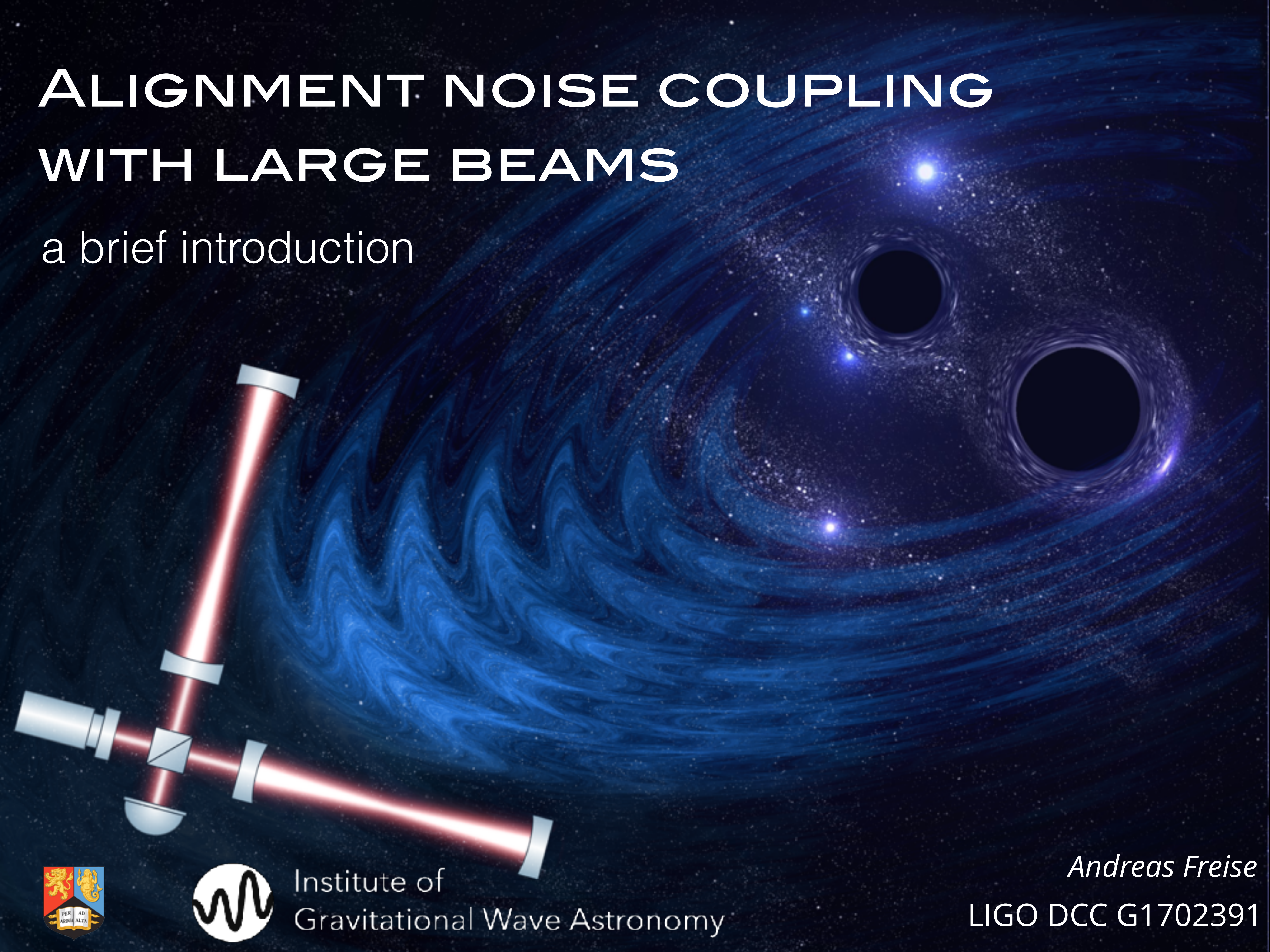


ALIGNMENT NOISE COUPLING WITH LARGE BEAMS

a brief introduction

Institute of Gravitational Wave Astronomy

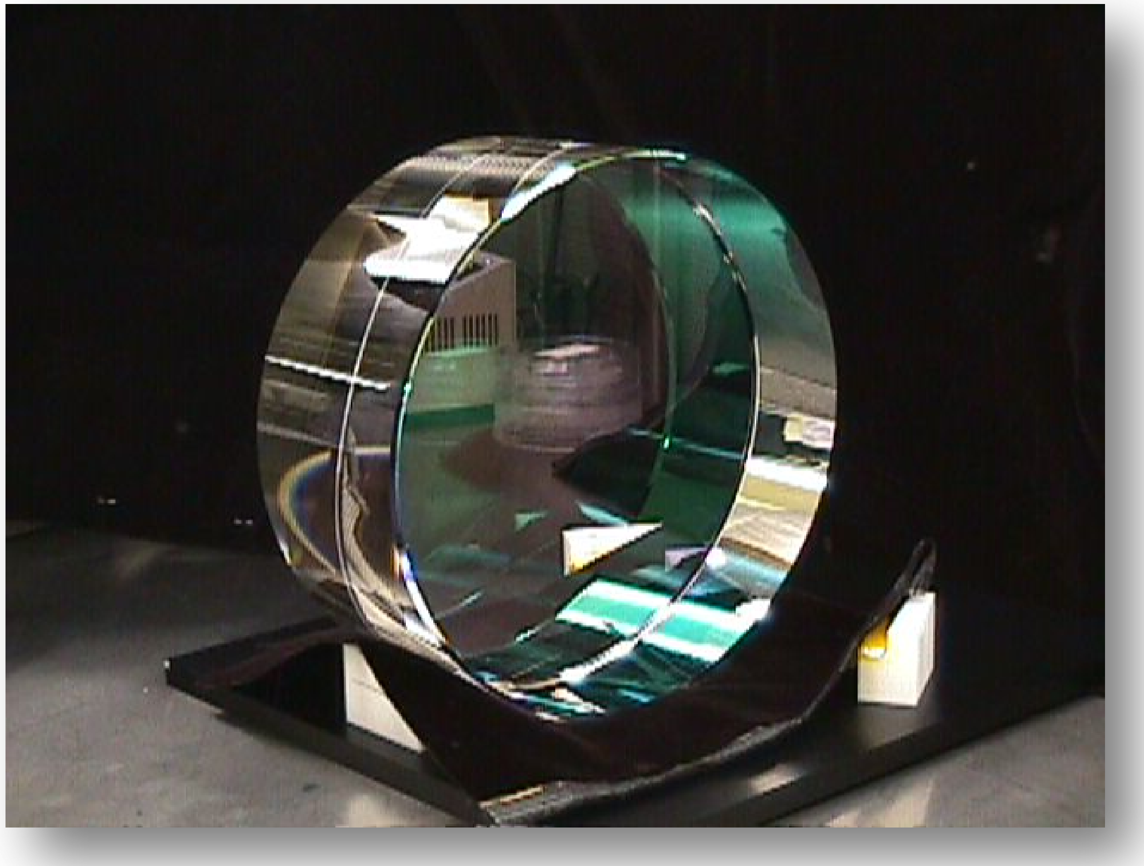
Andreas Freise

LIGO DCC G1702391

Aim

- It has been said that large beams will be a problem because the `alignment noise scales with the beam size to the power of six’.
- In this presentation I want to briefly review this statement and what it means.

Why large beam sizes?



Long arms make large beams

- Laser beams cannot be fully collimated. They are diverging due to diffraction
- For a given interferometer arm, there is a minimal beam width

