest h_0 that would produce such $2\mathcal{F}^*$?

GW amplitude upper limits and populations



- The GW amplitude h_0 is one of the factors determining the strength of a signal at the detector ($2\mathcal{F}^*$):
 - Frequency (sensitivity of the detector)
 - Position in the sky
 - Inclination angle, ι
 - Polarization
- When we measure a $2\mathcal{F}^*$ and want to determine the smallest h_0 that would produce it, we have to factor-in the effect of these other variables:
 - **We imagine a population of sources**
 - ✦ In a small frequency range where det sens is \sim constant
 - ✦ All-sky
 - ✦ Uniformly distributed $\cos \iota$
 - ✦ Uniformly distributed pol angle
 - At fixed h_0 we determine the corresponding distribution of $2\mathcal{F}$
 - We find the h_0 such that a large fraction, say 90%, of the $2\mathcal{F}$ values are larger than the measured $2\mathcal{F}^*$
 - That is the 90% confidence GW amplitude upper limit

This is a frequentist upper limit



- We set up a population of sources
- We find the h_0 such that 90% of them (frequency of occurrence) would have yielded a value of the detection statistic higher than the highest one that we measured
- We call that 90% , our confidence

Bayesian upper limit



- In Bayesian theory one computes the posterior probability for a given signal, given the data:

$$p(a | \{x\}) \propto p(a) \cdot p(\{x\} | a)$$

posterior prob
on signal

prior

prob of data given signal

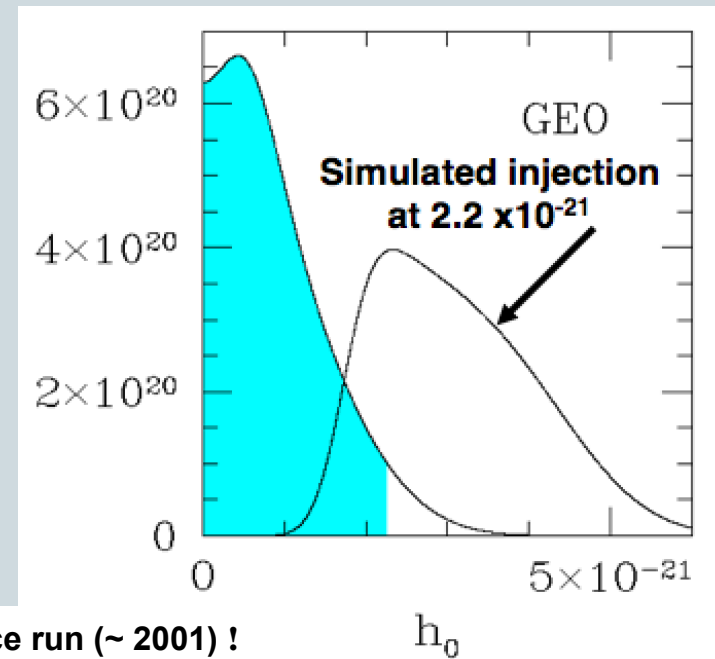
Bayesian upper limit



- We marginalize over the unknown parameters $\phi_0, \psi, \cos\iota$

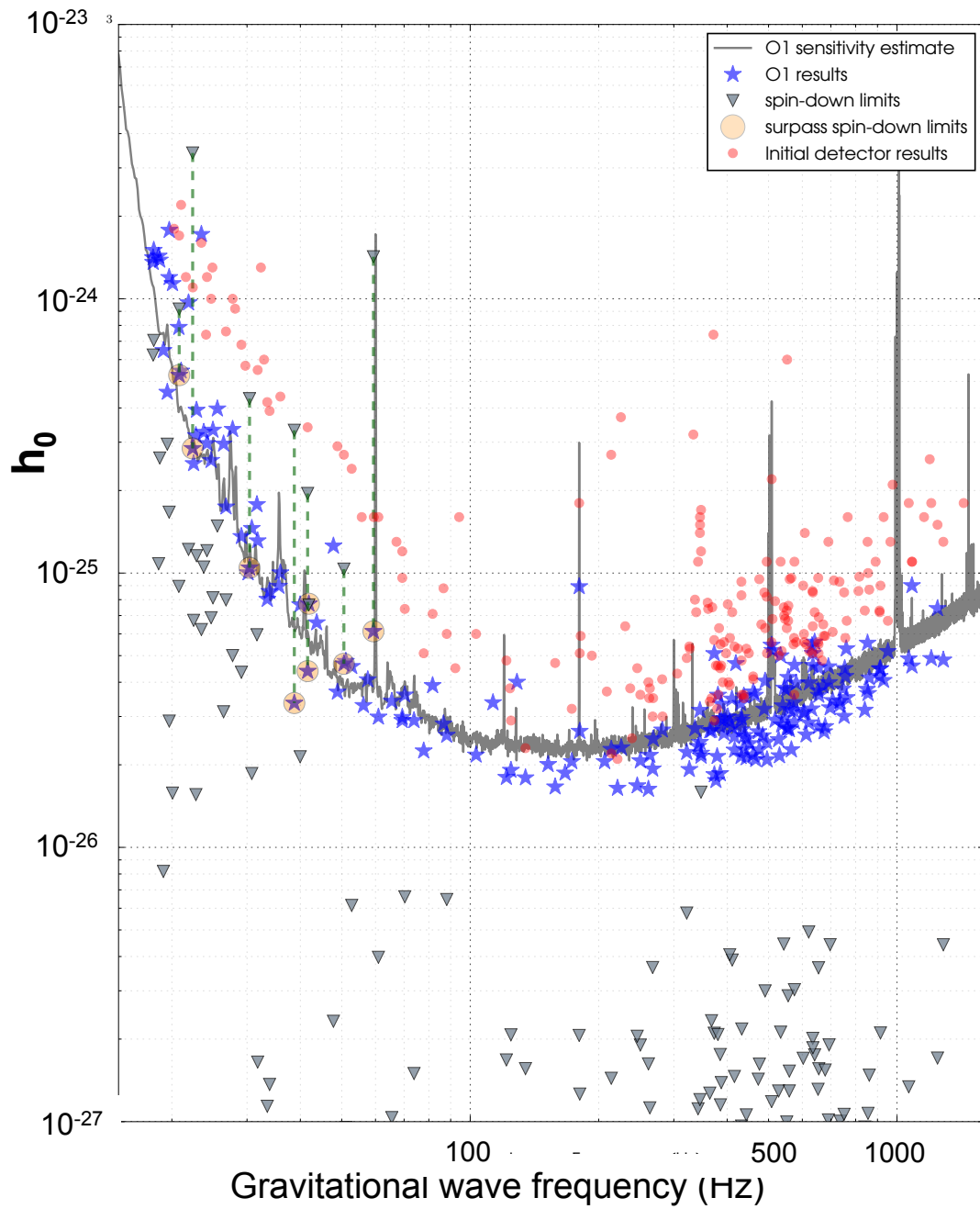
$$p(h_0 | \{x\}) \propto \iiint p(\{x\} | h_0, \phi_0, \psi, \cos\iota) d\phi_0 d\psi d\cos\iota$$

- integrate to the required total probability (confidence) level and read-off the h_0 upper limit value



from LIGO's first science run (~ 2001) !

O1 (first Advanced LIGO science run) results



Targeted searches



- We know of thousands of pulsars
- We know their positions, rotation frequency and spin down
- Obvious to start searching for continuous GWs from these objects
- Searches are fairly straightforward: need to search only for a single waveform. A coherent, highly sensitive search possible.

The spin-down limit GW amplitude



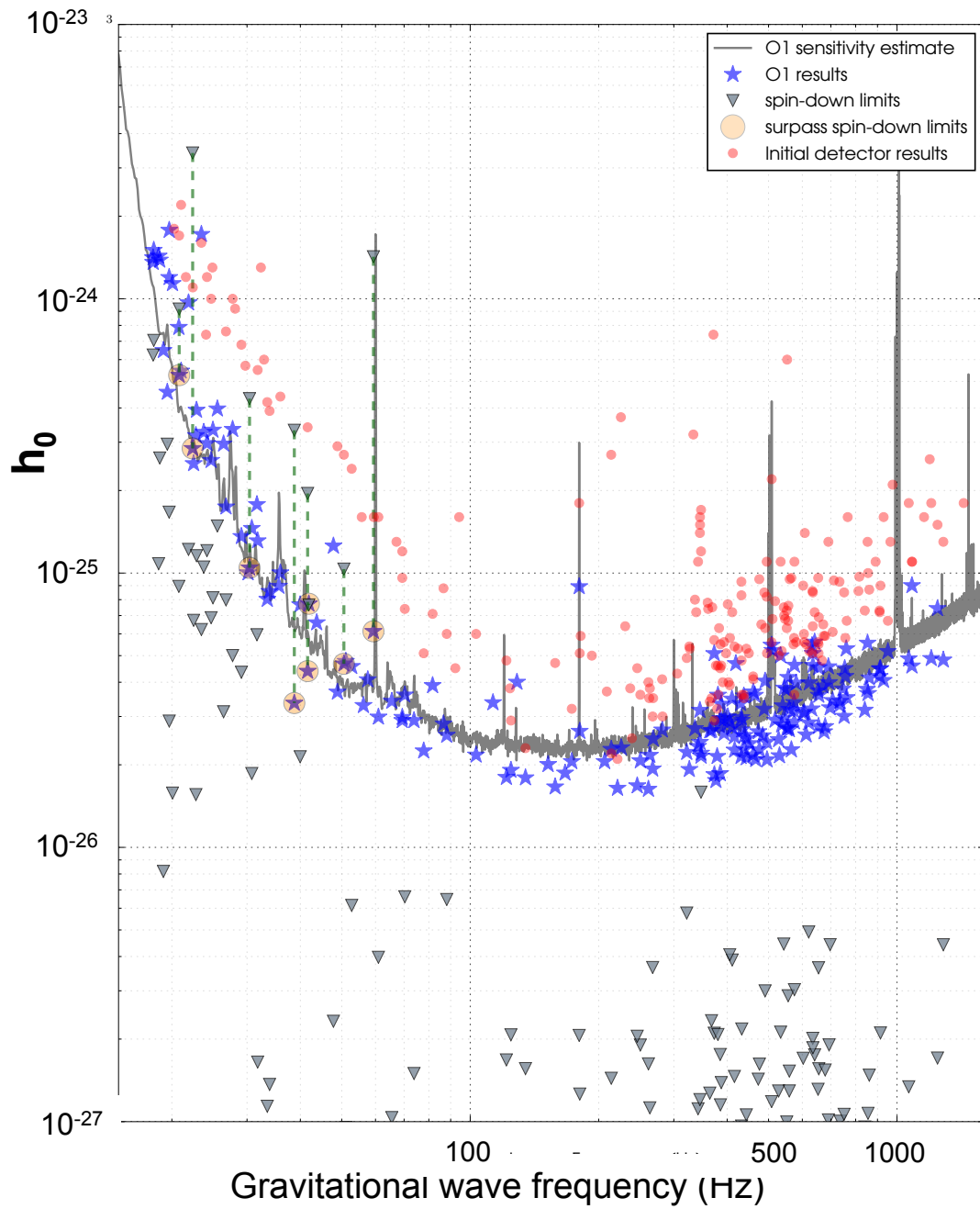
- Periods and period-evolution of pulsars are routinely measured
- In particular pulsars spin-down rates are known
- If all lost rotational energy goes in GWs, knowing the distance D , we can derive the corresponding GW amplitude:

$$h_0 = \frac{4p^2 G I_{zz} \epsilon \dot{f}_{\text{gw}}^2}{c^4 D}$$

$$h_0^{\text{spindown}} = \frac{1}{D} \sqrt{\frac{5 G I_{zz} x |\dot{f}|}{2 c^3 f}}$$

$$\text{with } x := \frac{E_{\text{GW}}}{E_{\text{SPINDOWN}}} = 1$$

O1 (first Advanced LIGO science run) results

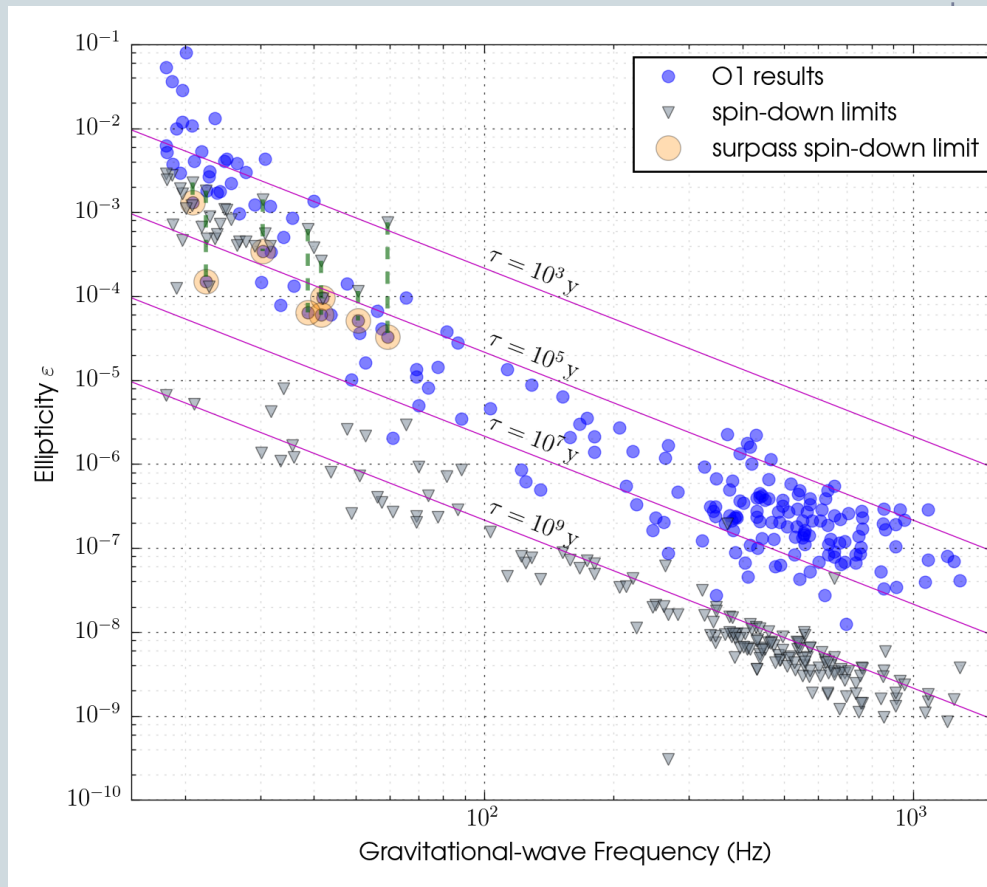


GW amplitude h_0 \leftrightarrow ellipticity ε : $h_0 = \frac{4p^2 G I_{zz} \varepsilon f_{\text{gw}}^2}{c^4 D}$



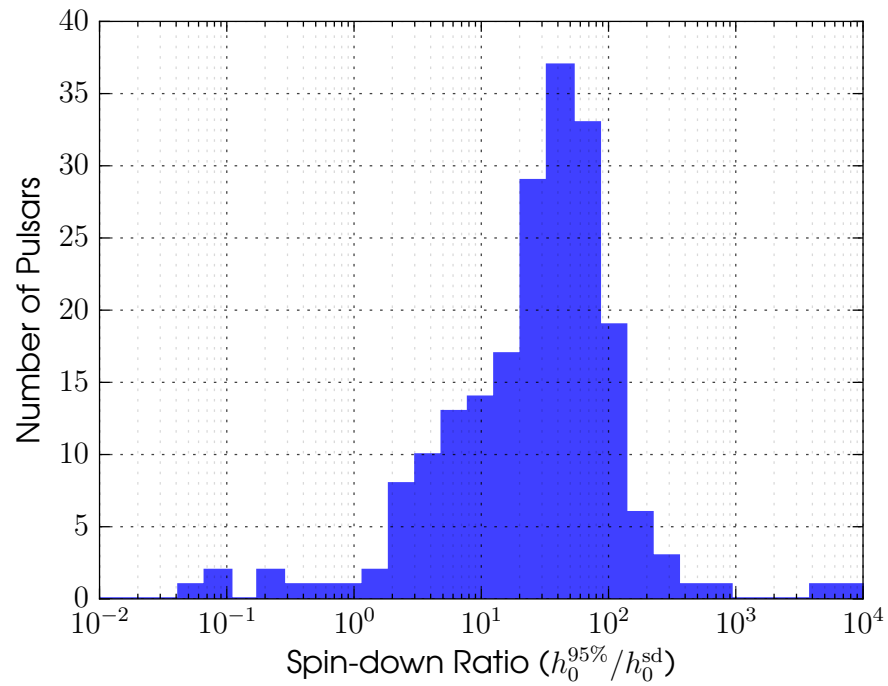
Ellipticity upper limits, look at dots

$$h_0 = \frac{4p^2 G I_{zz} \epsilon f_{gw}^2}{c^4 D}$$

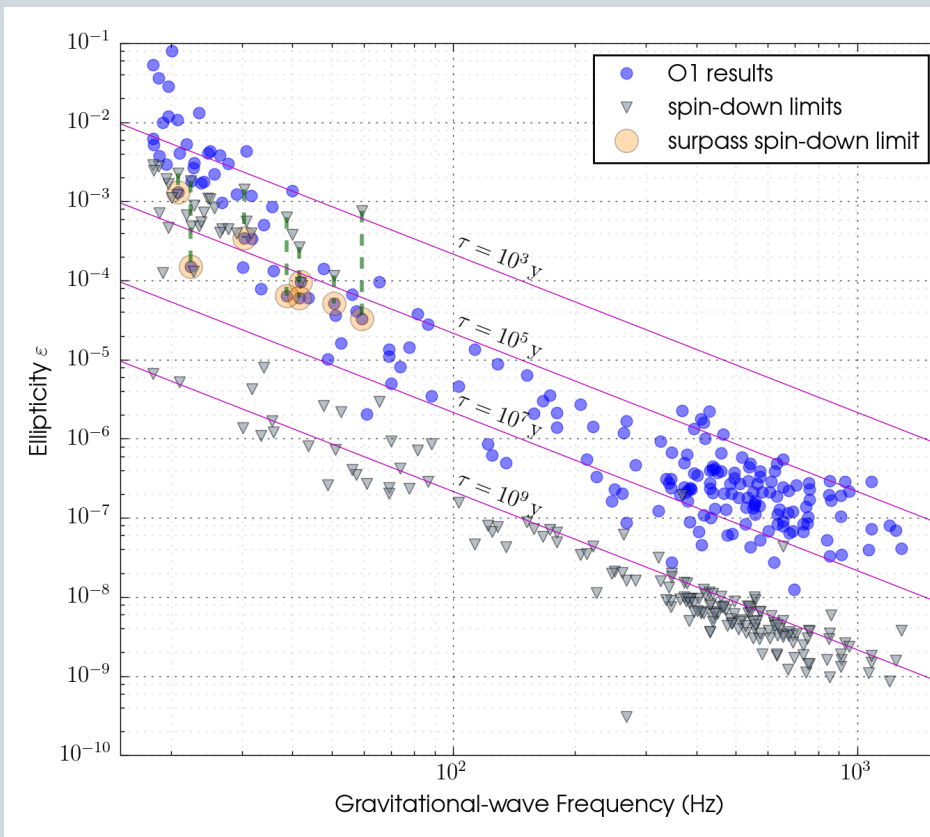


- Most constraining e UL is 1.3×10^{-8} for J0636+5129
 - 200pc
 - ~ a few above spindown limit
 - ~ 700 Hz
- @ > 300 Hz, the bulk below 10^{-6} , well within maximum predicted values
- I_{zz} taken 10^{38} kg m^2 , but higher values are possible

Distribution of spin-down ratios x



Ellipticity upper limits, now look at the lines:



$$h_0 = \frac{4p^2 G I_{zz} \epsilon f_{\text{gw}}^2}{c^4 D}$$

$$h_0^{\text{spindown}} = \frac{1}{D} \sqrt{\frac{5GI_x |\dot{f}|}{2c^3 f}}$$

with $x := \frac{E_{\text{GW}}}{E_{\text{SPINDOWN}}}$

$$\tau = \frac{f}{\dot{f}(n-1)} \quad \text{char. age} \Rightarrow$$

$$\epsilon^{\text{spindown}} = 7.64 \times 10^5 \frac{\sqrt{(n-1) I \tau}}{f^2}$$

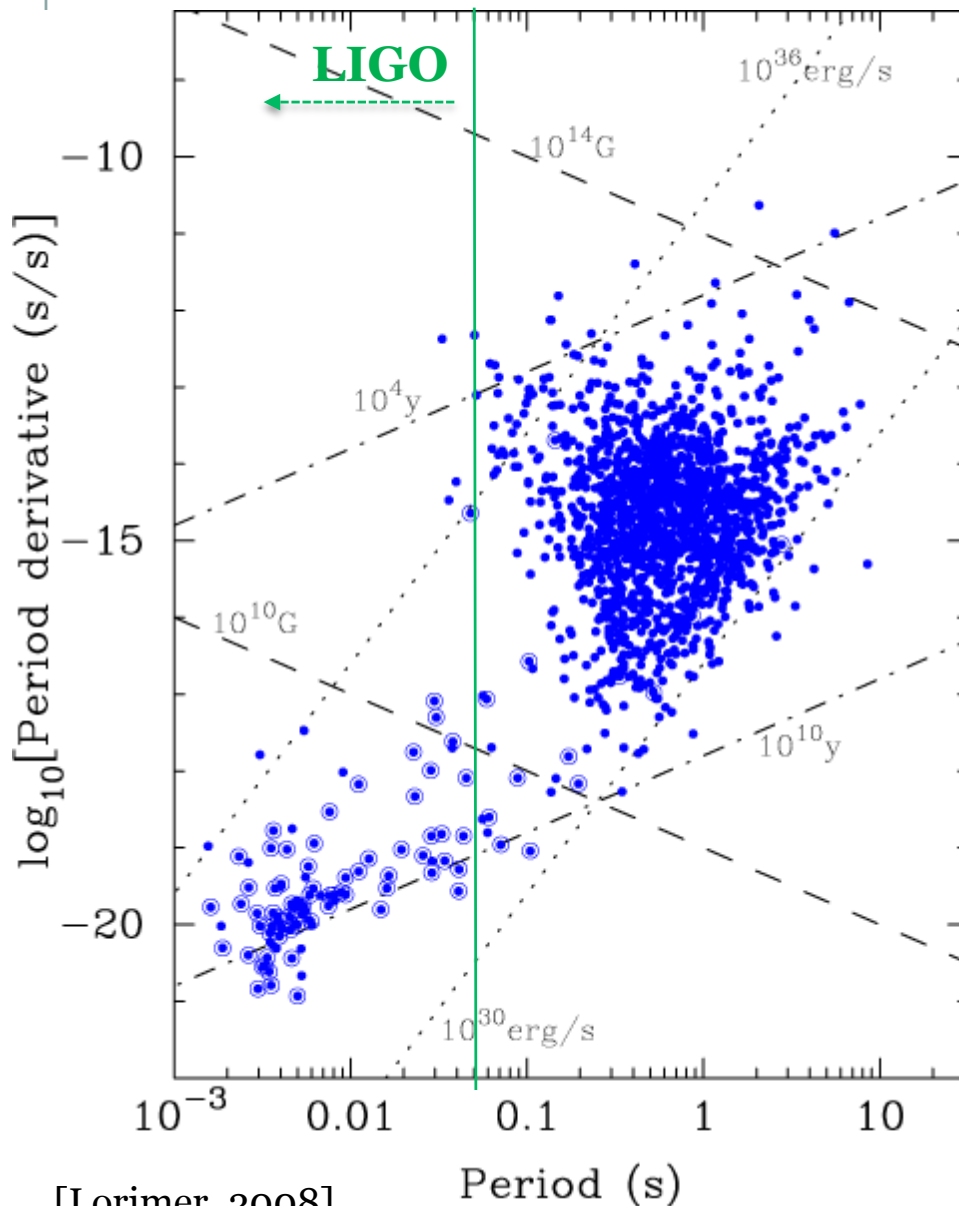
Should we look further ? Yes !