$$w_{\mathrm{min}} = \sqrt{L\lambda/\pi}$$

- I.e. for L=10km, lambda = 1.5 um, $w_{\mathrm{min}} \approx 7$ cm

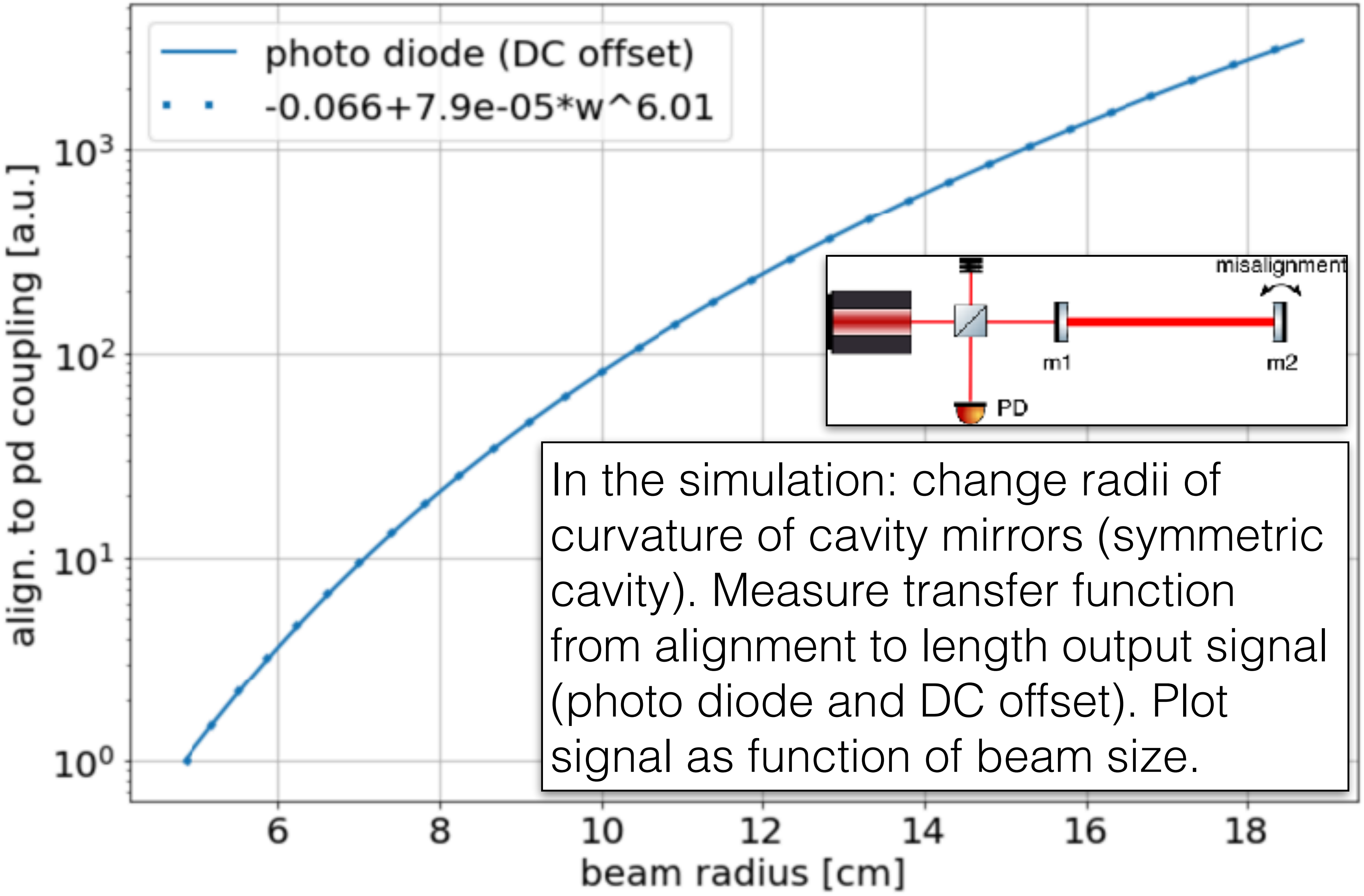
Larger beams reduce thermal noise

- Several proposals for future interferometers have suggested larger beams (or alternative beam shapes) to reduce thermal noise
- Coating thermal noise scales as ~1/*w*