- The large majority of the objects that we know of, are not detectable (spin-down limit)
- But: there might be a compact object that we do not observe
- That is close enough
- That is bumpy enough
- That it could be emitting at a detectable level

Blind All sky Searches

31



[Lorimer, 2008]

few thousand known pulsars

40,000 millisecond pulsars in our galaxy [Lorimer, Living Rev. Relativity, 11 2008]

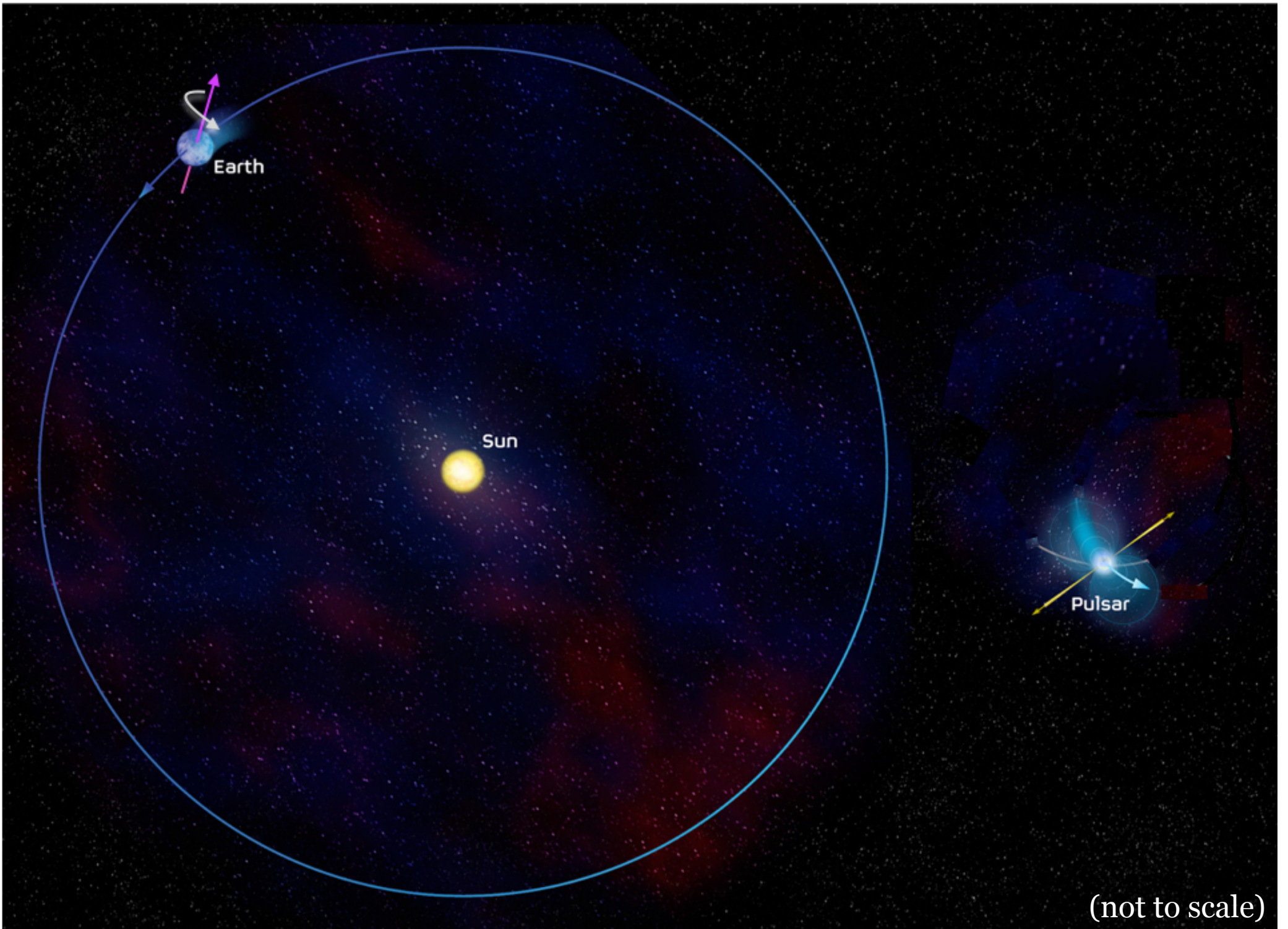
$O(10^6 - 10^7)$ undiscovered EM quiet NS within 5kpc [Narayan. *ApJ*, 1987]

Potential to discover off-axis pulsars or gravitars

All-sky surveys



- **Matched filtering**
 - Different waveforms
 - Have to search explicitly over frequency, spindowns, sky positions

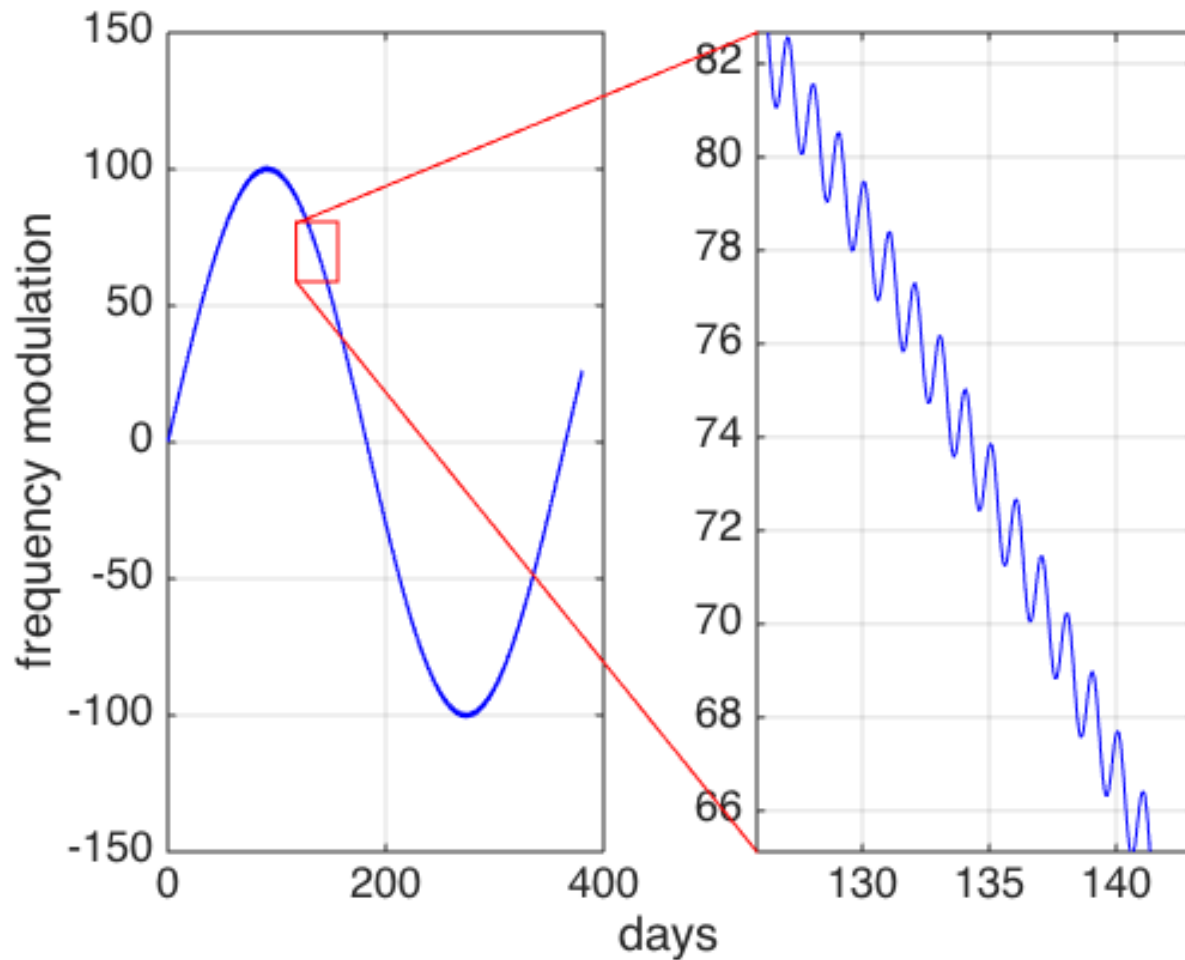


(not to scale)

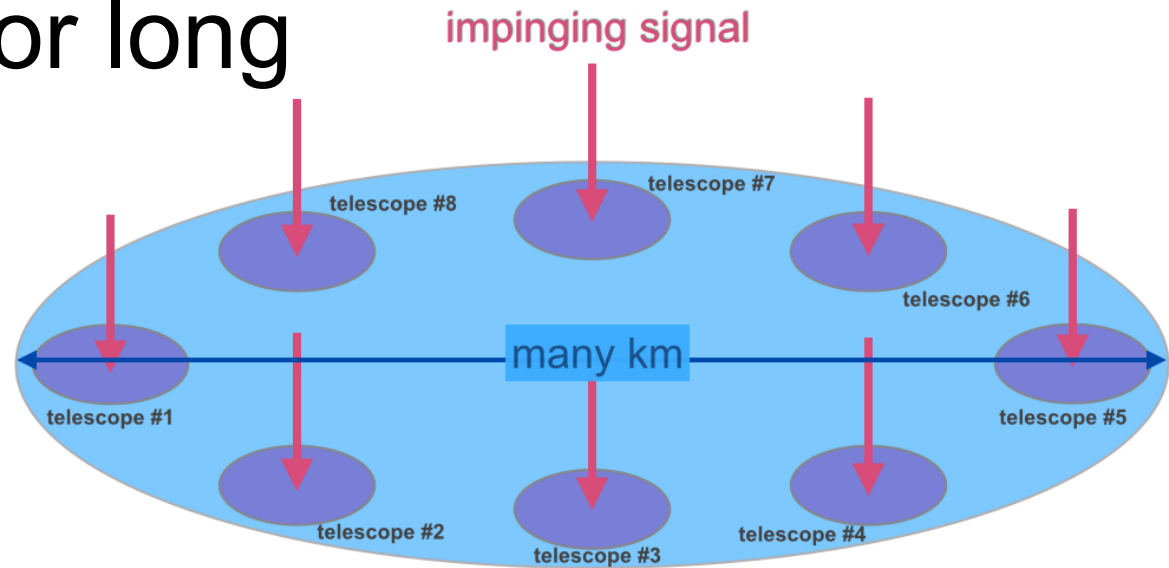
Observed signal



- frequency-modulated
- amplitude-modulated



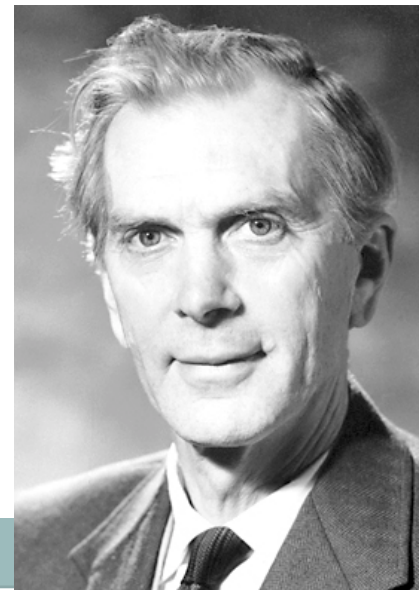
The resolution for long observations



- like aperture synthesis for radio telescopes

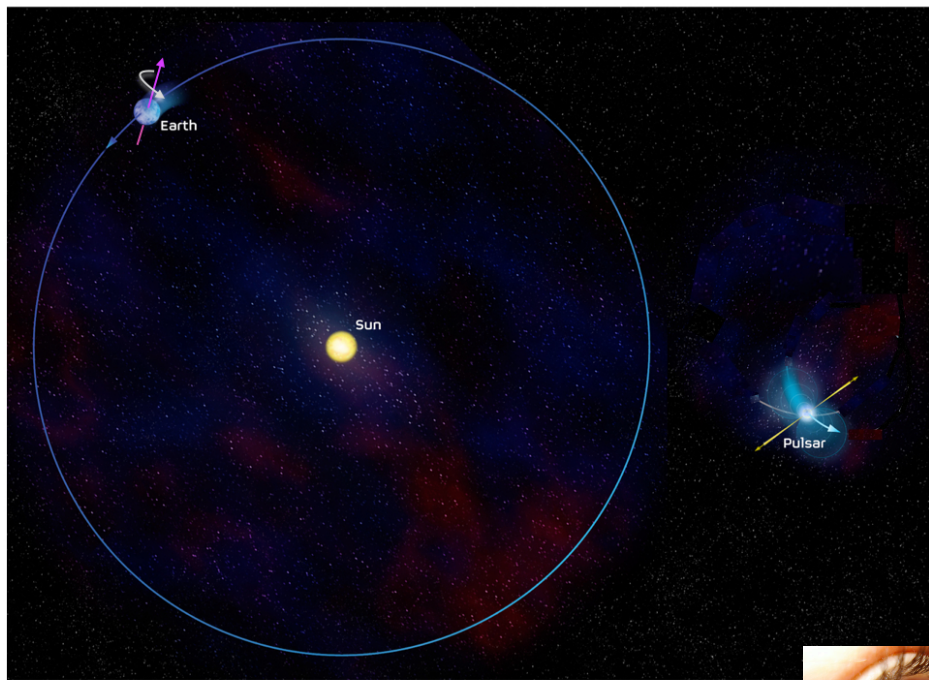
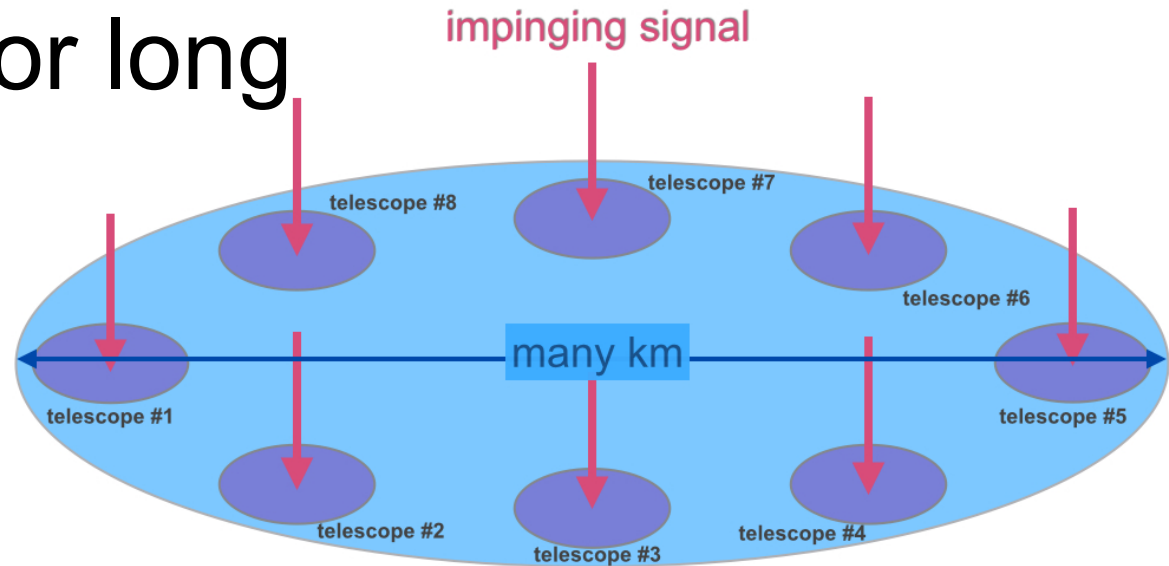


Part of VLA, Socorro, New Mexico
<https://tau0.wordpress.com/category/space/>



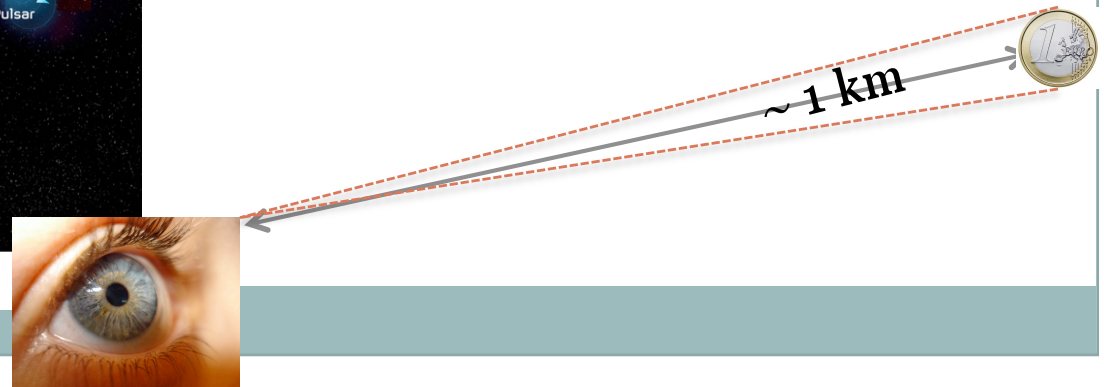
Sir M. Ryle,
Nobel Laureate 1974

The resolution for long observations



- like aperture synthesis for radio telescopes

- the baseline in this case is the diameter of the Earth's orbit around the Sun, hence yielding resolutions < 4 arcsec (@100Hz)



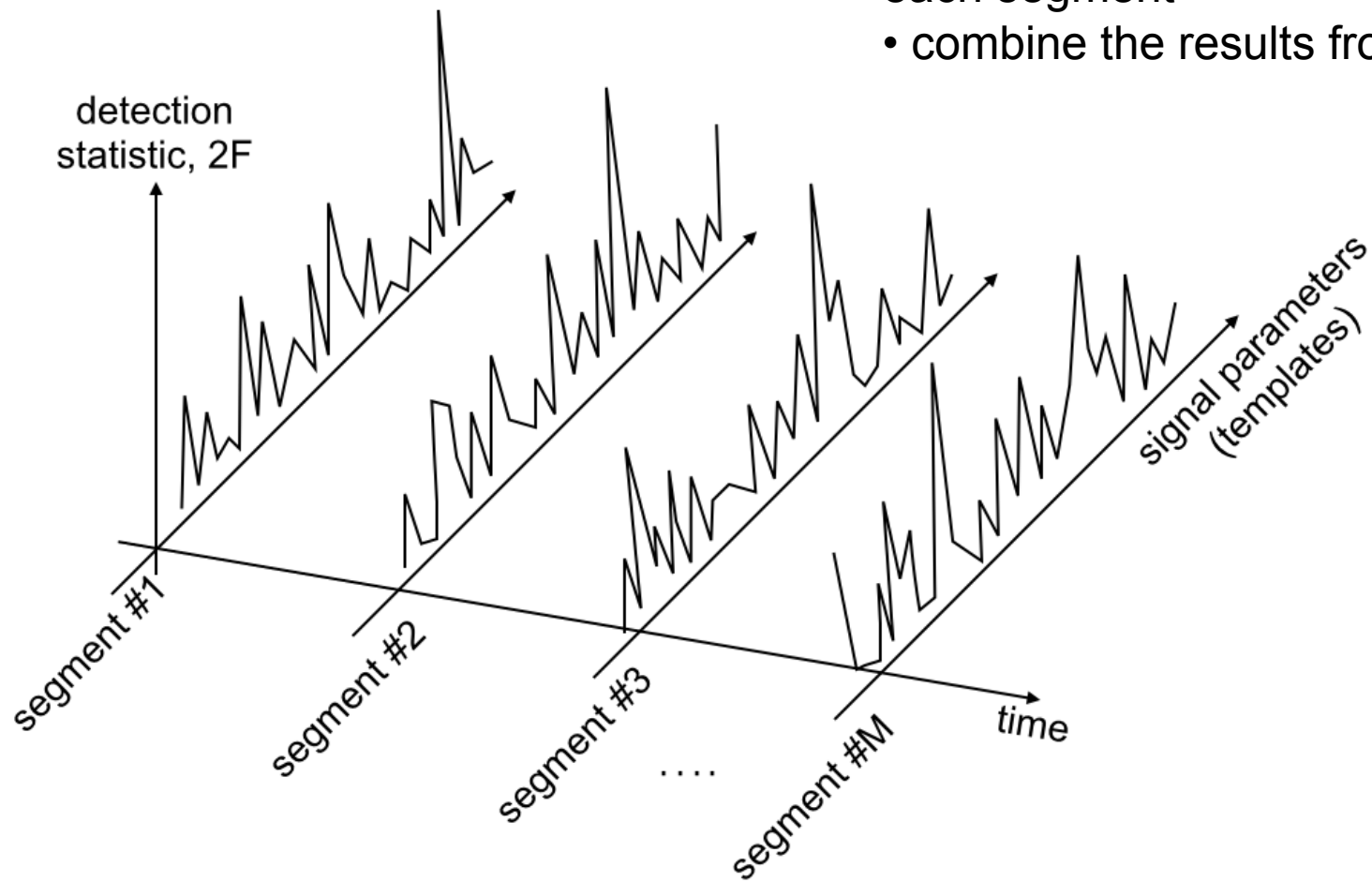
The problem of searching for unknown CW sources



- Most sensitive method (coherent) :
 - The so-called F -statistic : standard coherent matched filtering, which is a *maximum-likelihood detection* method (PRD 58, 063001, 1998).
 - Post-analysis ρ^2 (signal-to-noise ratio squared) increases with observation time, but the number of resolvable waveforms grows much faster
 - Already with a few months observing time the computational burden would be unmanageable
- Resort to hierarchical/semi-coherent search strategies, alternating coherent and non-coherent detection techniques.