⇨ Beams are likely to get larger in future interferometric detectors

Alignment coupling

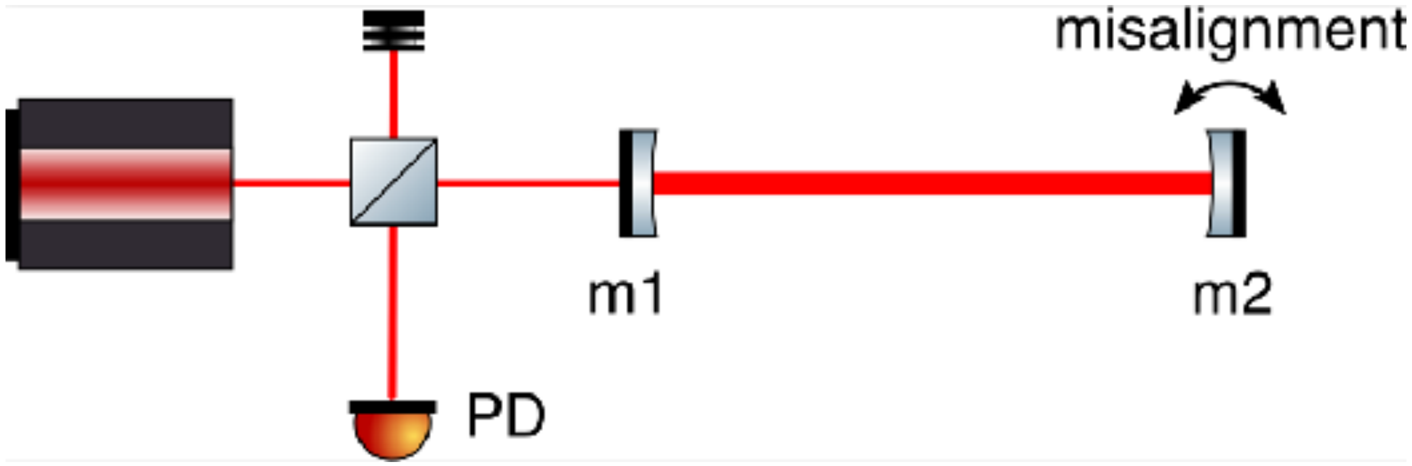
Finesse model, arm cavity



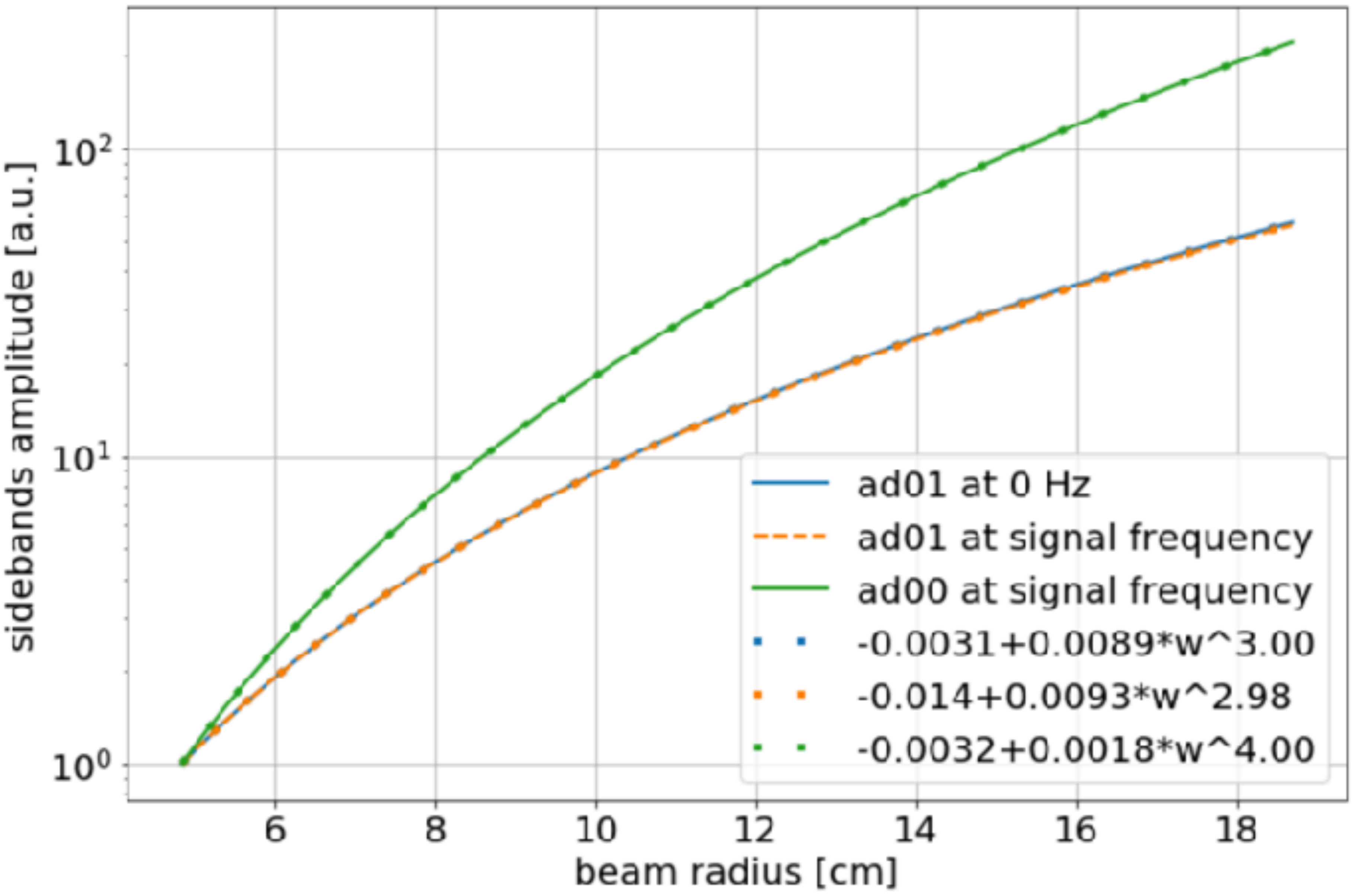
In the simulation: change radii of curvature of cavity mirrors (symmetric cavity). Measure transfer function from alignment to length output signal (photo diode and DC offset). Plot signal as function of beam size.