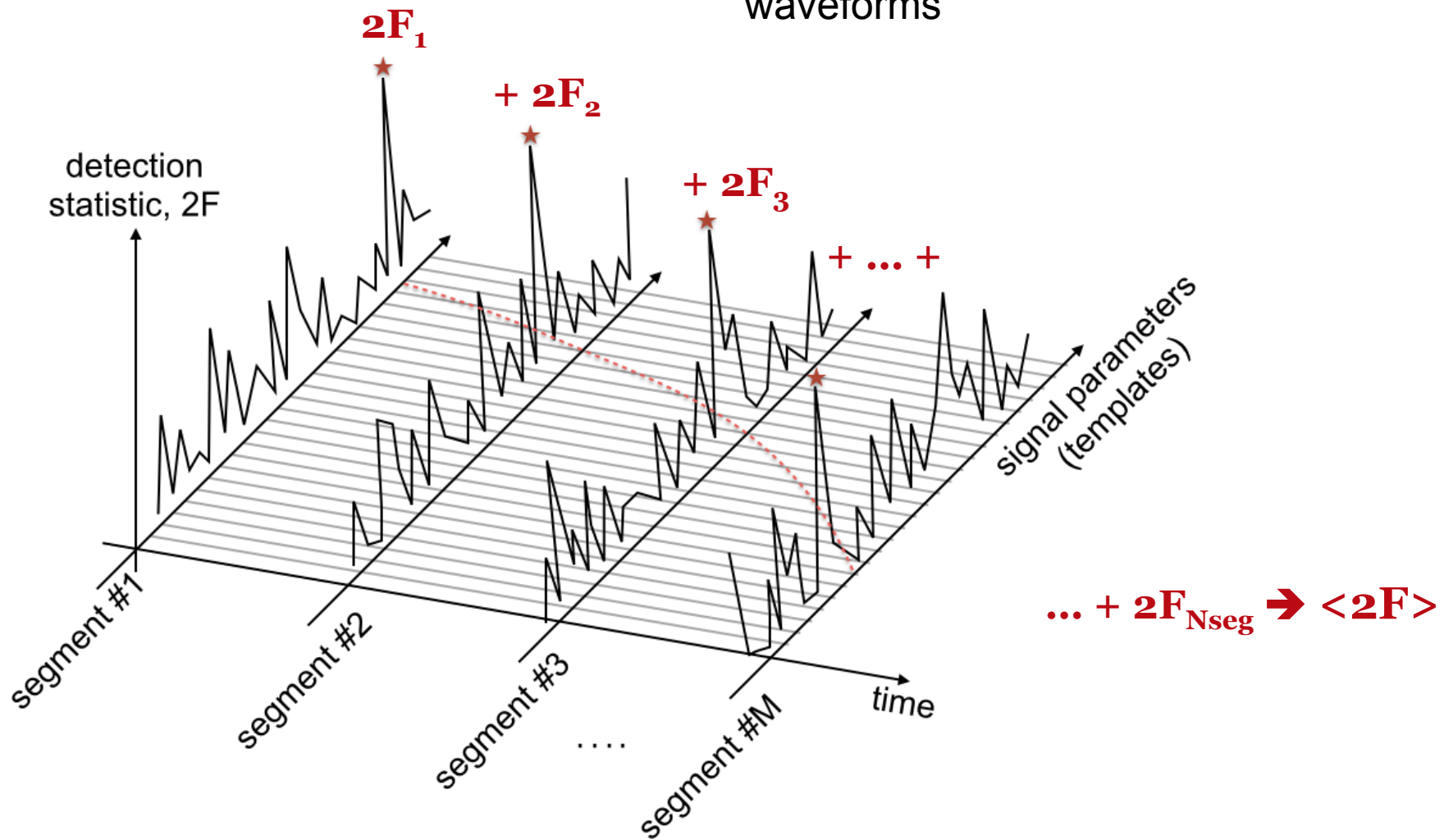
Semi-coherent search methods

- divide the data set in segments
- perform a coherent (F-stat) search on each segment
- combine the results from the segments



Semi-coherent search methods

- we sum the $2F$ -values along different tracks corresponding to different possible waveforms



Semi-coherent methods

$$\text{SNR} \propto \frac{h_o}{\sqrt{S_n}} T_{\text{coh}}^{1/2} N_{\text{seg}}^{1/4} w, \\ w(N_{\text{seg}}, p_{\text{FA}}) \text{ range } [1, \approx 3.5]$$

[Prix & Shaltev, PRD85,
2012]

Computationally limited

Two different types of surveys



- Broad, fast-turn around, robust
- More limited in breadth, deepest

Two different types of surveys in O1



- **Broad, fast-turn around, robust**
 - 20-475 Hz
 - $[-1, +1] \times 10^{-8}$ Hz/s
 - LVC, [arXiv:1707.02667](https://arxiv.org/abs/1707.02667)

- **More limited in breadth, most sensitive**
 - 20-100 Hz
 - $[-2.6, 0.3] \times 10^{-9}$ Hz/s
 - LVC, [arXiv:1707.02669](https://arxiv.org/abs/1707.02669)

Two different types of surveys in O1



- **Broad, fast-turn around, robust**
 - 20-475 Hz
 - $[-1,+1] \times 10^{-8}$ Hz/s
- **More limited in breadth, most sensitive**
 - 20-100 Hz
 - $[-2.6, 0.3] \times 10^{-9}$ Hz/s

Things I am going to tell you about this search

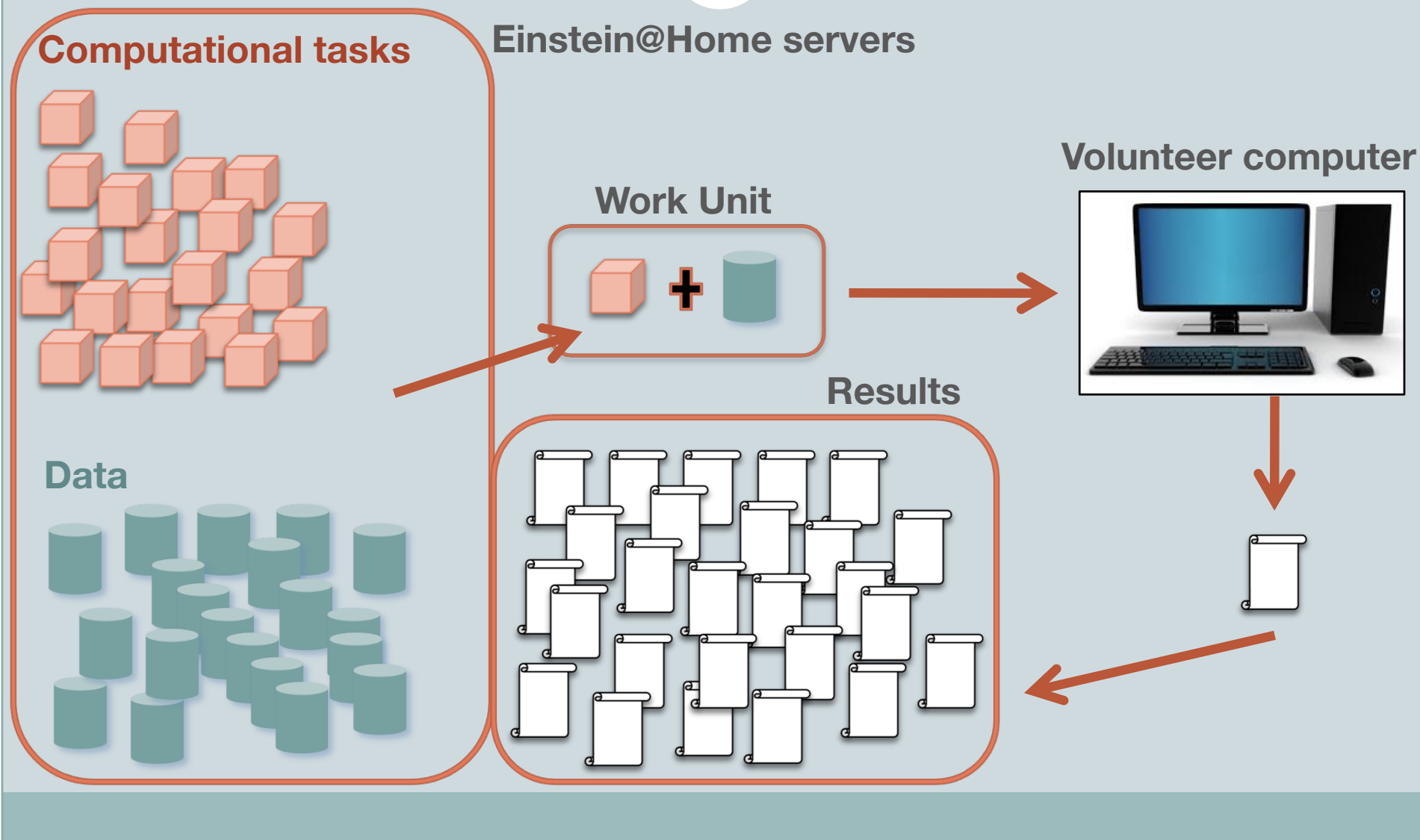


- Einstein@Home
- The problem of instrumental artefacts
 - Robust statistics
 - Hierarchical follow-ups
 - Clever vetoes
- Results



<https://einsteinathome.org/>

- Public distributed computing project: people donate idle cycles of their machines to some scientific project.
 - Public get a screensaver and get to take part in research
 - We get their compute cycles
- Like SETI@home, but for GW data and EM data.
- APS has publicized this as part of World Year of Physics 2005 activities.
- Use infrastructure and help from SETI@home developers for the distributed computing parts (BOINC).
- Support for Windows, Mac OSX, Linux clients.

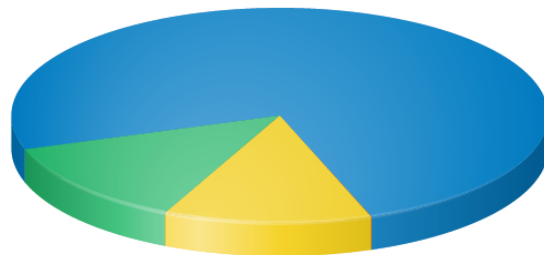


“heavy lifting” machine



- Over 1.6 million hosts and 1 million participants have done work for E@H
- ~ 50 000 hosts (33 000 participants) active in the past 2 weeks
- > 5 Pflops sustained 24 x 7
- Would be in the top-500 list

Hosts by CPU brand
Einstein@Home



Intel
Other
AMD
PowerPC

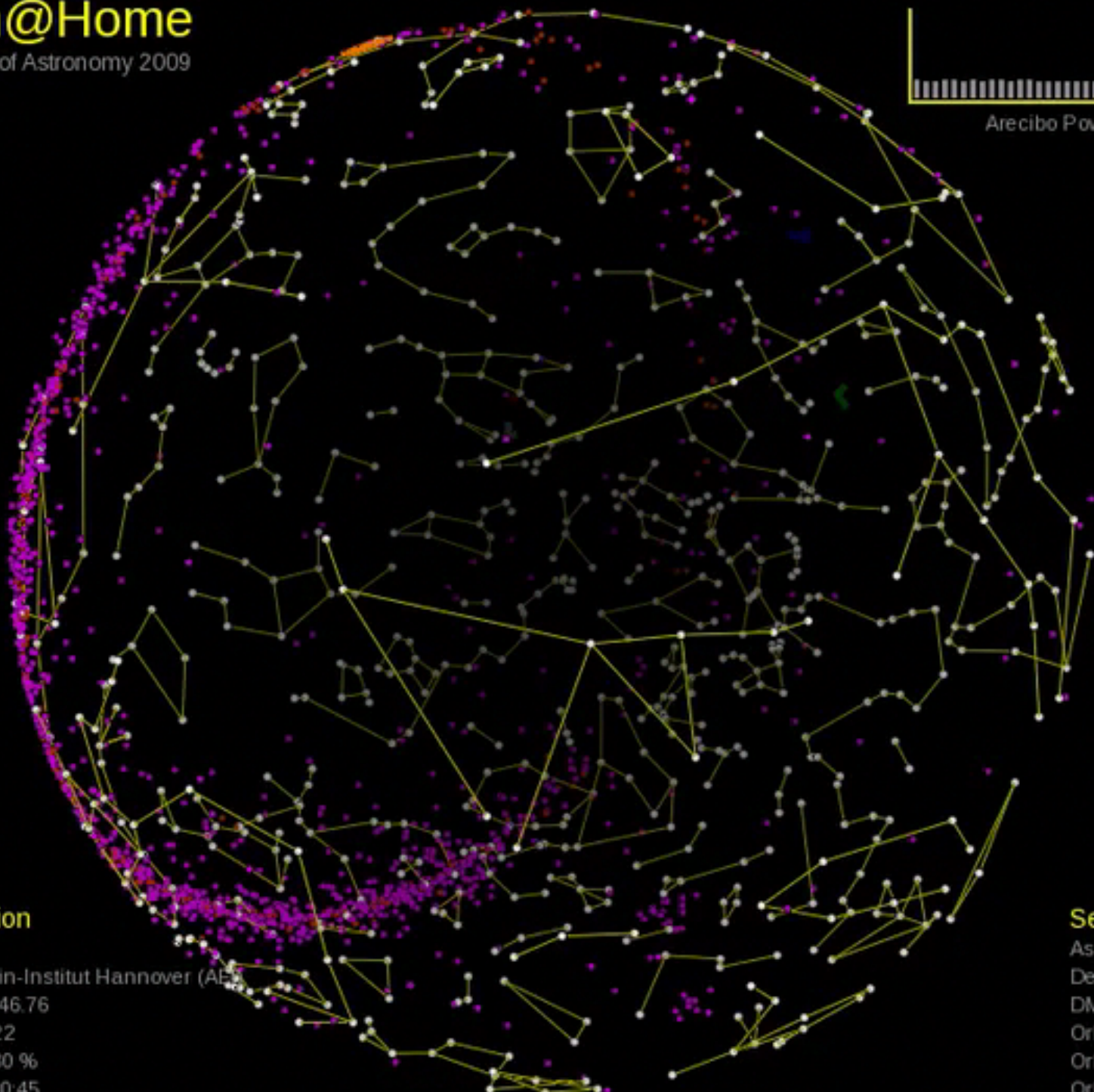
Hosts by Operating System
Einstein@Home



Windows
Linux
Other
MAC Intel
MAC Power

Einstein@Home

International Year of Astronomy 2009



BOINC Information

User: Oliver
Team: Albert-Einstein-Institut Hannover (AEI)
Project Credit: 330046.76
Project RAC: 1266.22
WU Completed: 15.80 %
WU CPU Time: 00:20:45

Search Information

Ascension: 300.40 deg
Declination: 25.10 deg
DM: 498.40 pc/cm³
Orb. Radius: 0.183 ls
Orb. Period: 1003 s
Orb. Phase: 3.85 rad

Einstein@Home

International Year of Astronomy 2009



Arecibo Power Spectrum

**Please sign up your computers to
Einstein@Home
<https://einsteinathome.org/>**

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Robust statistics



Robust statistics



- Fstat tests signal hypothesis against Gaussian noise hypothesis.
- Its values can be increased due to noise (a disturbance) that looks more like a signal than Gaussian noise, e.g. a line in one of the detectors
- We develop a statistic that tests against noise that can be either Gaussian or line-dominated:
 - Performance is comparable to Fstat in Gaussian noise
 - Is as good as or outperforms the Fstat +Fstat consistency veto in disturbed bands

- The new statistic is the odds ratio O_{SGL} :

$$F = \frac{P(H_S | \mathbf{x})}{P(H_G | \mathbf{x})} \longrightarrow O_{\text{SGL}} = \frac{P(H_S | \mathbf{x})}{P(H_{\text{GL}} | \mathbf{x})}$$

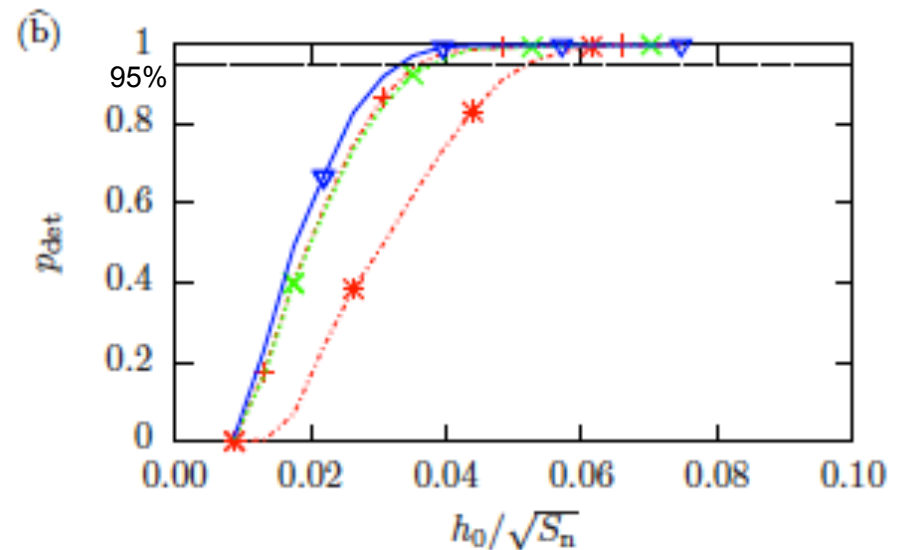
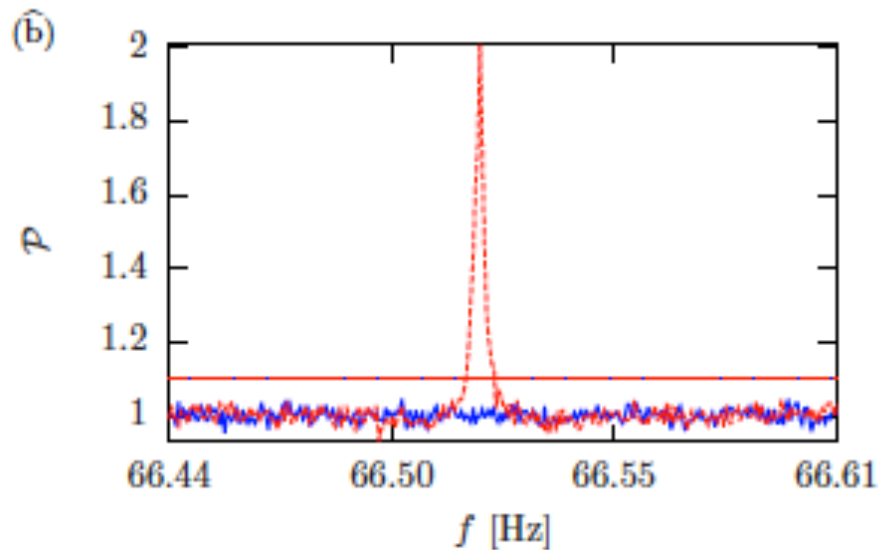
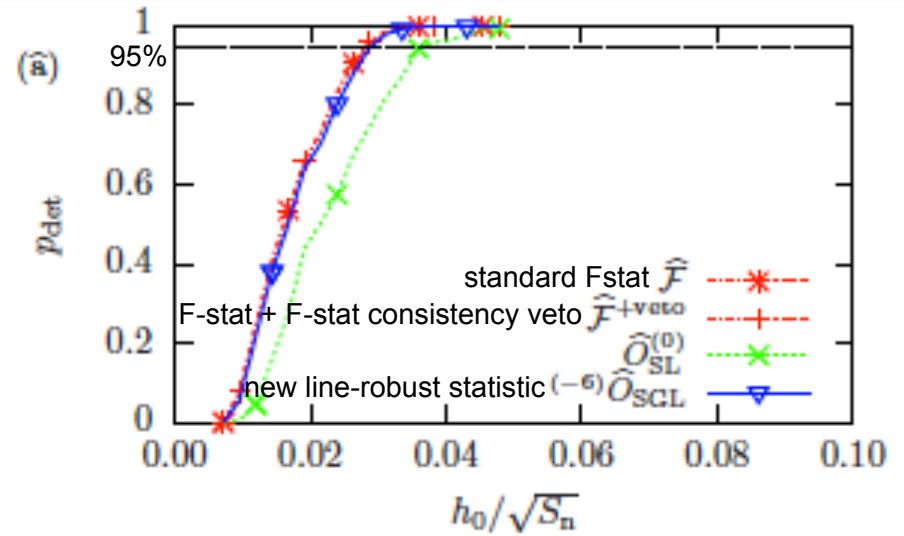
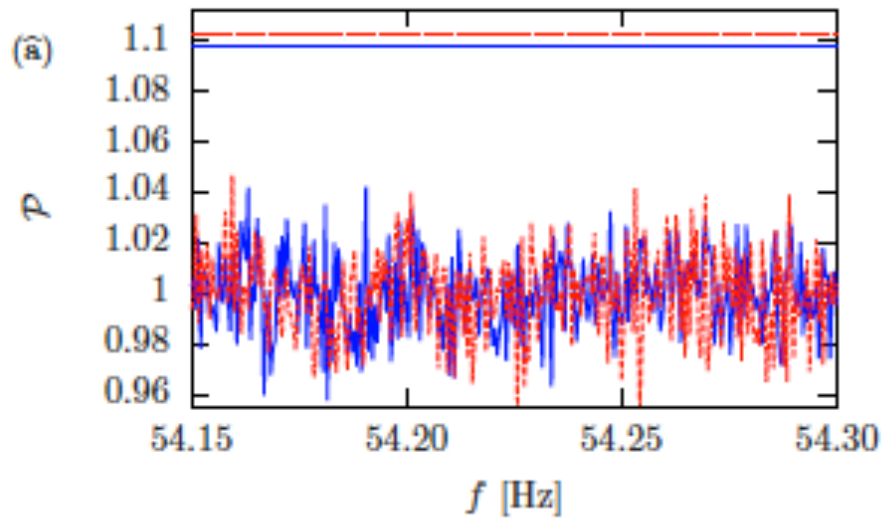
- H_S is the signal hypothesis : signal + gaussian noise
- H_{GL} is the noise hypothesis : H_G or $H_L \rightarrow P(H_{\text{GL}} | \mathbf{x}) = P(H_L | \mathbf{x}) + P(H_G | \mathbf{x})$
- H_L is the line-noise hypothesis (defined as a signal in only one detector).

$$\ln \hat{O}_{\text{SGL}}(\mathbf{x}) = \ln \hat{o}_{\text{SL}} + \hat{\mathcal{F}}(\mathbf{x}) - \hat{\mathcal{F}}''_{\text{max}}(\mathbf{x}) - \ln \left(e^{\hat{\mathcal{F}}_* - \hat{\mathcal{F}}''_{\text{max}}(\mathbf{x})} + \left\langle \hat{r}^X e^{\hat{\mathcal{F}}^X(x^X) - \hat{\mathcal{F}}''_{\text{max}}(\mathbf{x})} \right\rangle \right)$$

prior (pointing to \hat{o}_{SL})
 average F-stat (pointing to $\hat{\mathcal{F}}(\mathbf{x})$)
 line-hypothesis transition scale (pointing to $\hat{\mathcal{F}}_*$)
 single-detector prior line prob (pointing to \hat{r}^X)
 single-detector F-statistic (pointing to $\hat{\mathcal{F}}^X(x^X)$)

$$\hat{\mathcal{F}}''_{\text{max}}(\mathbf{x}) \equiv \max \left(\hat{\mathcal{F}}_*, \hat{\mathcal{F}}^X(x^X) + \ln \hat{r}^X \right)$$

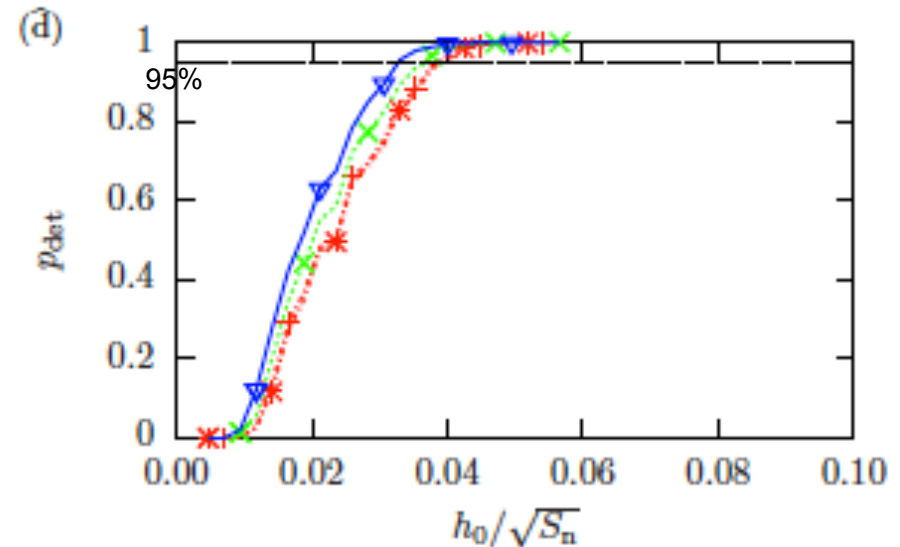
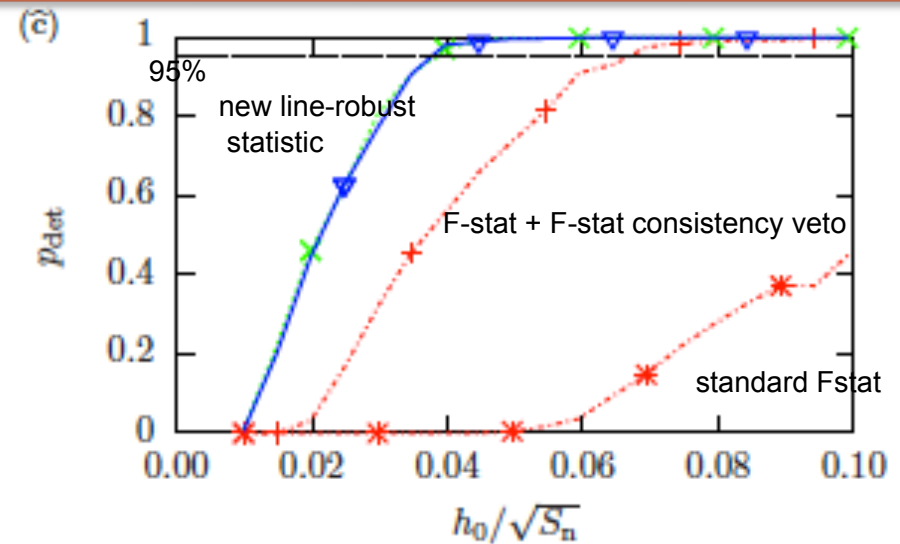
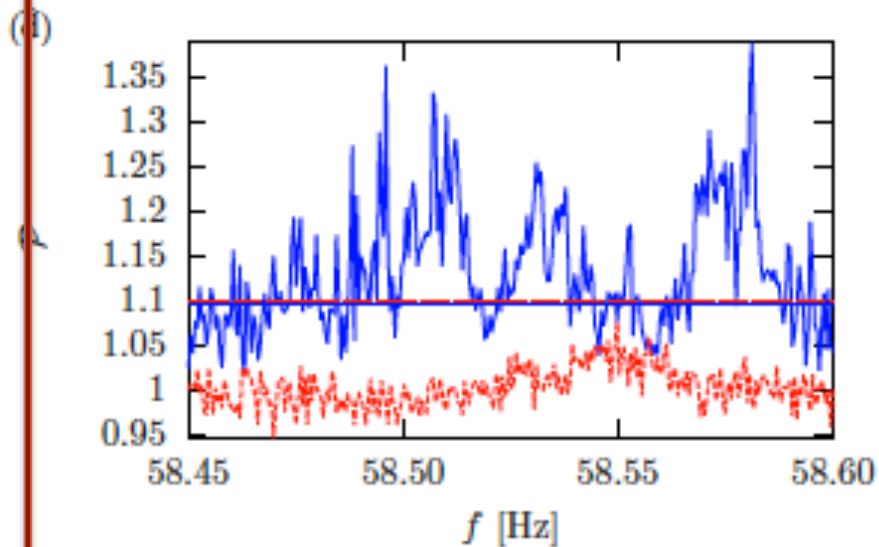
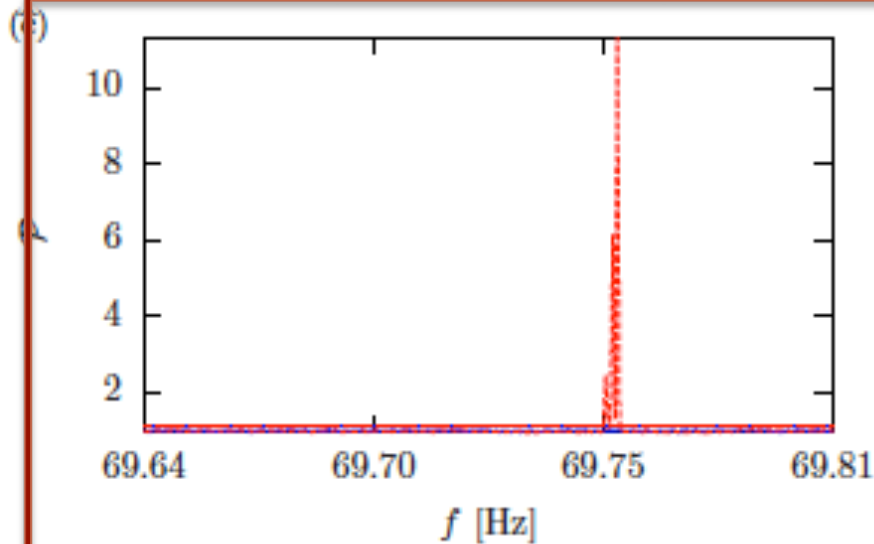
Performance in different noise conditions



Real detector data (noise): L1
in red, H1 in blue

Detection probability for
injected signals of different
amplitudes in that noise.

Performance in different noise conditions



Real detector data (noise): L1
in red, H1 in blue

Detection probability for
injected signals of different
amplitudes in that noise.

Important, because of top lists...

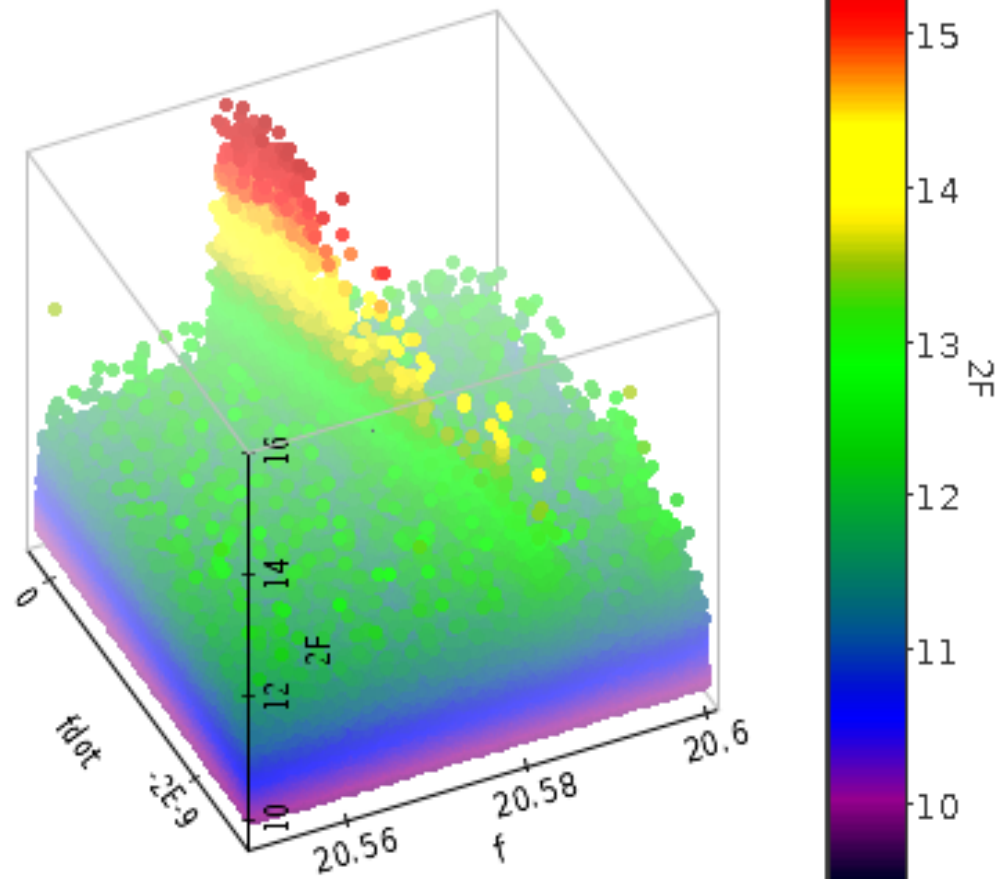


- Remember that E@H returns top candidates from every WU (work unit)
 - Important to not fill this top-list with junk
- First step when we look at the results is to gather all the results in 50 mHz signal-frequency bands
- We see that robust statistics work pretty well :

Original F-statistics



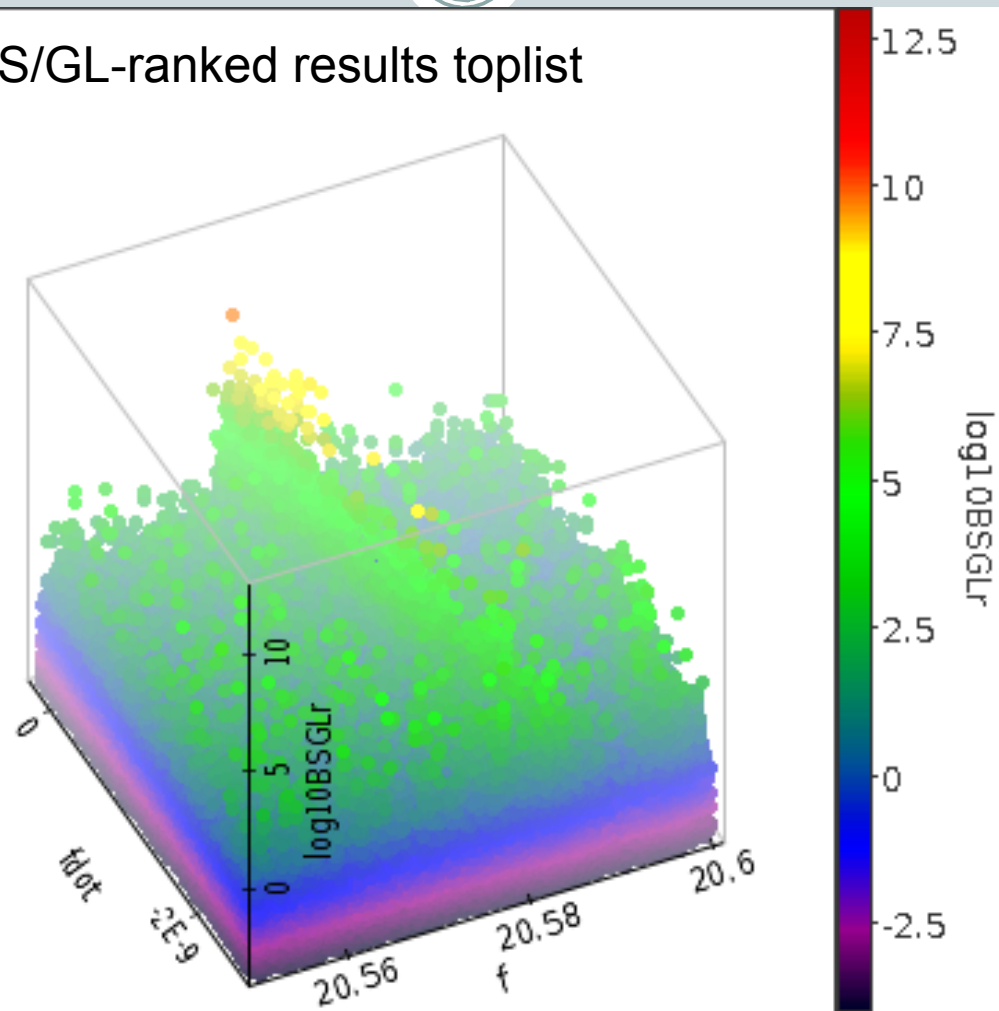
<2F>-ranked results toplist



Robust statistics



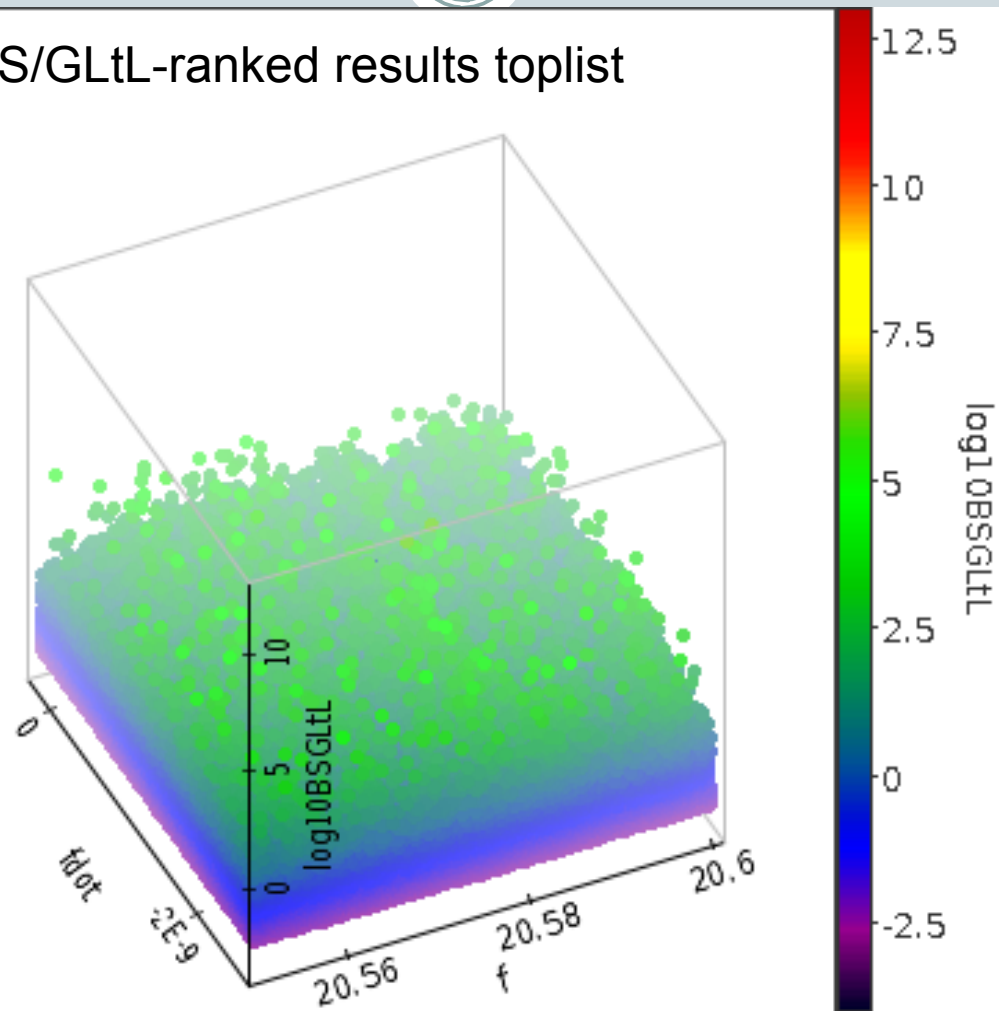
BS/GL-ranked results toplist



Robust statistics



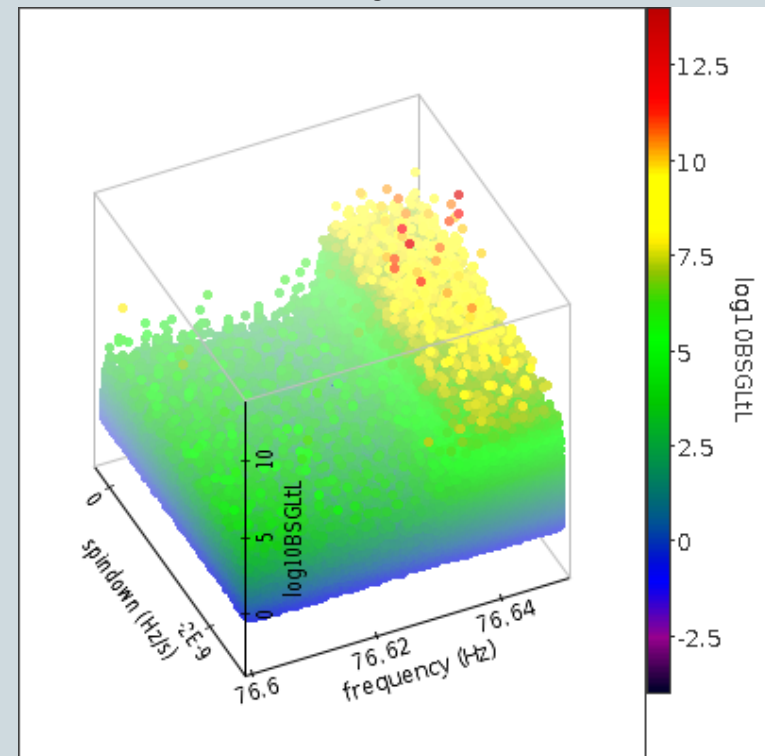
BS/GLtL-ranked results toplist



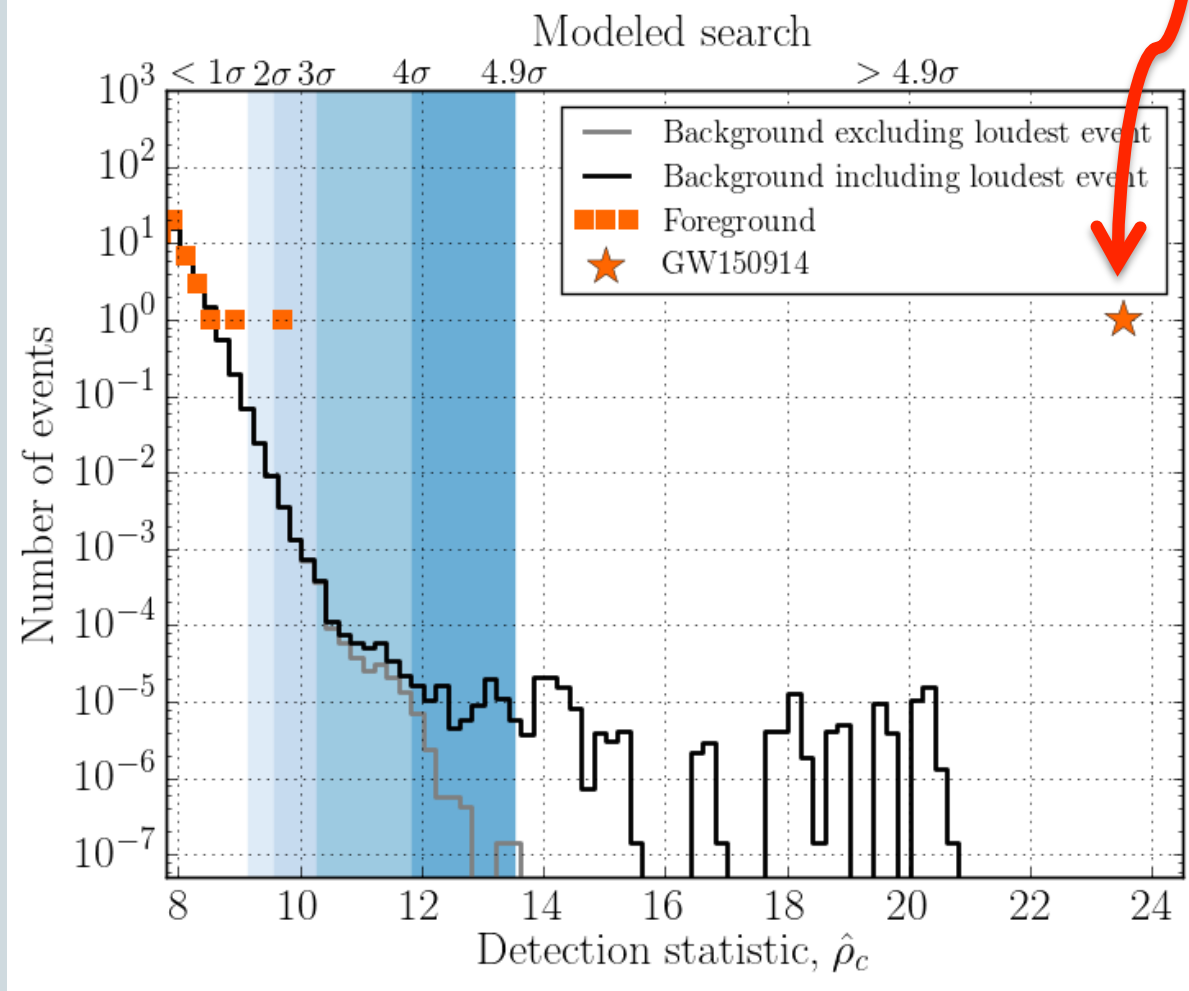
Even these robust statistics are not perfect...