Alignment to GW channel

- Mis-alignment at ‘m2’ is a sum of a static DC misalignment and an alignment oscillation at the GW signal frequency f_{GW}.
- Generares modes $a_{01,0}=u_{01}(f=0\text{ Hz})$ and $a_{01,f}=u_{01}(f=f_{GW})$
- Photo diode signal detects signal ~ $a_{01,0}\ a^*_{01,f}$



Finesse model: sideband amplitudes

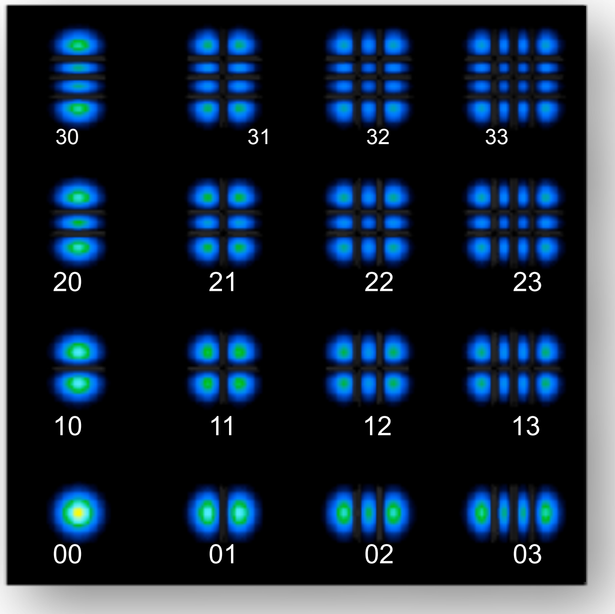


Why ~ w^3 ?

a) coupling coefficients

Coupling of higher-order modes

- Small mis-alignment causes coupling from fundamental mode (u_{00}) into first order mode (u_{01} or u_{10})



- Coupling coefficients *k* defined via:

$$u_{nm}(q_1)\exp\Big(\mathrm{i}\,(\omega t - kz)\Big) = \sum_{n',m'} k_{n,m,n',m'}\,u_{n'm'}(q_2)\exp\Big(\mathrm{i}\,(\omega t - kz')\Big),$$

Alignment coupling coefficient

- For u_{00} -> u_{01} (small angle γ):

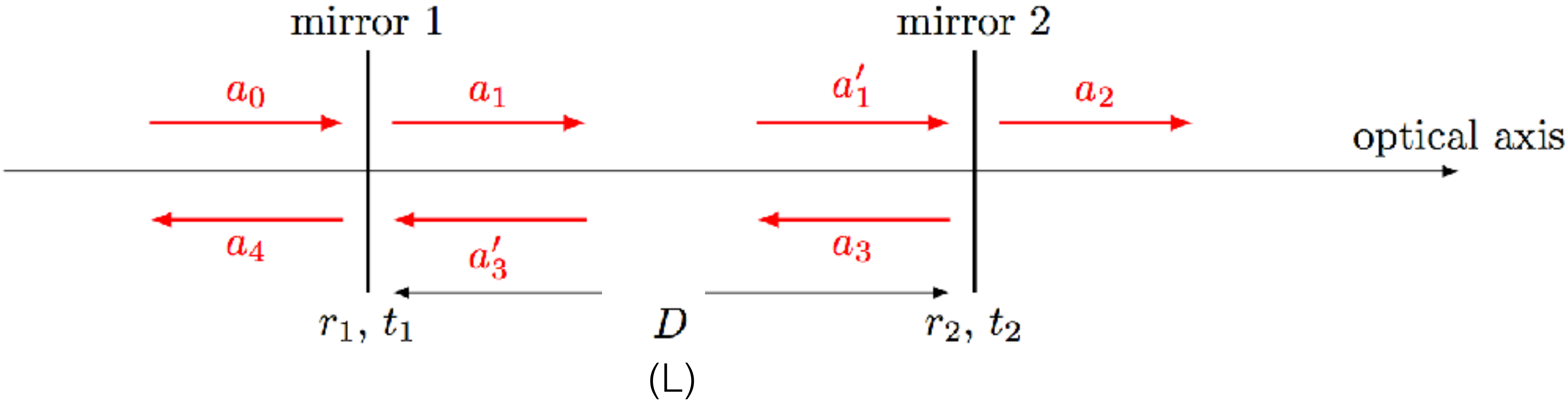
$$|k| = \left| \frac{(z - \mathrm{i}\, z_R) \sin\gamma}{w_0} \right|$$

$$\begin{aligned} |k| &= \frac{z_r}{w_0^2} w |\sin\gamma| \\ &= \frac{\pi}{\lambda} w |\sin\gamma| \\ &\sim w \end{aligned}$$

Why ~ w^3 ?

b) resonant enhancement/ suppression

Compute cavity fields



Set of linear equations, for example:

$$\frac{a_2}{a_0} = \frac{-t_1 t_2 \exp(-\mathrm{i}\,kL)}{1 - r_1 r_2 \exp(-\mathrm{i}\,2kL)}$$

Resonance factor

$$d = \frac{1}{1 - r_1 r_2 \exp\left(-\mathrm{i}\,2kD + (1+n+m)\,\Psi_{rt}\right)}$$

(D=L)

$$|d| = \sqrt{\frac{1}{1 - R_1 R_2 - 2 r_1 r_2 \cos\left(-2kD + (1+n+m)\,\Psi_{rt}\right)}}$$

$$|d|_{\mathrm{HG10}} = \sqrt{\frac{1}{1 - R_1 R_2 - 2 r_1 r_2 \cos\left(\Psi_{rt}\right)}}$$

(now assuming high finesse)

$$|d|_{\mathrm{HG10}} \approx \sqrt{\frac{\pi}{2L\lambda} w^4} \sim w^2$$

(for given L and λ)

Summary

- Coupling into u_{01} mode increased linearly with beam size
- Larger beams cause the higher-order modes to be suppressed less, field amplitude rises with beam size squared

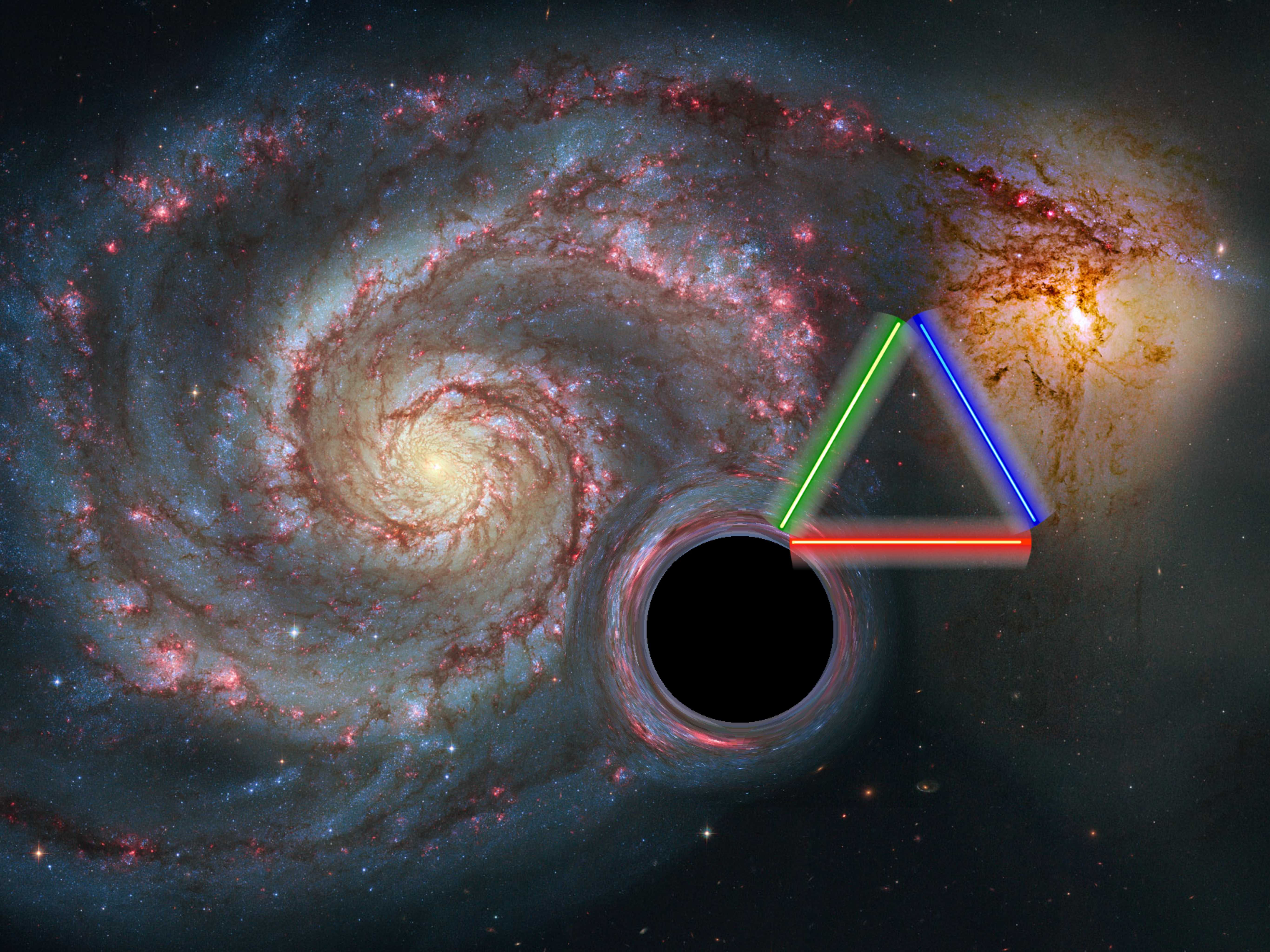
- The above is true for high finesse, medium frequency range, and fixed L and λ. In general the behaviour is complex but this example is useful as an order of magnitude estimate.