- However, still some bands (a few percent) remain highly disturbed, even just upon visual inspection. We typically exclude them from the analysis
- After doing so, we look at the results and check if there is something outstanding



What is outstanding ? This:

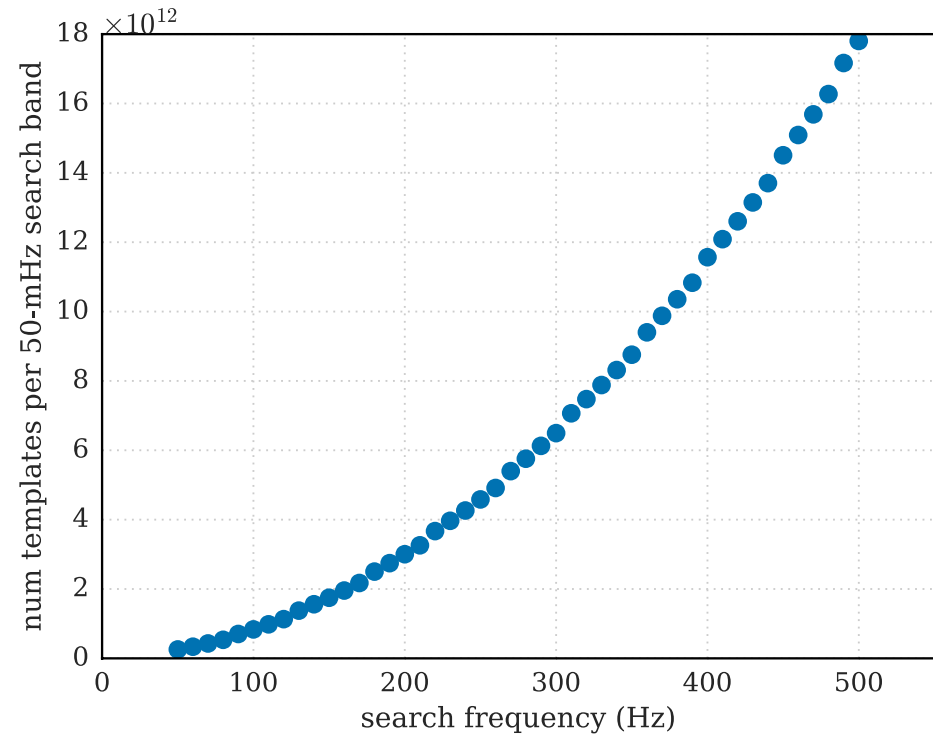
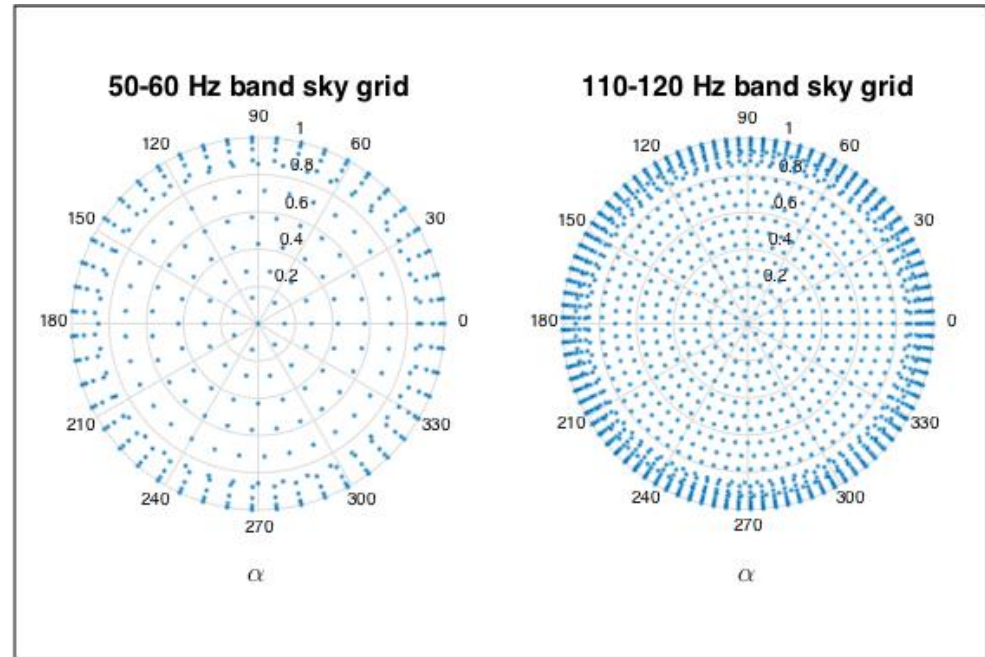
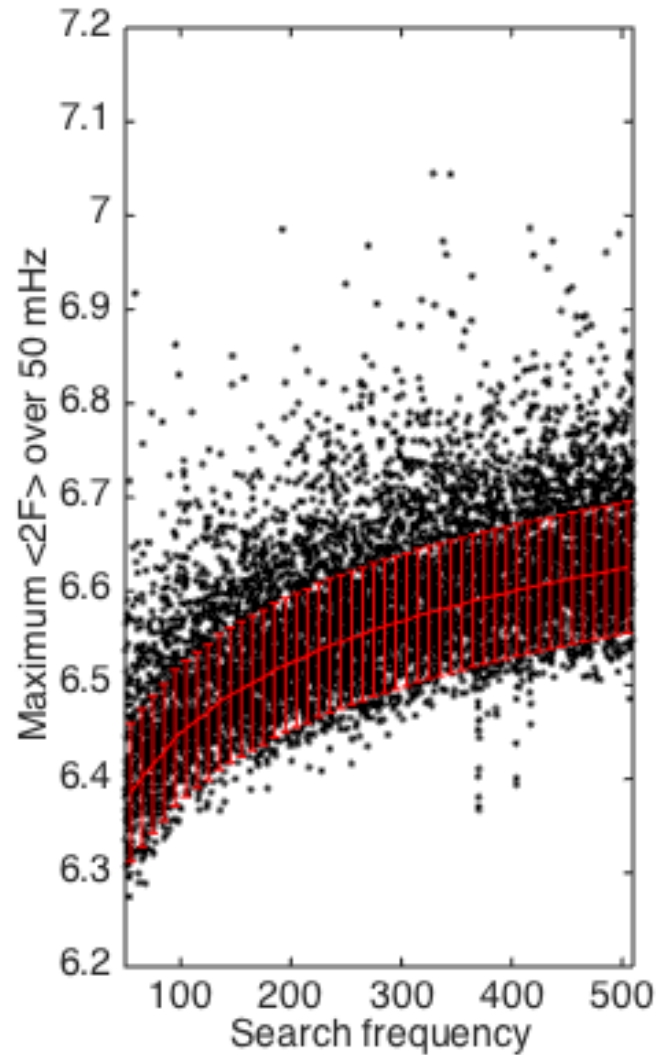


**“Observation of GWs from a binary black hole merger”,
Phys.Rev.Lett. 116 (2016) no.6, 061102**

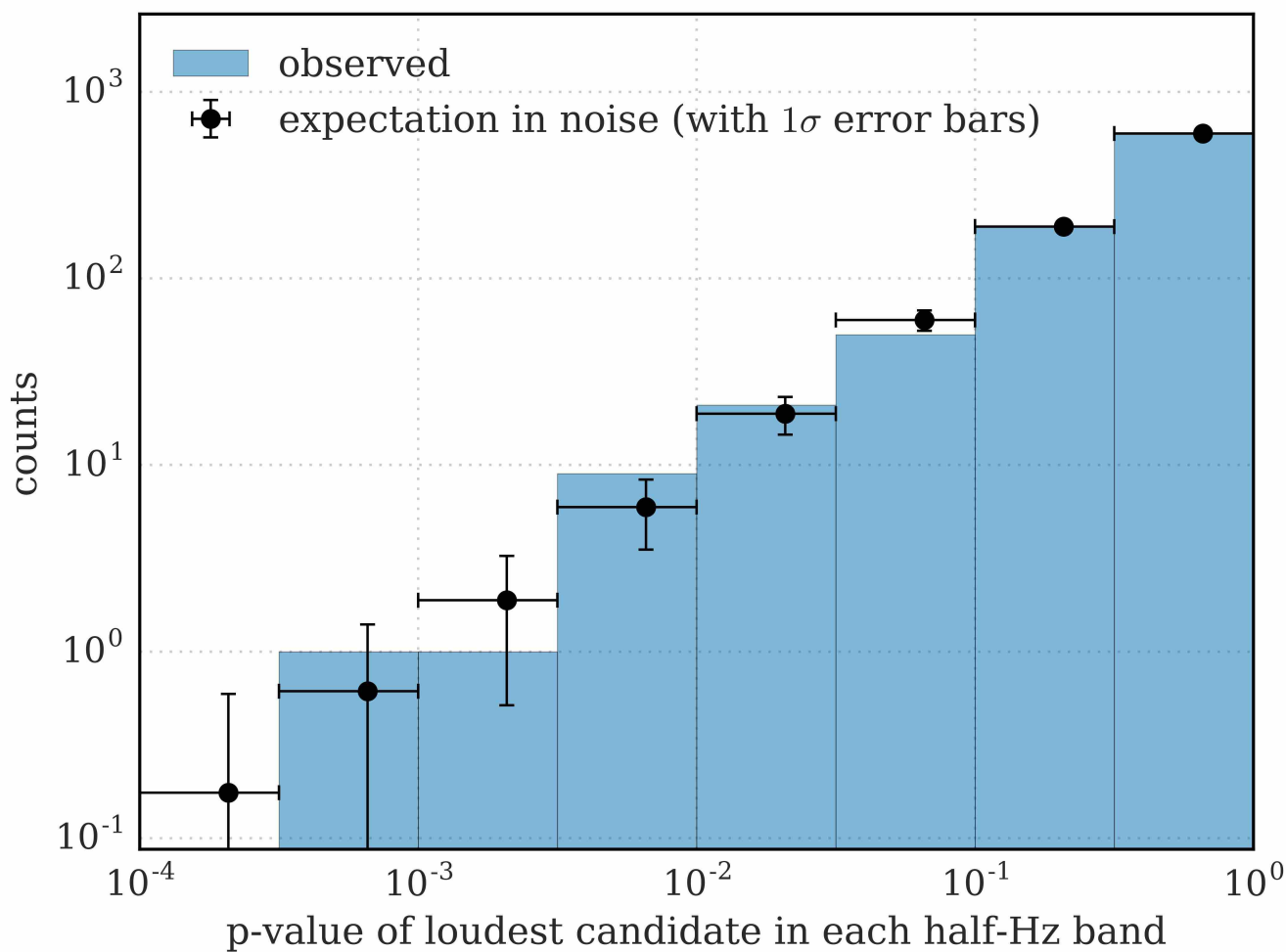
LIGO and Virgo

“Results of the deepest all-sky survey for CW waves in S6 LIGO data”

Phys Rev D 94 nr.10, 102002 , (2016)



P-value distribution



What is a P-value ?



- It is the false alarm prob value associated to a detection statistic value, say $2\mathcal{F}^*$:

$$P(2\mathcal{F}^*) = \int_{2\mathcal{F}^*}^{\infty} p_0(x) dx, \quad p_0 \text{ being the noise pdf}$$

- The detection stat $2\mathcal{F}^*$ is the loudest over N detection stat values (# templates in 50 mHz)

$$p_0(2\mathcal{F}^*) = N p(\chi_4^2; 2\mathcal{F}^*) \left[\int_0^{2\mathcal{F}^*} d(2F) p(\chi_4^2; 2F) \right]^{N-1} *$$

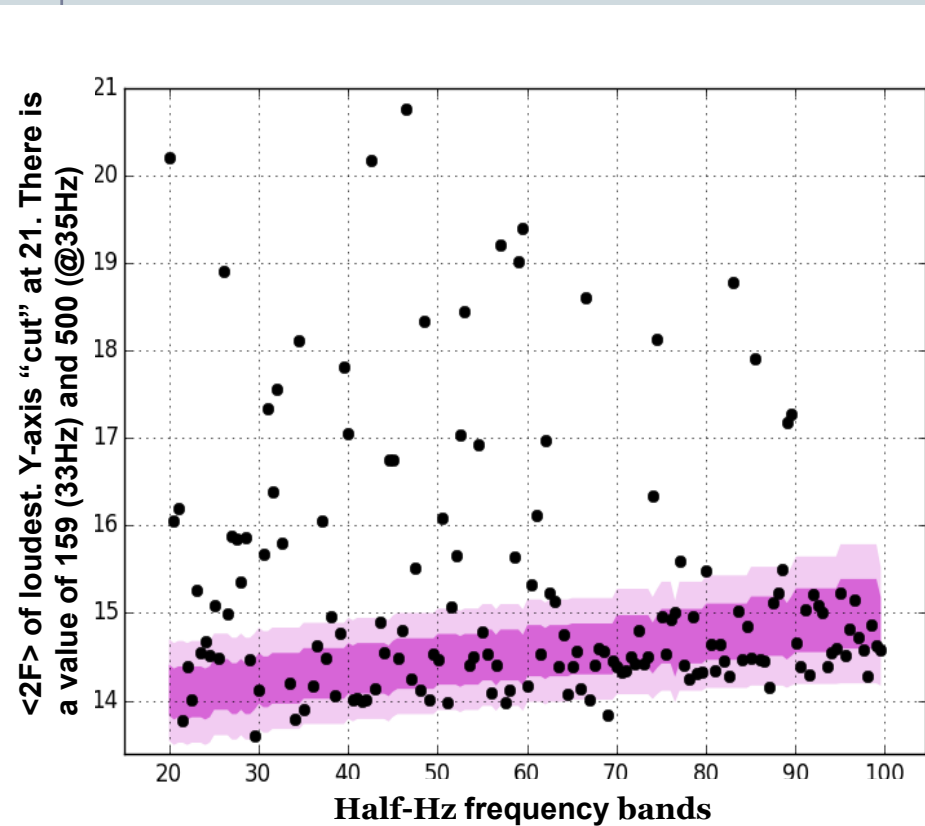
* Typically we look at $\langle 2F \rangle$, with “ $\langle \rangle$ ” over N segments. In this case we change variable to $N\langle 2F \rangle$, so $\chi_4^2 \rightarrow \chi_{4N}^2$

The first O1 E@H search

(LVC, [arXiv:1707.02669](https://arxiv.org/abs/1707.02669))



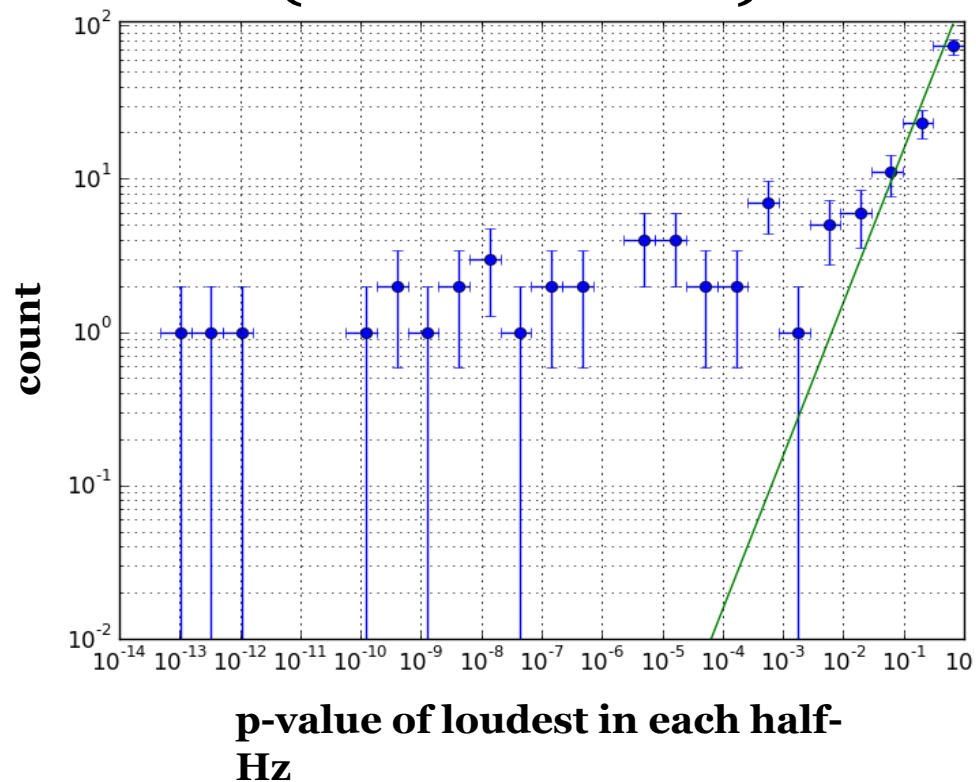
- All-sky search
 - 12 segments, each 210 hr
 - 20-100 Hz
 - $[-2.6, 0.3] \times 10^{-9}$ Hz/s
 - A few E@H-months
- In spite of having used a robust statistic and having removed the clearly disturbed bands, still many outliers



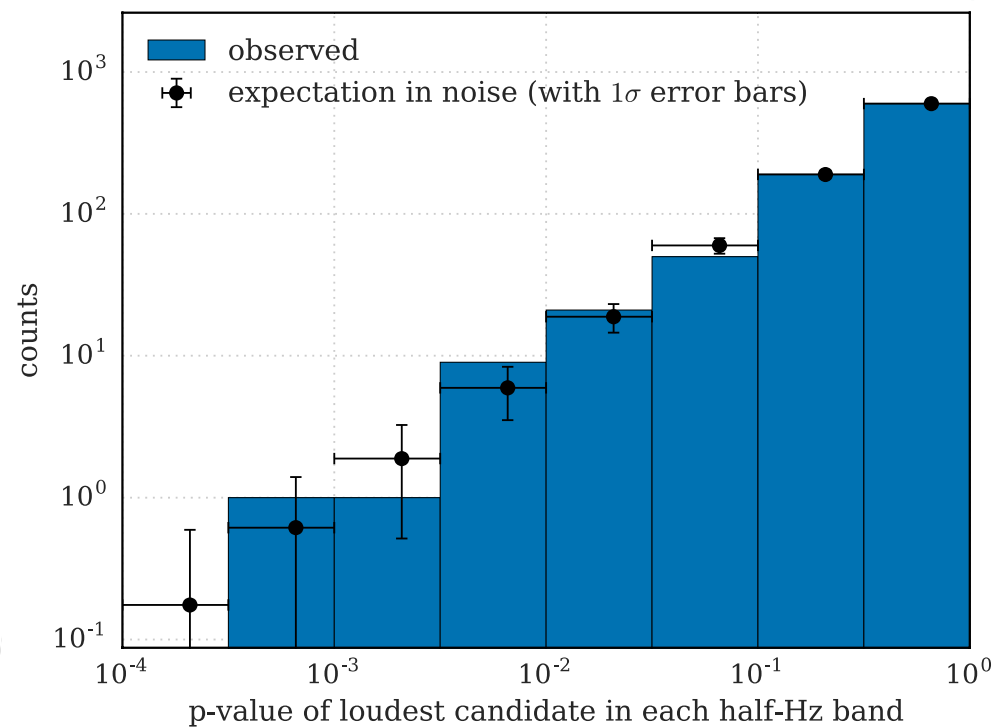
Compared with previous search



This search (over 80 Hz)



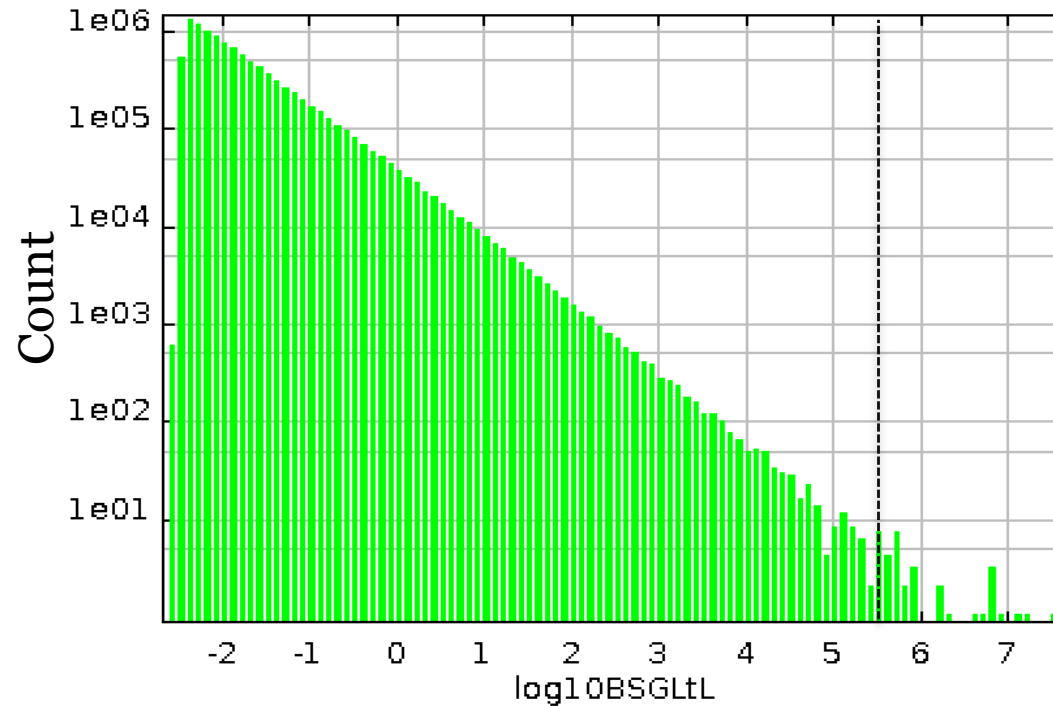
S6 search (over 500 Hz)



Candidate Selection



Example of results from an undisturbed 50 mHz band



- Select candidates with $\text{BSGLtL} > 5.5$
- Expect of $\text{o}(2000)$ above threshold in Gaussian noise
- Find 15 million candidates above threshold → disturbed data

we do a mild sub-threshold search (a virtue of necessity)



- Threshold at 5.5 means a few Gaussian noise candidates from each 50 mHz band
- In total we'd expect < 2000 candidates over 80 Hz, just due to noise accidentally exceeding the threshold
 - At this stage, Gaussian noise candidates would not be significant
 - With a series of follow-ups if one of them were a signal, we'd detect it

Same scheme as used in deep sub-threshold follow-up of S6 candidates

(M.A. Papa et al, "[Hierarchical follow-up of subthreshold candidates ...](#)", Phys.Rev. D94 (2016) no.12, 122006)

- ◆ The significance of marginal signal-candidates is increased with increasingly sensitive searches over a smaller waveform parameter spaces
- ◆ At each stage we reject more and more noise and the accuracy in signal parameter estimation increases
- ◆ Each stage is a semi-coherent search



Follow-up



	T_{coh} hr	N_{seg}	δf Hz	δf_c Hz/s	γ	m_{sky}
Stage 0	210	12	8.3×10^{-7}	1.3×10^{-11}	100	1×10^{-3}
Stage 1	500	5	6.7×10^{-7}	2.9×10^{-12}	80	8×10^{-6}
Stage 2	1260	2	1.9×10^{-7}	9.3×10^{-13}	30	1×10^{-6}
Stage 3	2512	1	6.7×10^{-8}	9.3×10^{-14}	1	4×10^{-7}

Resolution in the sky: $d(m_{\text{sky}}) = \frac{1}{f} \frac{\sqrt{m_{\text{sky}}}}{\pi \tau_E}$ with $\tau_E = 0.021s$

Three follow-up stages

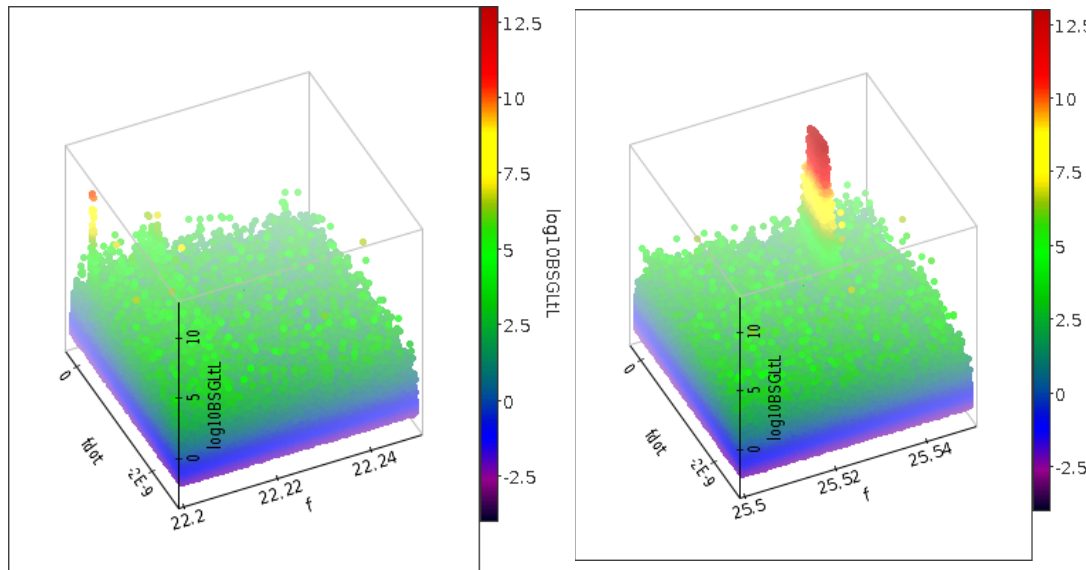
SNR of signal increases with T_{coh} , exclude more noise at each stage

Parameter uncertainty decreases after each stage

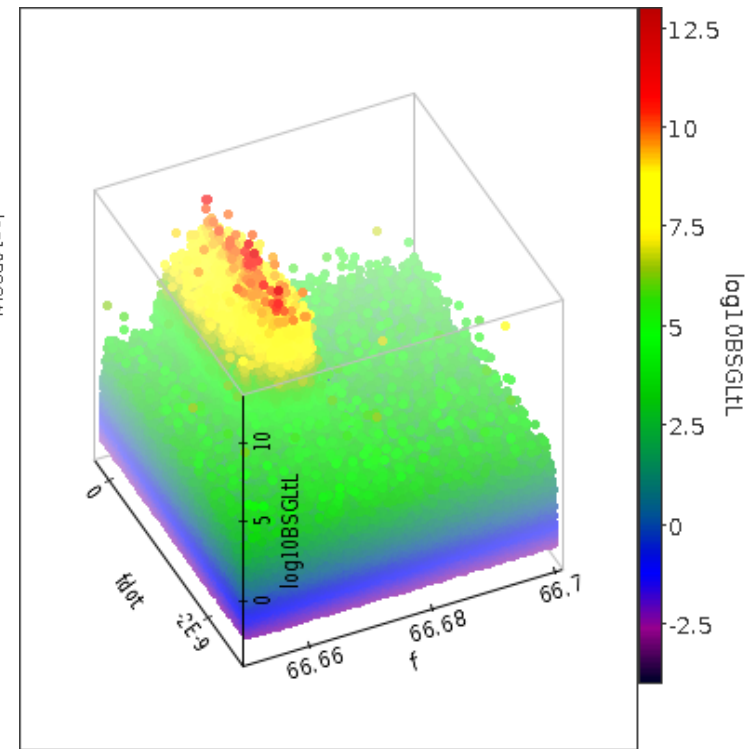
Clustering



Both signals and noise can yield high detection statistic values over large parameter space portions. Expensive to follow-up each point.

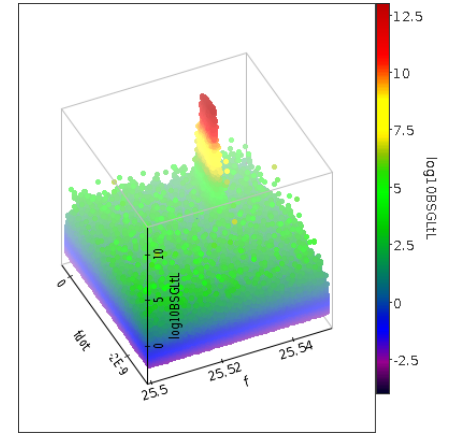
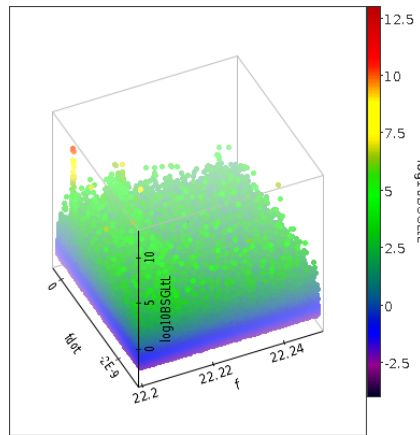
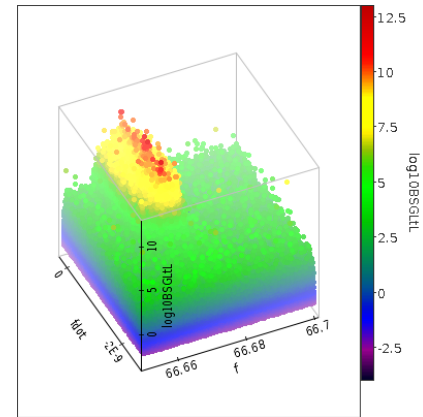
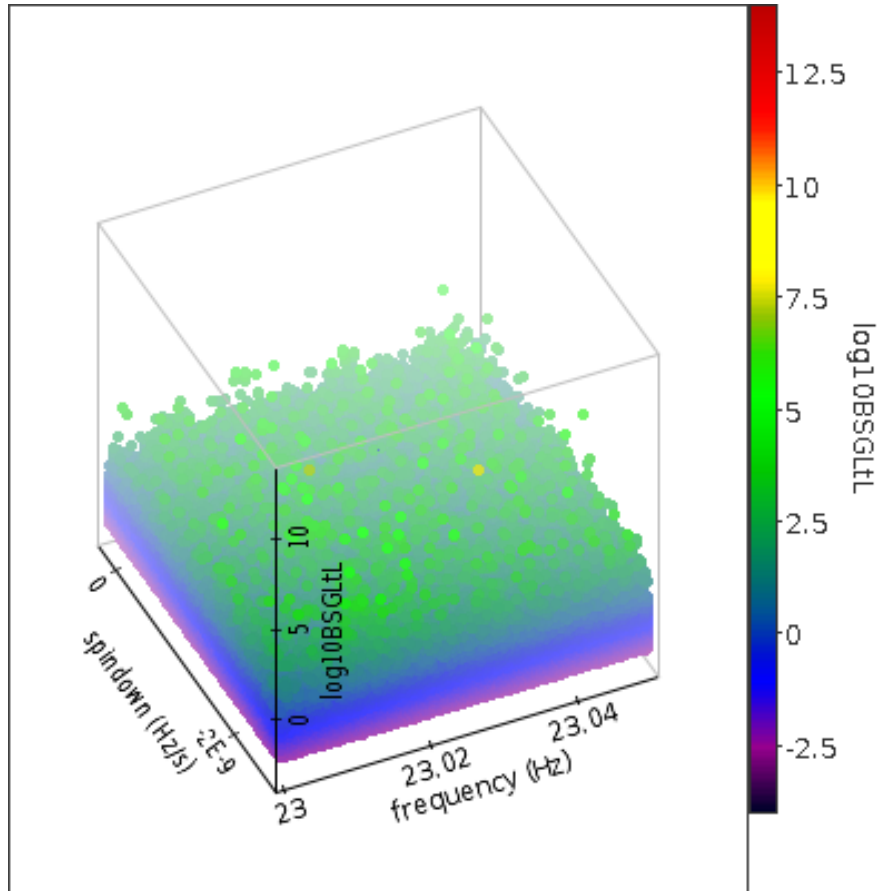


Fake signals + noise



Noise (containing a disturbance)

Undisturbed band



Clustering: a tricky business



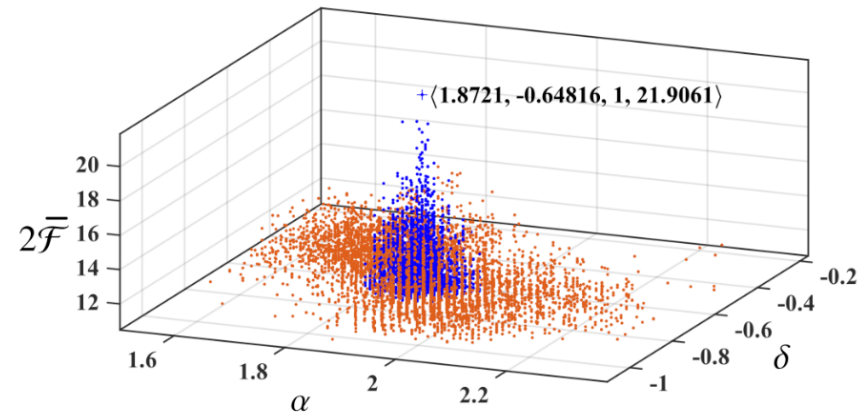
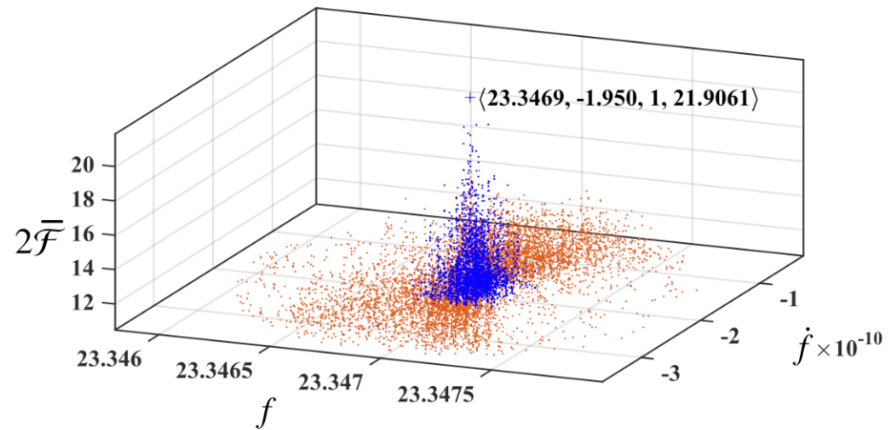
- Want to identify over-densities around significant candidates
- Associate them to a single root cause, and possible signal
 - Remove them from pool of candidates
 - Look for next highest detection statistic value
 - Identify over-densities of candidates around it
 - Associate them to the same root cause
 - ...
- How far do we let the over-density extend ?
- How do we measure over-density ?

Adaptive clustering

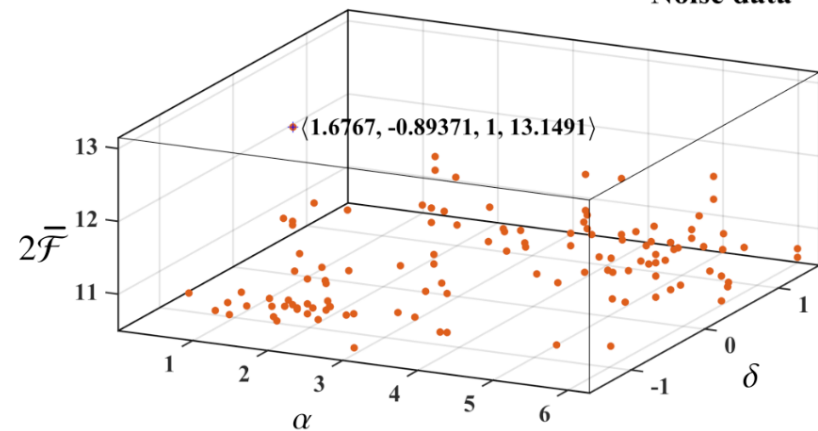
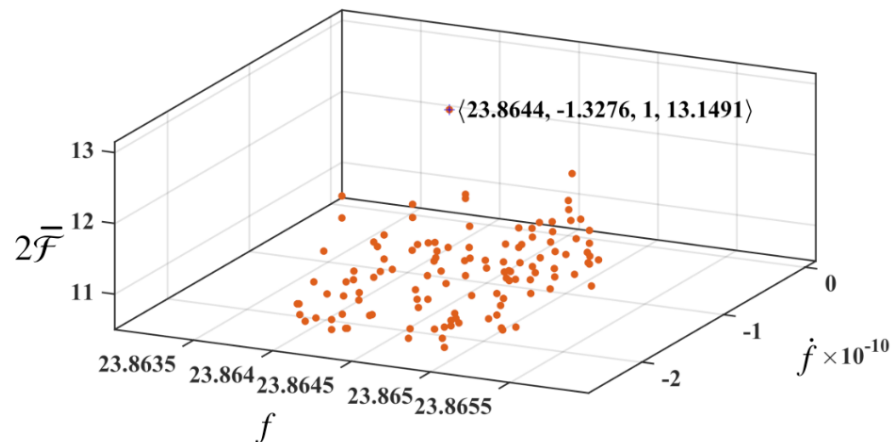
(A. Singh et al, "An adaptive clustering procedure...", [arXiv:1707.02676](https://arxiv.org/abs/1707.02676))



Fake Signal



Noise data



We require consistent over-density both in f - \dot{f} and sky

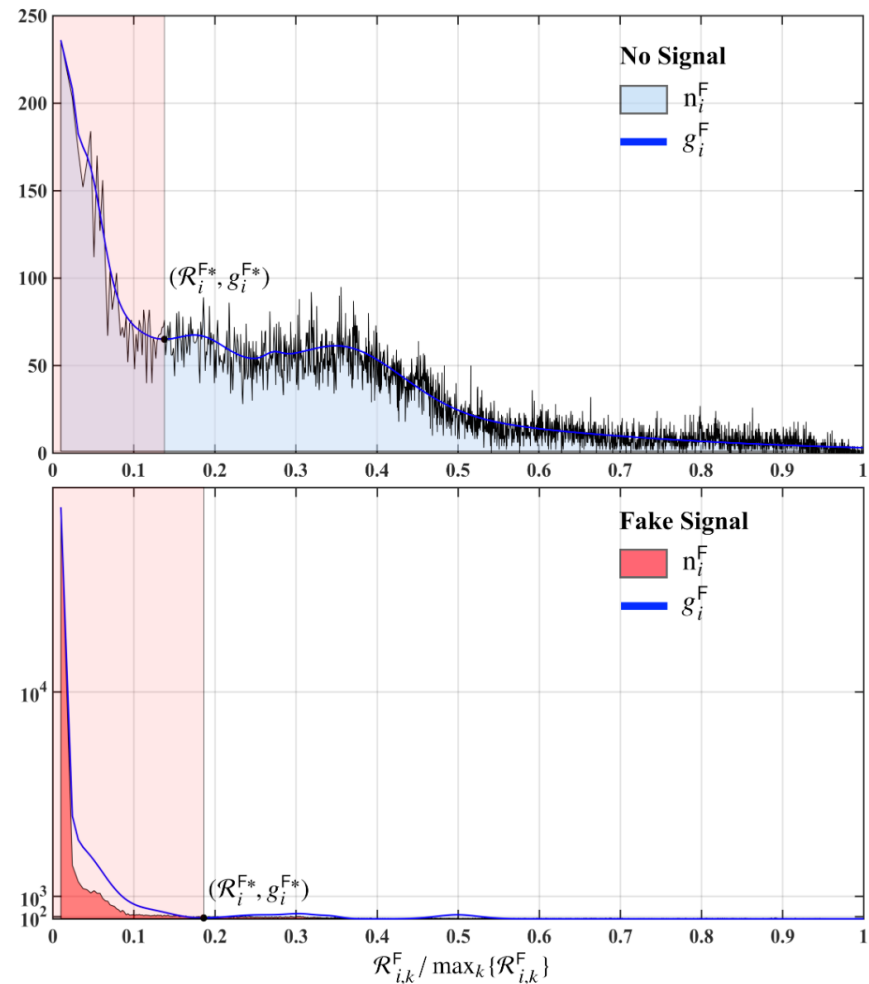
How densities are determined

- A distance is introduced

$$\mathcal{R}_{i,k}^F := \sqrt{\left[\frac{f_k - f_{\ell(i)}}{\delta f}\right]^2 + \left[\frac{\dot{f}_k - \dot{f}_{\ell(i)}}{\delta \dot{f}}\right]^2} \quad \forall \kappa_k \in \chi_i, \quad (2)$$

where, δf and $\delta \dot{f}$ are the frequency and spin-down grid spacings used in the search. Note that at fixed $\mathcal{R}_{i,k}^F$, (2) is an ellipse in F-space centered at $(f_{\ell(i)}, \dot{f}_{\ell(i)})$ and with axes of half-length $\delta f \times \mathcal{R}_{i,k}^F$ and $\delta \dot{f} \times \mathcal{R}_{i,k}^F$.

- The cluster size is determined based on the measured distribution of the distances



Performance



- Tuning parameters need to be set, depending on what type of search is being carried out:
 - Low threshold
 - High threshold
 - On the specific search set-up and the data
- Requires some MC-ing to understand clustering properties of noise
- In the end, O1 search performance:

		AdCI Procedure	Old Procedure
High-significance $\overline{2\mathcal{F}}$ search	NR	65.9%	$\leq 40.0\%$
	\mathcal{E}	97.6%	95.1%
Sub-threshold $\hat{\beta}_{S/GLtL}$ search	NR	90.5%	$\leq 74.1\%$
	\mathcal{E}	95.5%	$> 95.0\%$

Performance

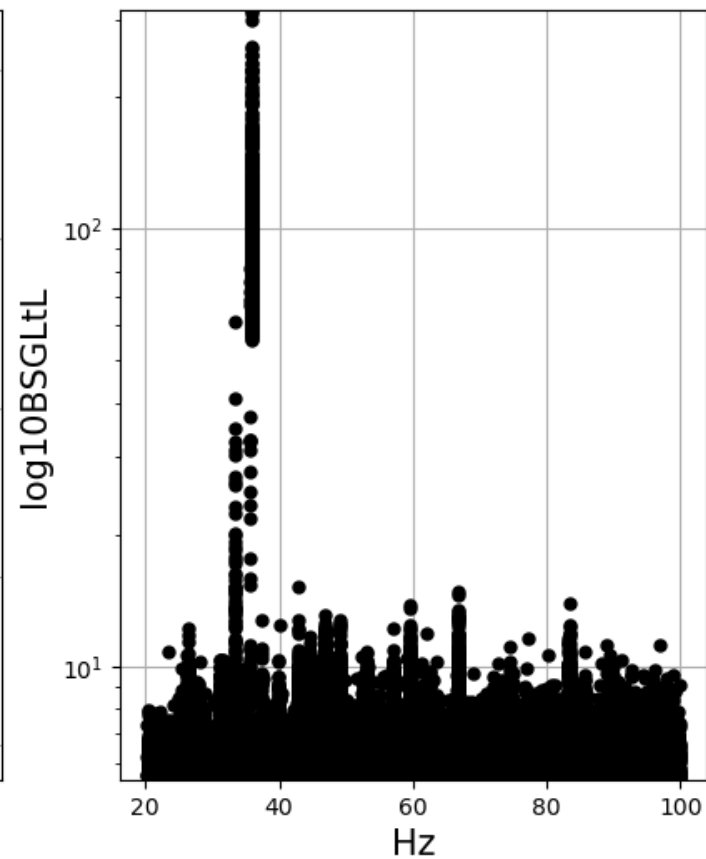
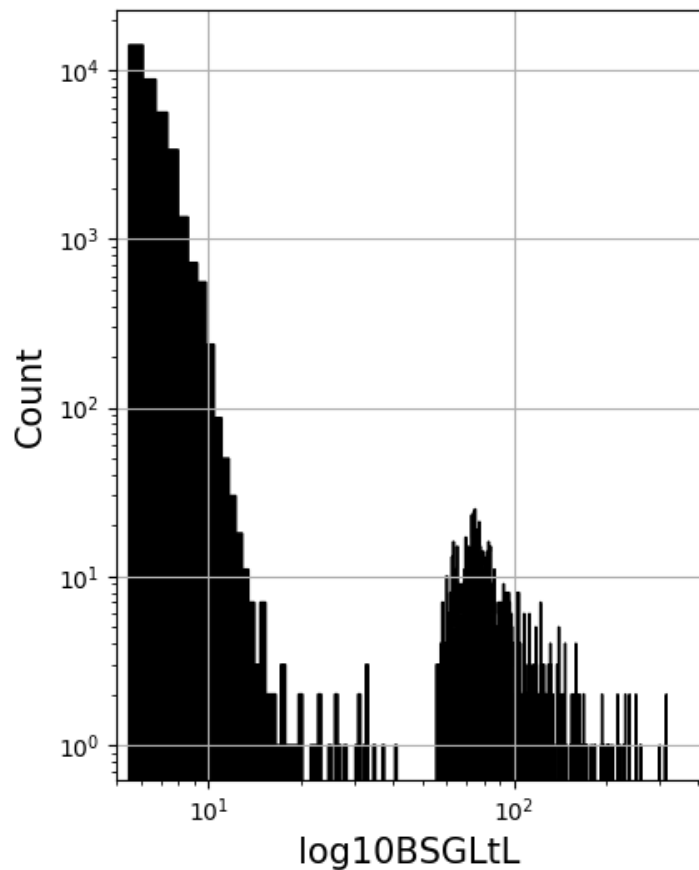


- In the O1 search:

15×10^6 candidates \rightarrow 36 000 clusters

i.e. “only” 36 000 candidates to follow-up

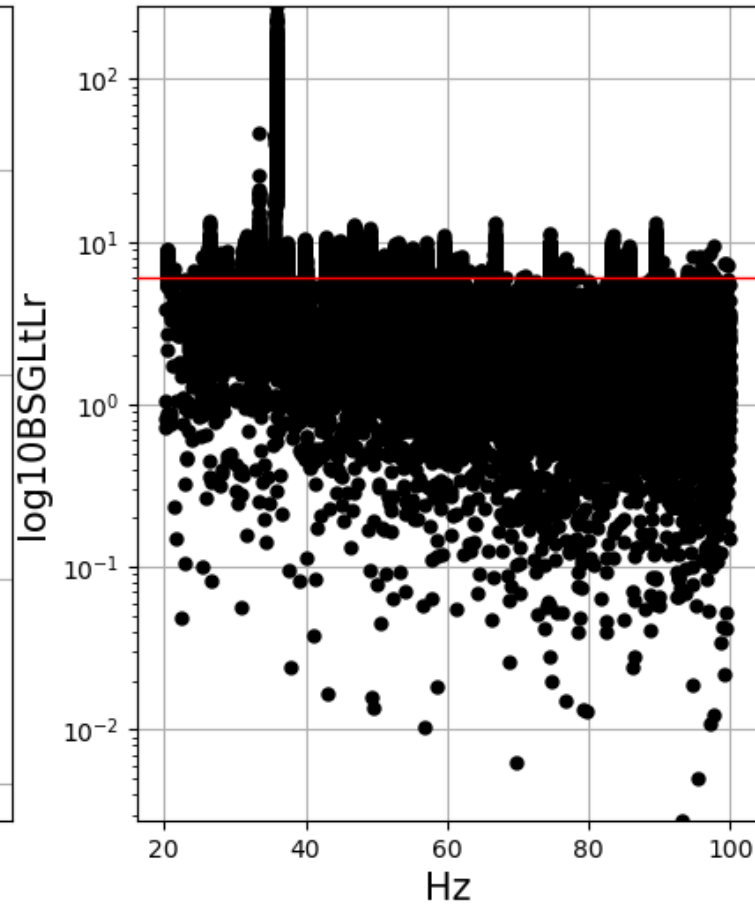
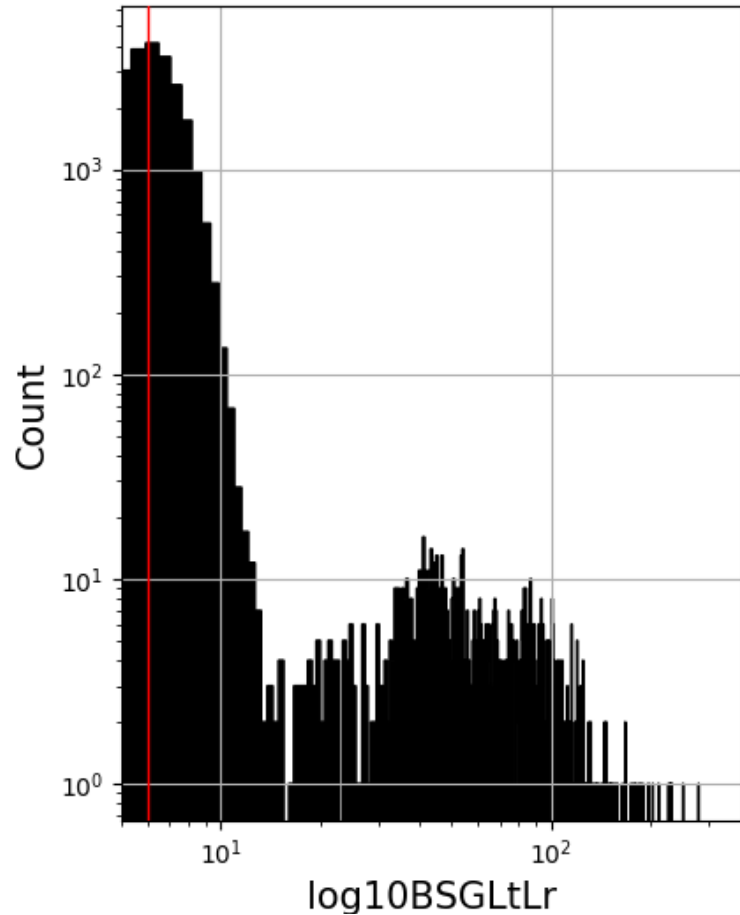
35693 candidates from clustering