Conclusion

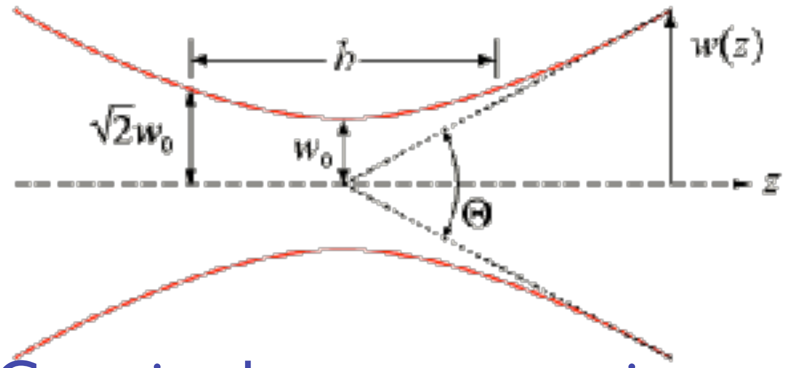
- Large beams are likely/useful/necessary in future detectors
- Large beam increase the coupling of alignment noise into the gravitational wave channel, in this example as ~w^6
- Combination of:
  - stronger coupling into a u_{01} mode (~w per field), means also we get stronger alignment signals
  - and lower suppression of the u_{01} mode in the cavity (~w^2 per field), could be fixed with higher finesse
- Work in progress!

References

- A.E. Siegman: `Lasers', University Science Books (1986)
- Bond, Brown, Freise, Strain: `Interferometer techniques for gravitational-wave detection', Living Reviews in Relativity,19, 3 (2017) arXiv:0909.3661
- Bayer-Helms: `Coupling coefficients of an incident wave and the modes of spherical optical resonator in the case of mismatching and misalignment', Appl. Opt., 23, 1369-1380 (1984)



Eigenmodes



Gaussian beam propagation

Cavity Eigenmodes

$$E(t,x,y,z) = \sum_j \sum_{n,m} \boxed{a_{jnm}}\,\boxed{u_{nm}(x,y,z)}\,\boxed{\exp\left(\mathrm{i}\left(\omega_j t - k_j z\right)\right)}$$