Uncertainty volume around each candidate:

$$\begin{cases} \Delta f & = \pm 9.25 \times 10^{-5} \text{ Hz} \\ \Delta \dot{f} & = \pm 4.25 \times 10^{-11} \text{ Hz/s} \\ \Delta_{\text{sky}} & \simeq 4.5 \text{ sky grid points} \end{cases}$$

35693 candidates after first follow-up (T_{coh} 500 hours)

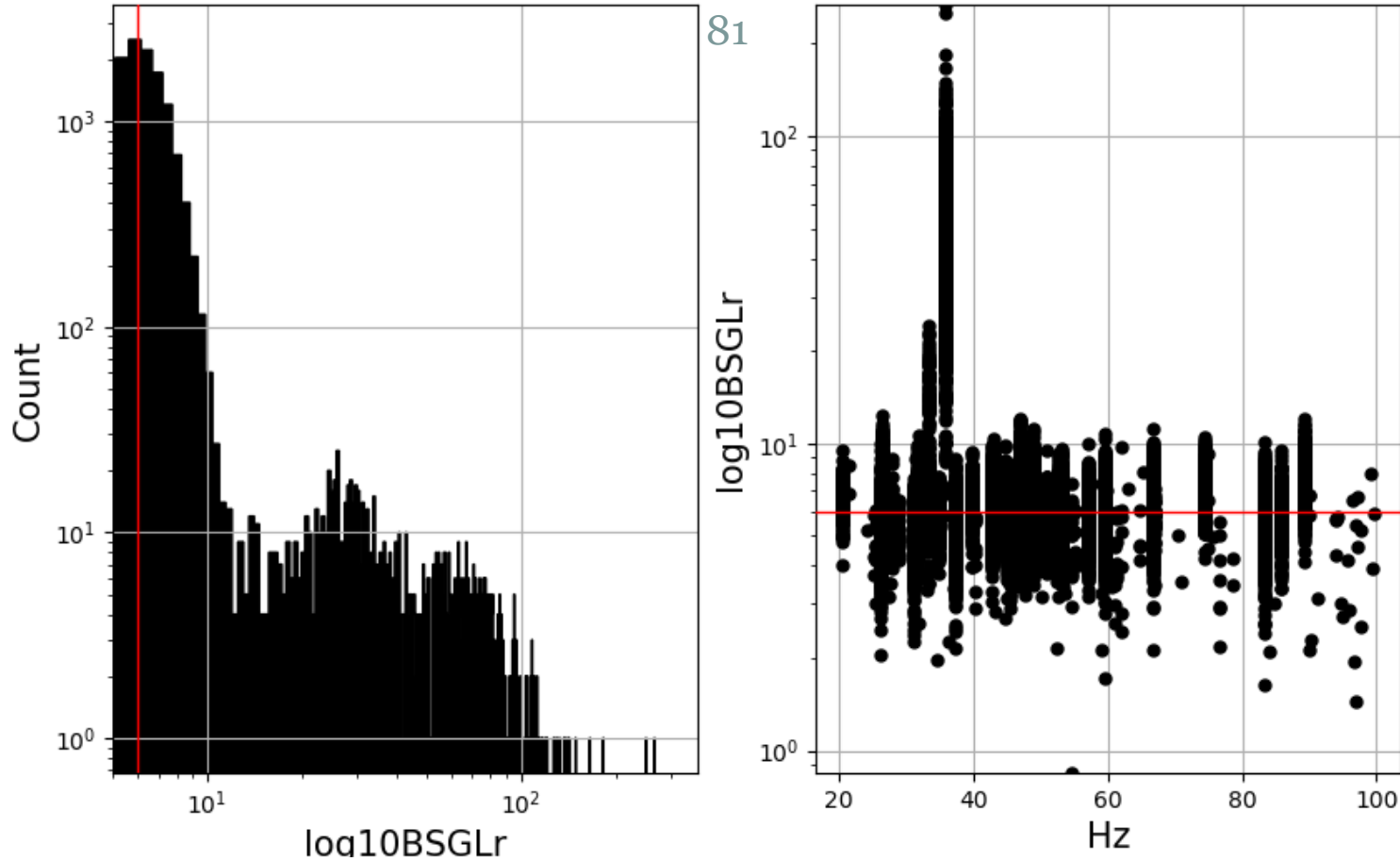


Threshold at 6.0 \rightarrow 14456 survive

Uncertainty volume around each candidate:

$$\begin{cases} \Delta f & = \pm 1.76 \times 10^{-5} \text{ Hz} \\ \Delta \dot{f} & = \pm 9.6 \times 10^{-12} \text{ Hz/s} \\ \Delta_{\text{sky}} & \simeq 0.23 \Delta_{\text{sky}}^{\text{Stage-0}} \end{cases}$$

14456 candidates after 2nd follow-up (T_{coh} 1260 hours)

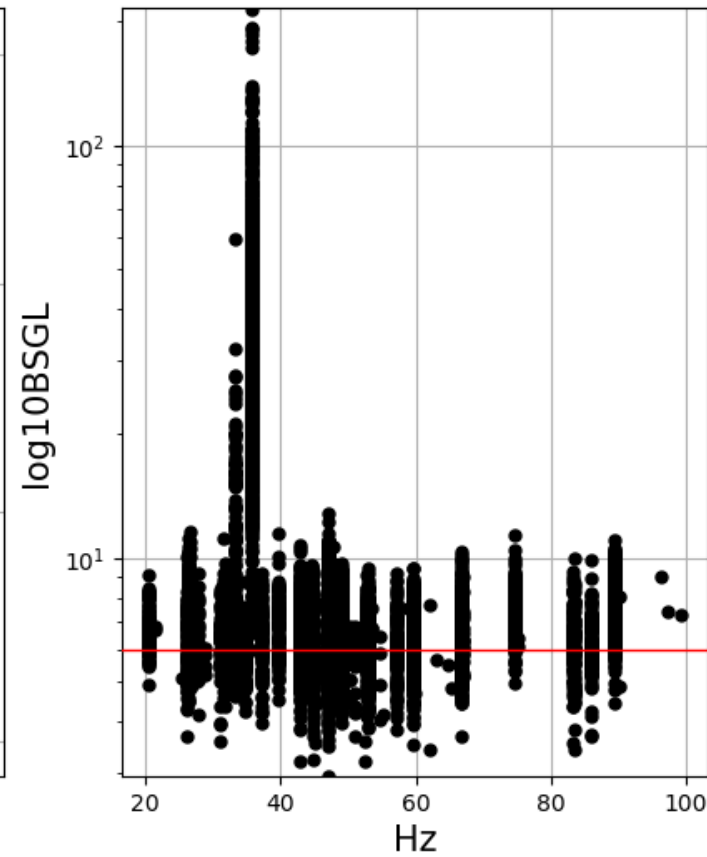
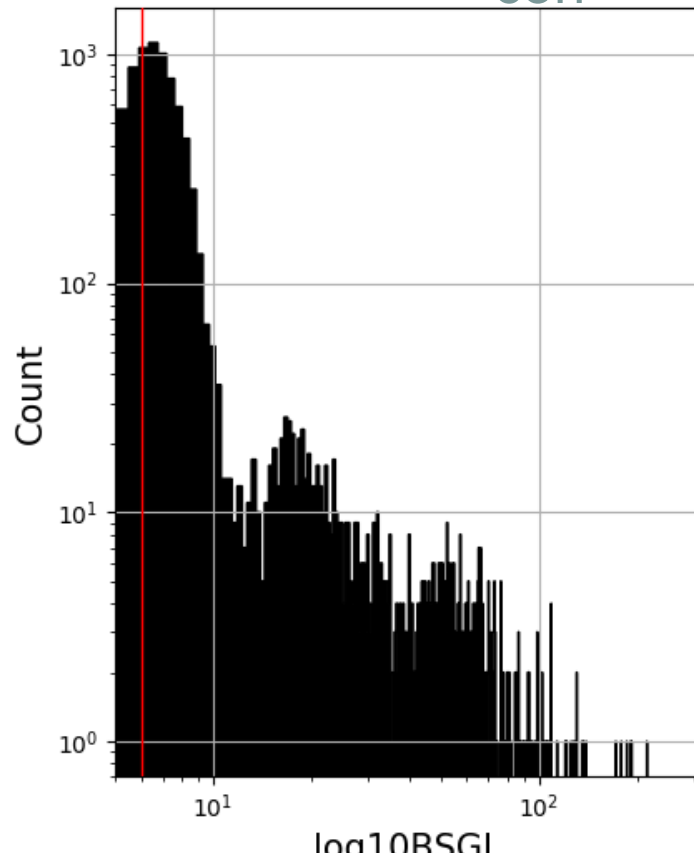


Threshold at 6.0 \rightarrow 8486 survive

Uncertainty volume around each candidate:

$$\begin{cases} \Delta f & = \pm 8.65 \times 10^{-6} \text{ Hz} \\ \Delta \dot{f} & = \pm 7.8 \times 10^{-12} \text{ Hz/s} \\ \Delta_{\text{sky}} & \simeq 0.81 \Delta_{\text{sky}}^{\text{Stage-1}}. \end{cases}$$

14456 candidates after 3rd follow-up (T_{coh} 2512 hours)



Threshold at 6.0 \rightarrow 6349 survive

Uncertainty volume around each candidate:

$$\begin{cases} \Delta f & = \pm 7.5 \times 10^{-6} \text{ Hz} \\ \Delta \dot{f} & = \pm 7 \times 10^{-12} \text{ Hz/s} \\ \Delta_{\text{sky}} & \simeq 0.99 \Delta_{\text{sky}}^{\text{Stage-2}} \end{cases}$$

Now what ?



We are left with > 6000 candidates that are not consistent with Gaussian noise fluctuations, and we have increased the time baseline of the coherent segments up to the entire observation time



“A new veto for continuous GW searches”

S. Zhu, M.A. Papa, S. Walsh, arxiv:1707:05268



- The survivors are not Gaussian fluctuations, these must be coherent disturbances
- Simple idea: long-lasting coherent disturbance is less likely to exhibit the astrophysical Doppler signature than a signal
- Let's filter for DM-off waveforms and compare with original results. If significance increases → candidate is disturbance

DM-off waveforms



- Switched off Doppler modulation
- Left amplitude modulation, so still maximizing over nuisance parameters
 - More flexibility in disturbance waveform
 - End up with same statistic as real search, which makes comparison simpler

Overlap between DM-off and astrophysical waveform families

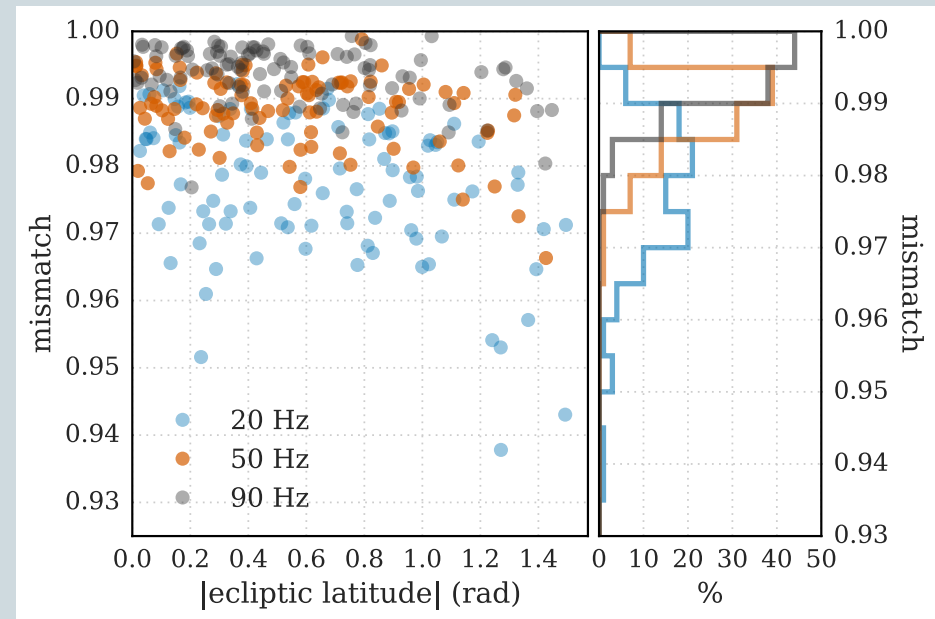
- Simulate astrophysical waveform
- Run an F-stat search (fully coherent) with DM-on (standard search) and with DM-off waveform. ρ^2 is the measured SNR*.
- Compute mismatch:

$$\mu = \frac{\rho_{\text{DM-on}}^2 - \rho_{\text{DM-off}}^2}{\rho_{\text{DM-on}}^2}$$

if overlap is small we expect

$$\rho_{\text{DM-off}}^2 \ll \rho_{\text{DM-on}}^2$$

* $E[2F] = \rho^2 + 4$



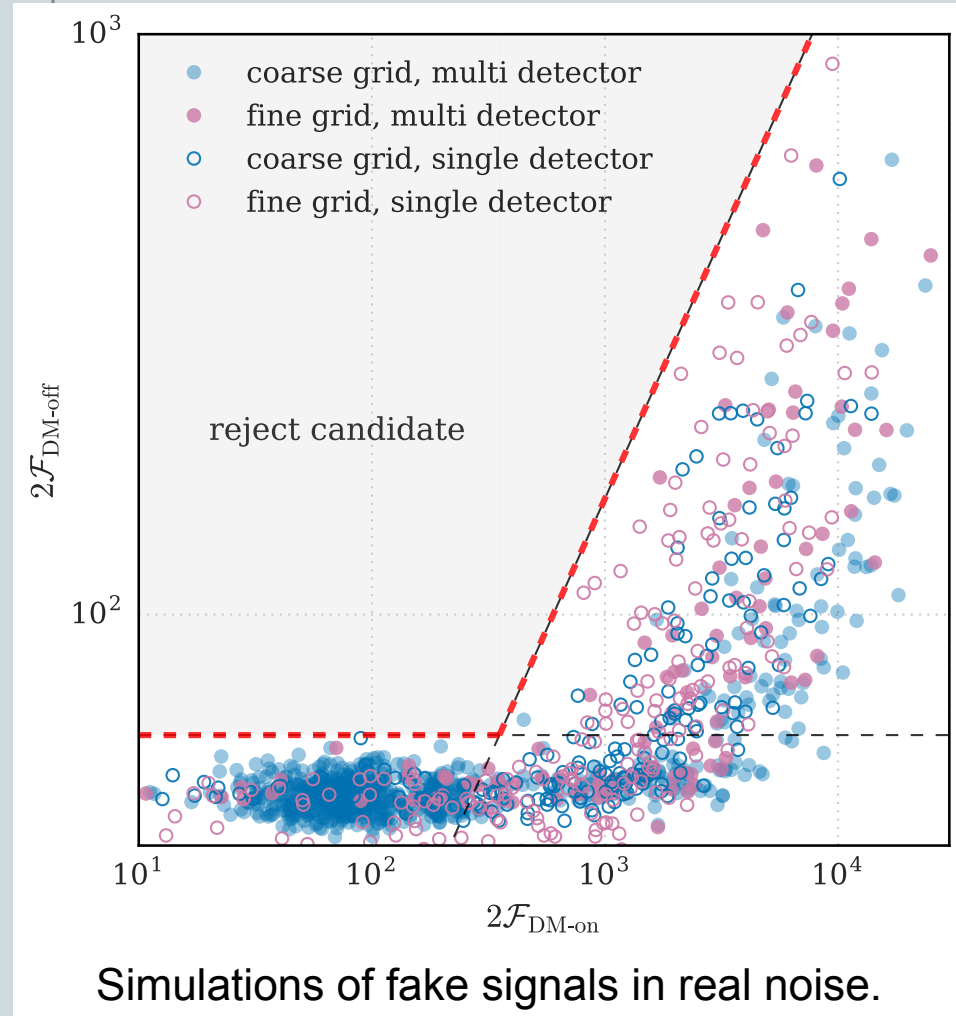
How do we construct this veto ?



- Now we know that the DM-off waveforms do not resemble very much the astrophysical ones → good
- This means that for a signal $2F_{\text{DM-off}} < 2F_{\text{DM-on}}$ and we hope that for disturbances the opposite is going to happen
 - Since $2F$ is the log likelihood, the ratio of two likelihoods would be the difference of $2F$

Tuning based on veto being safe on signals

- Cannot pick the noise-rejection because we do not know how to model the disturbances in the data
- We pick the thresholds (dashed red lines) to be safe and then see how much noise we reject

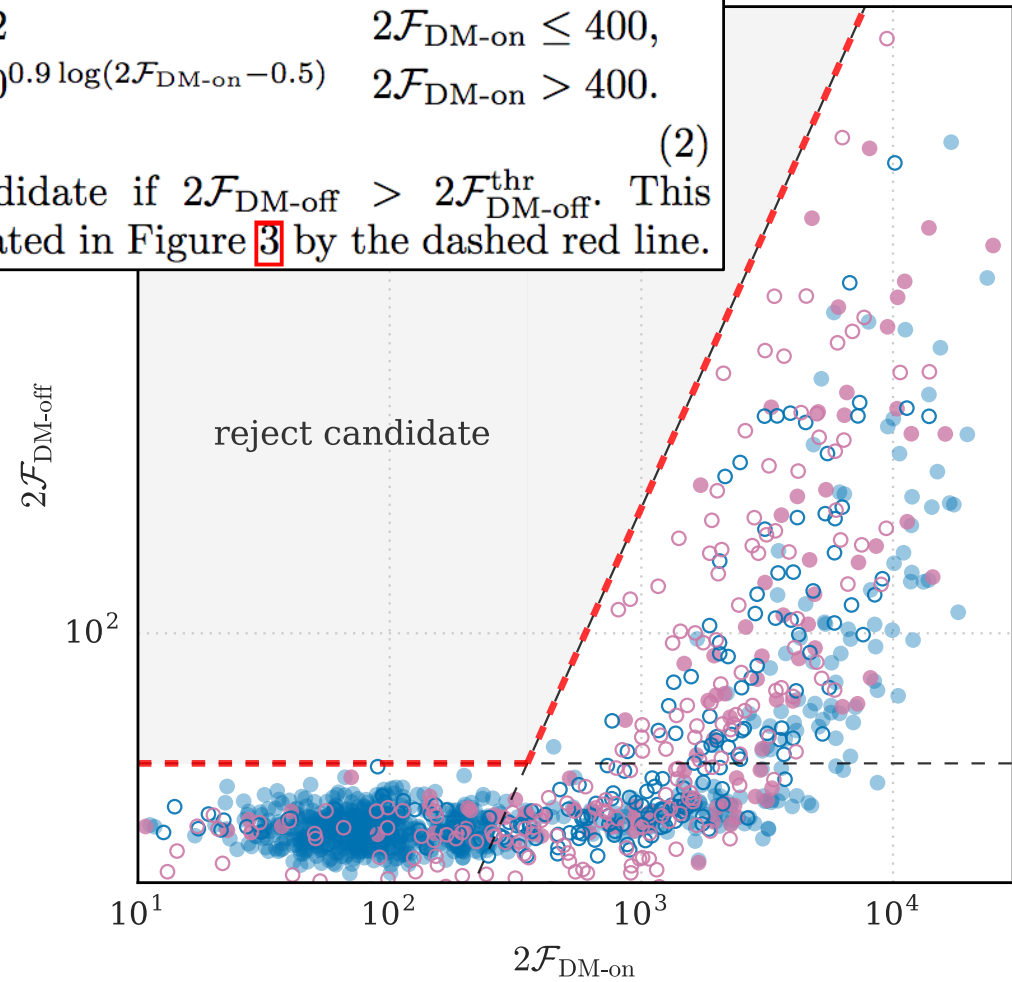


Thresholds



$$2\mathcal{F}_{\text{DM-off}}^{\text{thr}} = \begin{cases} 62 & 2\mathcal{F}_{\text{DM-on}} \leq 400, \\ 10^{0.9 \log(2\mathcal{F}_{\text{DM-on}} - 0.5)} & 2\mathcal{F}_{\text{DM-on}} > 400. \end{cases} \quad (2)$$

We reject a candidate if $2\mathcal{F}_{\text{DM-off}} > 2\mathcal{F}_{\text{DM-off}}^{\text{thr}}$. This threshold is indicated in Figure 3 by the dashed red line.

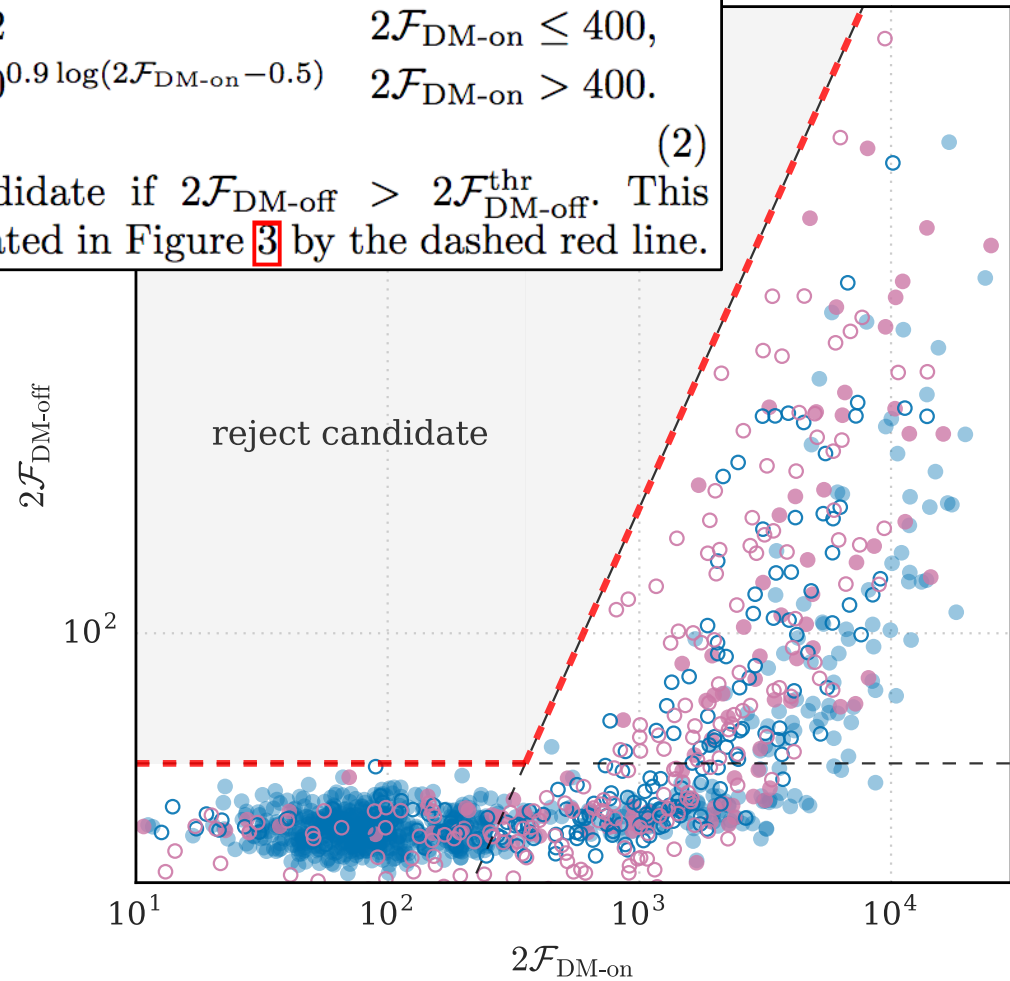


Thresholds



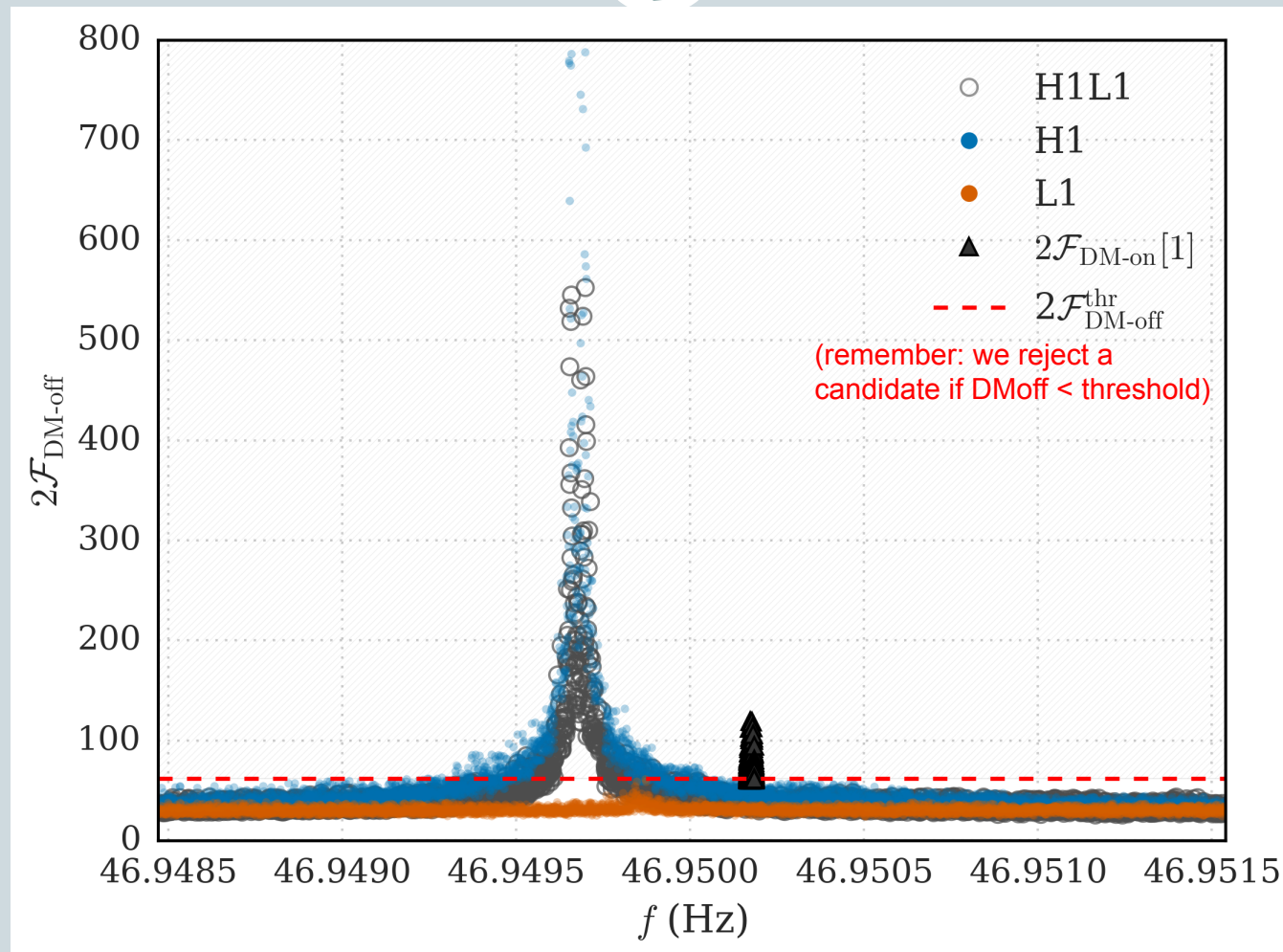
$$2\mathcal{F}_{\text{DM-off}}^{\text{thr}} = \begin{cases} 62 & 2\mathcal{F}_{\text{DM-on}} \leq 400, \\ 10^{0.9 \log(2\mathcal{F}_{\text{DM-on}} - 0.5)} & 2\mathcal{F}_{\text{DM-on}} > 400. \end{cases} \quad (2)$$

We reject a candidate if $2\mathcal{F}_{\text{DM-off}} > 2\mathcal{F}_{\text{DM-off}}^{\text{thr}}$. This threshold is indicated in Figure 3 by the dashed red line.



- with such large number of candidates the computational cost of the DM-off is not negligible
- hierarchical 3-step approach

Example of a stationary line in Hanford detector

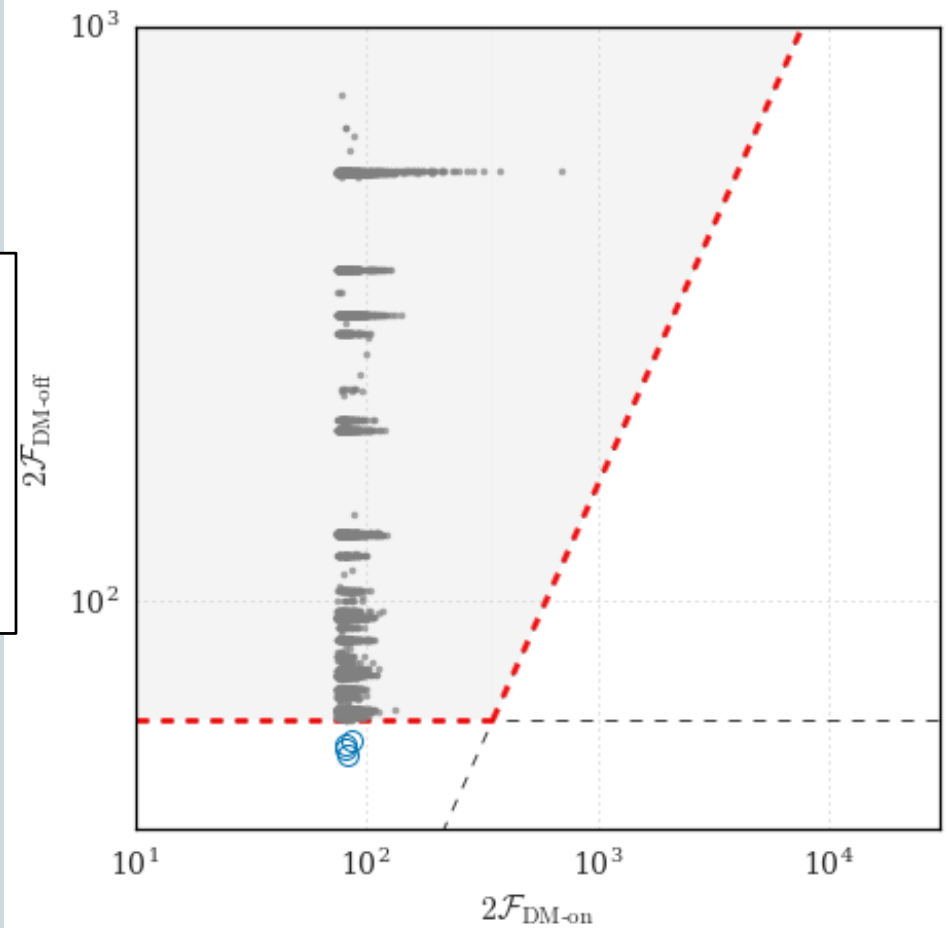


Results when applied to 6349 candidates



Stage	num surviving candidates	num remaining frequency bands
DM-on Stage 3	6349 (201)	57 (1)
DM-off Step 1	653 (8)	22 (1)
DM-off Step 2	101 (5)	10 (1)
DM-off Step 3	4 (1)	4 (1)

Very effective : only 4* candidates survive !



* +1 from a fake-signal inserted in the data for validation purposes.

The 4 surviving candidates



ID	f [Hz]	α [rad]	δ [rad]	\dot{f} [Hz/s]	$2\bar{\mathcal{F}}$	$2\bar{\mathcal{F}}_{H1}$	$2\bar{\mathcal{F}}_{L1}$	$2\mathcal{F}_{DM-off}$
1	58.970435900	1.87245	-0.51971	-1.081102×10^{-9}	81.4	48.5	33.4	55
2	62.081409292	4.98020	0.58542	-2.326246×10^{-9}	81.9	45.5	39.0	52
3	97.197674733	5.88374	-0.76773	2.28614×10^{-10}	86.5	55.0	31.8	58
4	99.220728369	2.842702	-0.469603	-2.498113×10^{-9}	80.2	41.4	45.8	55

- 4 months of O2 data (2nd science runs of Advanced LIGO) was available
- Used these for a fully-coherent search:

Parameter	Value
T_{coh}	2160 hrs
T_{ref}	1168447494.5 GPS sec
N_{seg}	1
δf	9.0×10^{-8} Hz
$\delta \dot{f}_c$	1.1×10^{-13} Hz/s
γ	1
m_{sky}	4×10^{-7}

O2 follow-up results

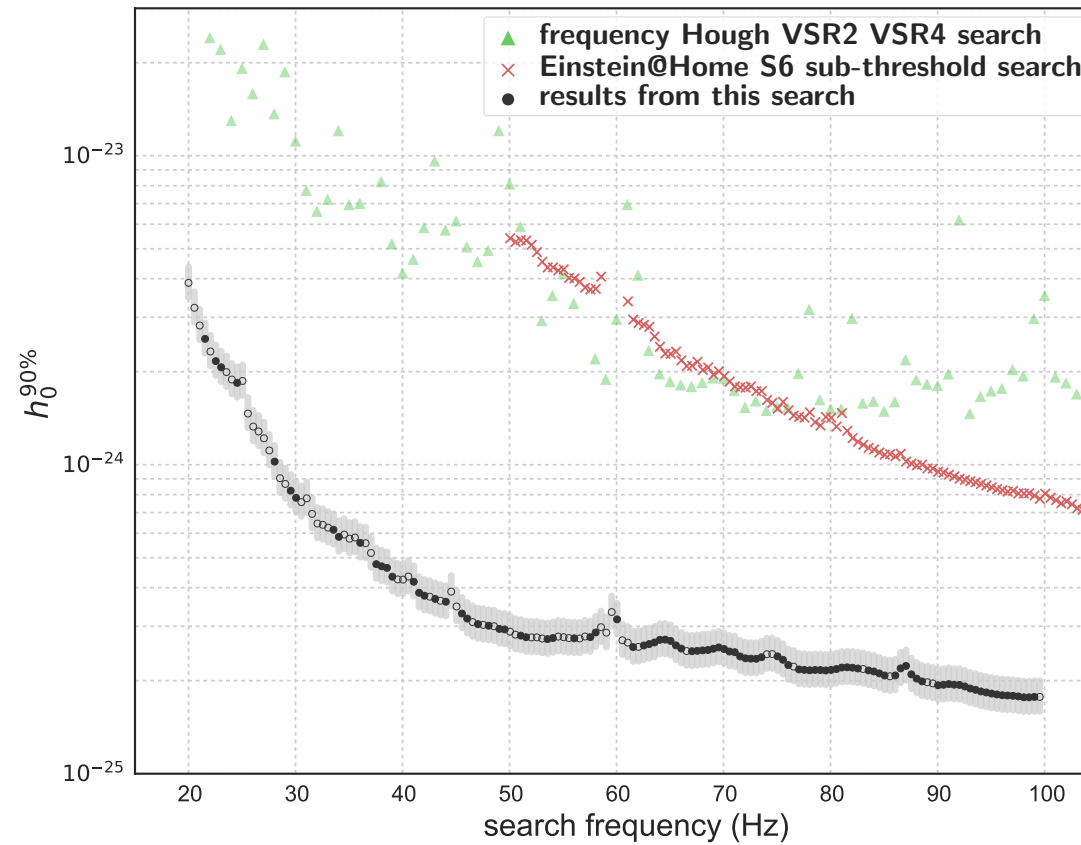


Candidate	Expected $2\bar{\mathcal{F}} \pm 1\sigma$	Loudest $2\bar{\mathcal{F}}$ recovered
1	85 ± 18	44
2	90 ± 19	52
3	84 ± 18	49
4	77 ± 17	47

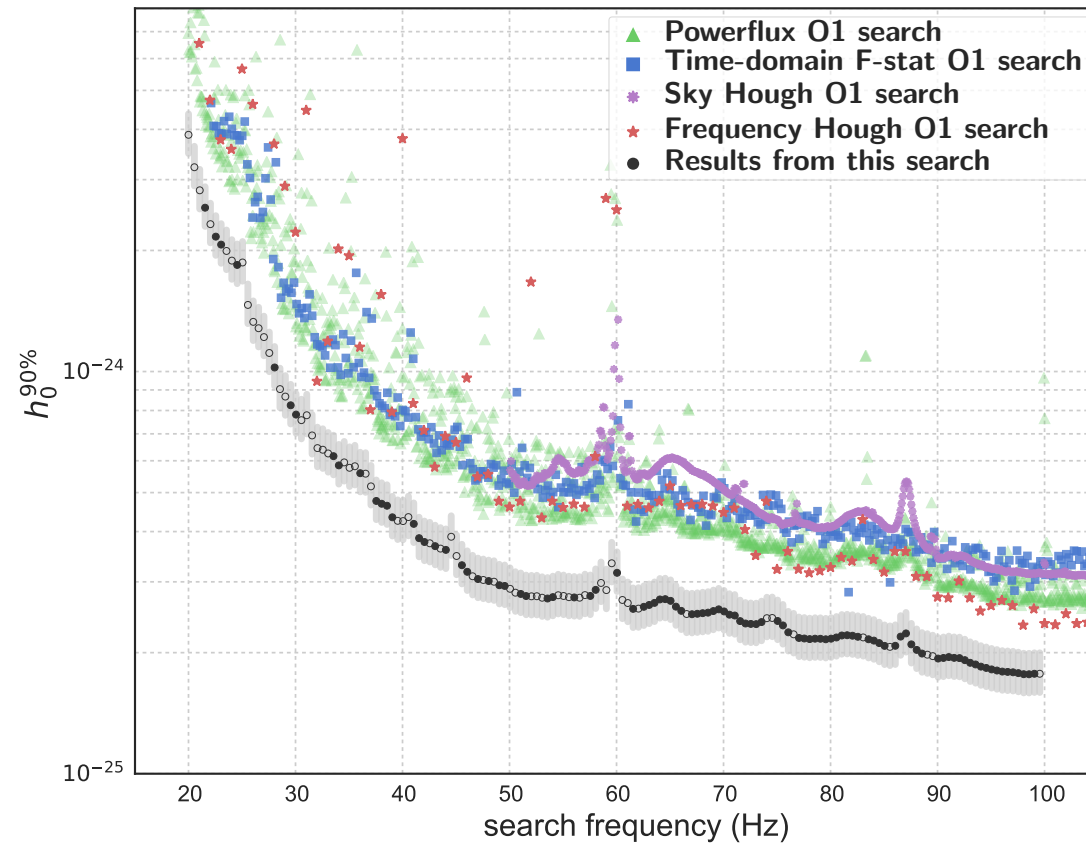
Assuming Gaussian noise the expected value is 52 ± 3

Conclusion: it is unlikely that any of these 4 candidates stems from a continuous GW signal.

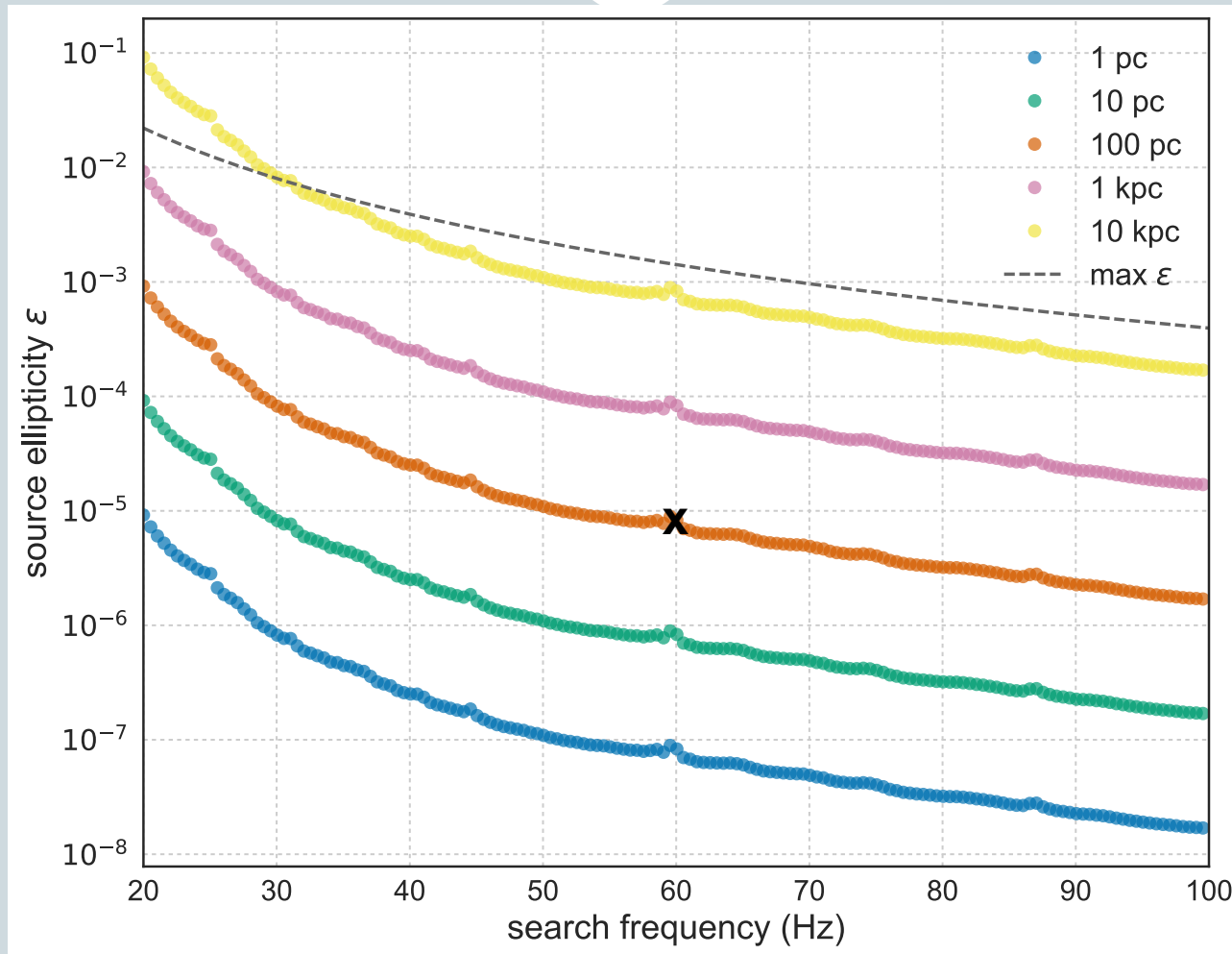
90% confidence upper limits



90% confidence upper limits



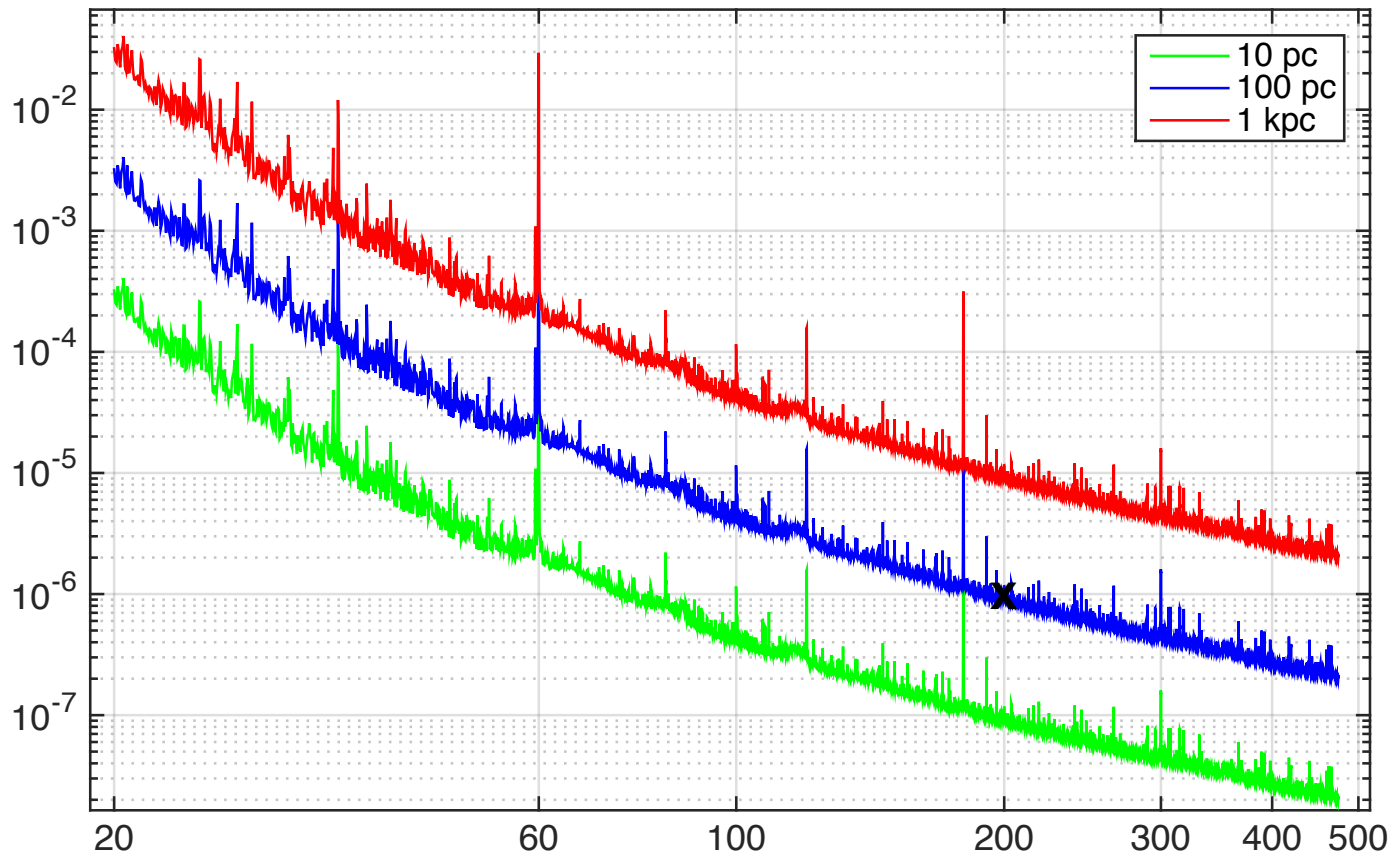
Astrophysical reach



We can exclude objects with ellipticities $\geq 10^{-5}$ within a distance of 100 pc of Earth at frequencies ≥ 60 Hz.

Astrophysical reach

(other search results, arxiv:1707:02667)



We can exclude objects with ellipticities $\geq 10^{-6}$ within a distance of 100 pc of Earth at frequencies ≥ 200 Hz.

Recap



Recap



- Basic concepts of signal detection
 - ✦ False alarm prob, p-values
 - ✦ Trials factors and large parameter space searches
 - ✦ Upper limits, spin-down limit
- Broad (blind) surveys
 - ✦ Einstein@Home
 - ✦ Illustration of recent results
 - ✦ Robust detection statistics
 - ✦ Multi-stage hierarchical approaches
 - ✦ A new veto
 - ✦ Astrophysical reach

Thank you !



Take some time to read LIGO's observational results papers

Sign-up at einsteinathome.org !



<https://einsteinathome.org/>