complex amplitude factor

spatial properties

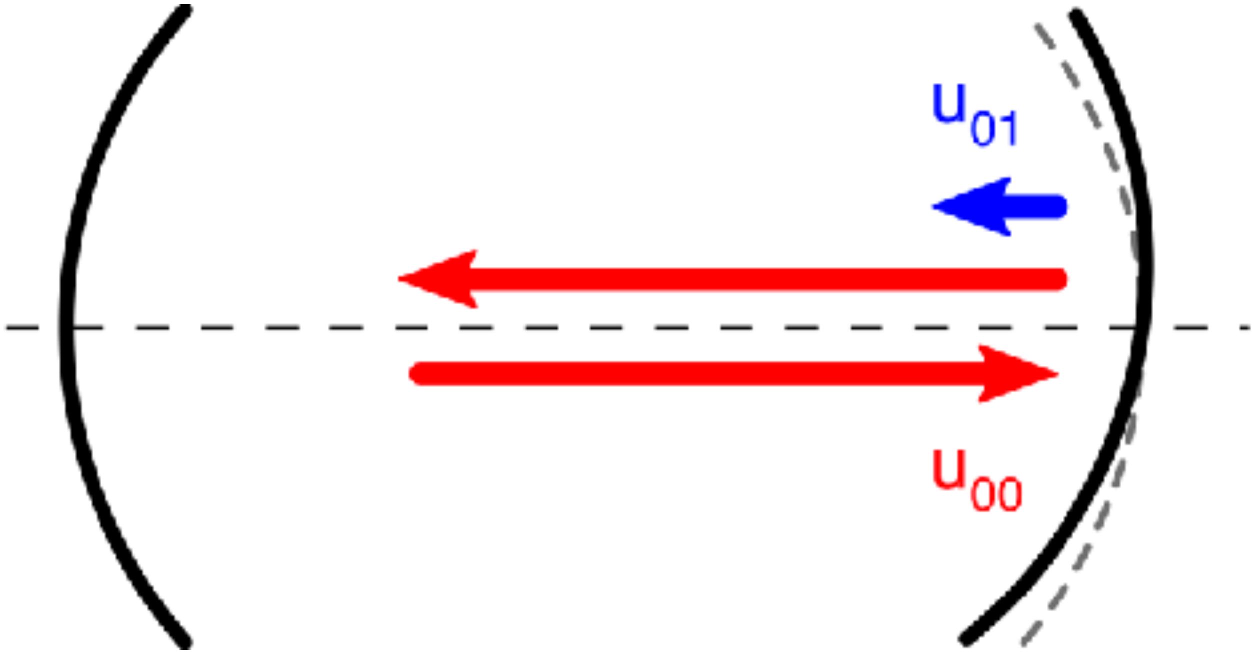
propagation

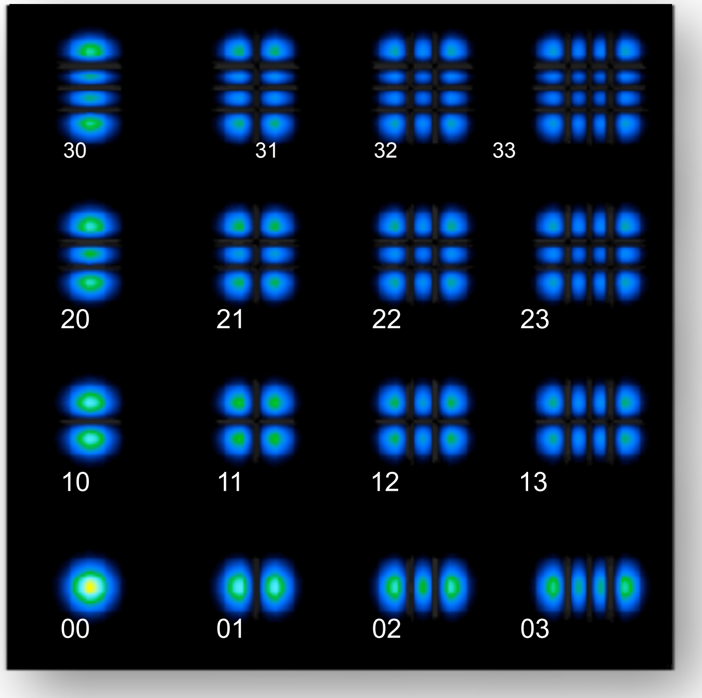
Hermite-Gauss modes

$$\begin{aligned} u_{\mathrm{nm}}(x,y,z) &= \left(2^{n+m-1} n! m! \pi\right)^{-1/2} \frac{1}{w(z)} \exp\left(\mathrm{i}\,(n+m+1)\Psi(z)\right) \\ &\times \quad H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-\mathrm{i}\,\frac{k(x^2+y^2)}{2R_C(z)} - \frac{x^2+y^2}{w^2(z)}\right) \end{aligned}$$

Alignment in the mode picture

Small mis-alignment causes coupling from fundamental mode (u_{00}) into first order mode (u_{01} or u_{10})





Minimal mirror size